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Taras Svyshch[1], Oleksii Lanets[2], Oleksii Vambol[3], Iryna Derevenko[4]

[1]Department of Design Machine and Automotive Engineering, Lviv Polytechnic National University, Ukraine, Lviv, S. Bandery street 12, E-mail: Taras.V.Svyshch@lpnu.ua, ORCID 0009-0002-7783-871X
[2]Department of Aviation and Manufacturing Engineering, Lviv Polytechnic National University, Ukraine, Lviv, S. Bandery street 12, E-mail: Oleksii.S.Lanets@lpnu.ua, ORCID 0009-0005-9631-717X
[3]Department of Aviation and Manufacturing Engineering, Lviv Polytechnic National University, Ukraine, Lviv, S. Bandery street 12, E-mail: Oleksii.O.Vambol@lpnu.ua, ORCID 0000-0002-1719-8063
[4]Department of Strength of Materials and Structural Mechanics, Lviv Polytechnic National University, Ukraine, Lviv, S. Bandery street 12, Email: Iryna.A.Derevenko@lpnu.ua, ORCID 0000-0003-0132-8035

SUBSTANTIATION OF THE ADVANTAGES OF USING PLANE ELASTIC ELEMENTS IN THE RESONANT DESIGN OF A VIBRATING TABLE OF THE ECCENTRIC-PENDULUM TYPE

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Abstract. This article presents an analytical and engineering justification for the use of flat elastic elements in eccentric-pendulum-type resonant vibration tables. The study addresses the design, calculation, and structural optimization of resonant elastic nodes, which ensure directional oscillations, define dynamic performance, and determine the durability and reliability of the machine. Stiffness and strength calculation procedures are derived for elastic components of rectangular and circular cross-sections, considering material properties, geometric parameters, and stress constraints under cyclic loading. Analytical results indicate that circular elements experience higher bending stresses than rectangular ones for equivalent stiffness, requiring increased length to achieve comparable durability. Finite-element simulations validate these findings, confirming the superior structural efficiency, lower operating stresses, and enhanced geometric adaptability of flat elements. The research provides a rigorous basis for selecting flat elastic components in the design of high-performance resonant vibration systems.

Keywords: stiffness parameters, force parameters, elastic element, stress, geometric parameters of an elastic element, bending moment.

Introduction

Any vibration machine can be divided into three main components: vibrating masses, elastic units (both resonant elements and vibration isolators), and the drive. During engineering design, masses are assigned arbitrary geometric parameters and shapes based on the developer's subjective opinion and technical requirements. It is often possible to proceed without a careful calculation of their natural vibration modes, strength characteristics, and so on. The drive is often selected from vibration exciters unified by the manufacturer (mainly based on the frequency of forced oscillations and the excitation force), and it practically does not require additional recalculation. Resonant elastic units, however, require not only a verified calculation but also high-quality design solutions, careful manufacturing and fastening, and proper operation. Therefore, they constitute some of the most critical components of resonant vibration machines.

Elastic nodes comprise elastic links of vibration machines that connect oscillating masses elastically, maintaining them in a given position and providing flexibility between adjacent masses. Elastic nodes are designed for specific inertial parameters of masses. Only by properly coordinating the stiffness of elastic nodes with the inertial parameters of the interacting masses can an oscillating system with the necessary properties be formed.

During the rapid development of vibration technology, dozens of varieties of elastic units have been tested, whose components include elements of various types and designs, such as elastic elements with rectangular and circular cross-sections (rods and torsion bars), coiled springs, rubber gaskets, and pneumatic cylinders. There are vibration machines that utilize combined elastic units, which integrate steel and rubber elements, as well as other hybrid designs. A characteristic feature of elastic units is that their stiffness is significantly lower compared to the stiffness of the masses. A disturbed mechanical oscillatory system deforms primarily in these most compliant zones and can be considered as a system of absolutely rigid bodies connected by elastic bonds.

Problem Statement

The operational reliability and durability of vibration machines largely depend on the quality of the design and manufacturing of resonant elastic nodes, their positioning, and the mechanical properties attributed to them. The rigidity of such nodes is strictly defined. They serve to provide oscillatory motion to masses in near-resonant modes, playing the role of converters of the kinetic energy of mass motion into the potential energy of elastic deformation. The potential energy accumulated in them (when the spring is in a deformed state) is converted into kinetic energy of mass motion during the restoration of the elastic elements. The largest deformations occur in these nodes. No other component of a vibration machine can be compared to them in terms of the loads perceived per unit of stiffness. Therefore, when designing resonant vibration machines, the greatest attention is paid to calculating these critical nodes.

Until recently, elastic steels were mainly used as materials for elastic elements. Currently, fiberglass is becoming increasingly widespread. Fiberglass has proven itself well: although it is inferior to steel in terms of mechanical properties, its main advantage is that it does not require such expensive technological operations as grinding, hardening, or annealing to manufacture elastic elements. Additionally, this material tolerates alternating loads well, and its multilayer structure effectively prevents crack propagation. In the future, mechanics are expected to transition to composite materials whose mechanical properties will be equal to those of high-quality steels. This trend is already observed in leading industrial countries, especially in the production of high-tech products.

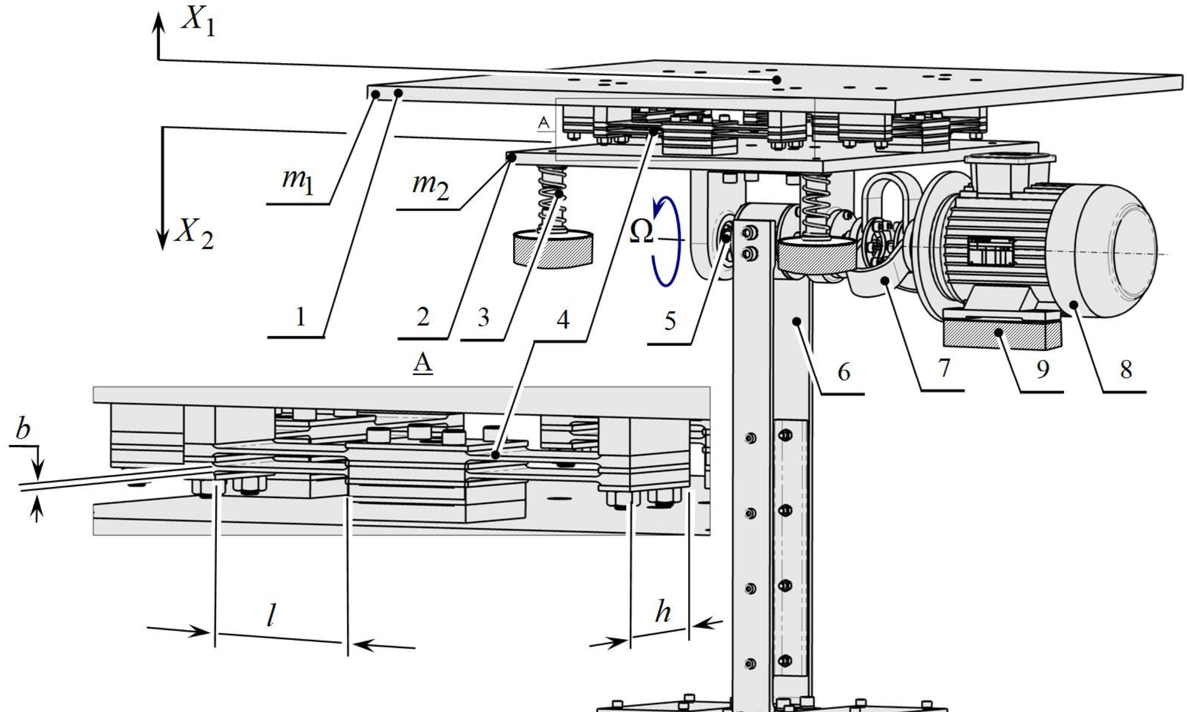

Fig. 1. Spatial model of an eccentric-pendulum-type vibrating table

Today, there is a need to create a vibrating table, the schematic diagram of which is shown in Fig. 1. The design of this vibrating table represents an improvement over the development presented in [1]. A distinctive feature of this design is the use of resonant elastic elements. The principle of operation is as follows. The working body of the vibrating table 1 is connected through resonant elastic elements 4 to the reactive body 2, which is mounted on the foundation 9 through vibration isolators 3. A drive shaft 5 is installed in the housing of the reactive body 2, on which a pendulum 6 is mounted with eccentricity ε. Shaft 5 is driven by an asynchronous electric motor 8 through a flexible coupling 7. The pendulum 6 and the reactive body 2 are periodically displaced relative to each other by an amount ε due to the rotation of the shaft 5. The working (active) body 1 is kinematically excited by the movement of the reactive body 2.

To implement a resonant structure, it is necessary to use elastic elements. The authors of this article will deal with the issues of analysis and determination of the optimal cross-section of resonant elastic elements.

Review of Modern Information Sources on the Subject of the Paper

The calculation and engineering of elastic elements – springs, rods, leaf springs, and continuous members – represent a central methodological axis in the design of high-performance vibratory machines. These works collectively advance the theoretical foundation, modeling strategies, and optimization of elastic components operating under resonant, inter-resonant, and vibro-impact regimes.

Lanets monograph "High-Efficiency Inter-Resonance Vibration Machines with Electromagnetic Drive" [2] establishes the fundamental analytical framework for determining stiffness characteristics of spring systems employed in dual- and inter-resonance configurations. The monograph emphasizes the decomposition of elastic structures into discrete and continuous components, applying linearized and nonlinear stiffness models depending on the operating regime. In this work, the derivation of equivalent stiffness for complex elastic assemblies enables closed-form expressions for natural frequencies and dynamic compliance – parameters essential for high-efficiency operation near resonance. His later textbook, "Fundamentals of Analysis and Design of Vibratory Machines" [3], formalizes these methods into a design workflow, emphasizing the importance of accurate elastic-element modeling when creating machines with narrow operational bandwidths. The textbook highlights analytical rod and beam theory, Rayleigh–Ritz approximations, and finite-element-based stiffness estimation as interchangeable tools, depending on geometric complexity.

The modeling perspective is significantly expanded in Kachur et al. [4], who investigate forced oscillations of continuous elastic members using PDE-based formulations. Their work utilizes distributed-parameter elastic models to capture the contributions of bending and axial stiffness under dynamic excitation. Modal decomposition techniques enable the reduction of high-order distributed systems to manageable computational models while preserving the fidelity of elastic behavior. The integration of elastic-member modeling with non-stationary excitation makes this study a relevant extension of Lanets earlier discrete-focused formulations.

Nazarenko and colleagues address elastic behavior under nonlinear and impact-type loading. In their monograph on material compaction systems [5], elastic elements are described within hybrid discrete–continuous assemblies, where stiffness nonlinearity arises from contact losses, prestressing, and geometric constraints. Their later article [6] highlights how higher harmonics of technological loads influence dynamic stress within elastic subsystems, necessitating refined stiffness modeling that is sensitive to amplitude-dependent effects. This contributes a crucial nonlinear correction to classical linear elastic analyses.

The contributions by V.Gursky and I.Kuzio [7-8] focus specifically on flat springs and rod systems, which are fundamental elastic components in vibratory machines. In [7], the authors conduct a comprehensive analysis of the strength and durability of flat springs subjected to vibro-impact loading. Their approach integrates dynamic stiffness, stress concentration modeling, and fatigue assessment, offering a detailed methodology for calculating elastic components operating under high-stress gradients.

The study [8] investigates rods with intermediate supports under dynamic action, using elastic-beam theory to determine stiffness distribution, modal characteristics, and transient deformation – key aspects when rods serve as continuous elastic guides or supports.

The elastic behavior of two-dimensional continuous members is further explored by Maistruk et al. [9], who derive the natural frequencies of plate-like elements using combined analytical and numerical procedures. Later optimization-oriented studies by Maistruk [12] and Maistruk, Lanets, and Maystruk [13] refine the determination of stiffness distributions in continuous sections of vibrating tables. Their research introduces shape-optimization frameworks that link geometric parameters to effective stiffness, stress fields, and frequency-response characteristics. These works leverage finite-element analysis (aligned with methodologies described in Adams and Abraham [10]) and incorporate damping-contact corrections inspired by continuous contact models such as those discussed by Poursina and Nikravesh [11].

Collectively, these publications form a cohesive, evolving methodology: beginning with foundational analytical stiffness modeling, progressing through distributed-parameter and nonlinear formulations, and culminating in modern optimization-driven elastic-element design.

Objectives and Problems of Research

It has been established that resonant elastic nodes are primarily implemented in the form of elastic elements with rectangular or circular cross-sections, as their ends are easily rigidly fastened and it is technologically simple to shape them to achieve a given stiffness. The main part of the article is devoted to establishing the advantages and disadvantages of such elastic elements.

Main Material Presentation

The deformation of the working area of the elastic elements is carried out by translational displacement (without rotation) of one end of the elastic element relative to the other (the places of attachment to the oscillating masses). The ends of the elastic elements in the oscillating masses are rigidly clamped (Fig. 2). Let us consider a flat model of the load of the working area of an elastic element with a length of l, which is shown in Fig. 3. The forces that arise in the attachment points when the ends of the element are displaced relative to each other without scrolling are [2, 3]:

- for the force $$R = F_{ben} = \frac{12 E J \delta}{l^3}; \quad (1)$$
- for the bending moment $$M = M_{ben} = \frac{6 E J \delta}{l^2}. \quad (2)$$

where E is the modulus of elasticity of the element material during tension; J is the moment of inertia of the cross-section of the elastic element; l is the length of the working section of the elastic element that is deformed.

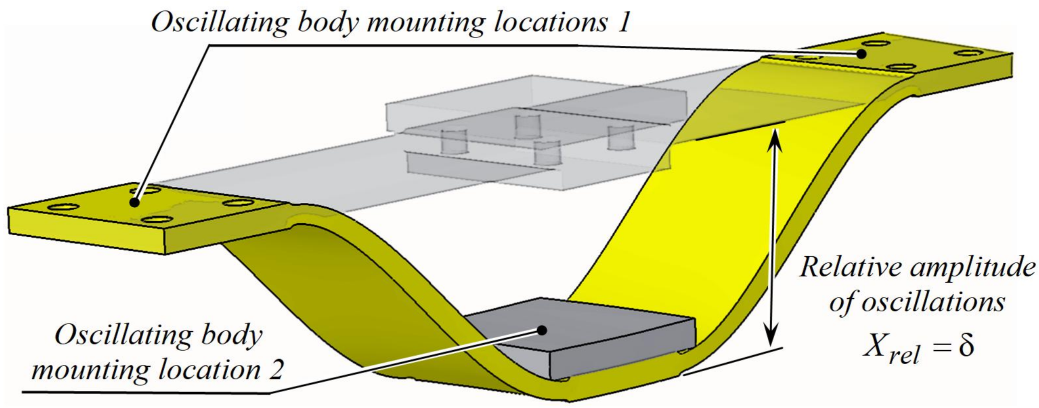

Fig. 2. Scaled diagram of the deformation of the resonant flat elastic element of the vibrating table, as shown in Fig. 1.

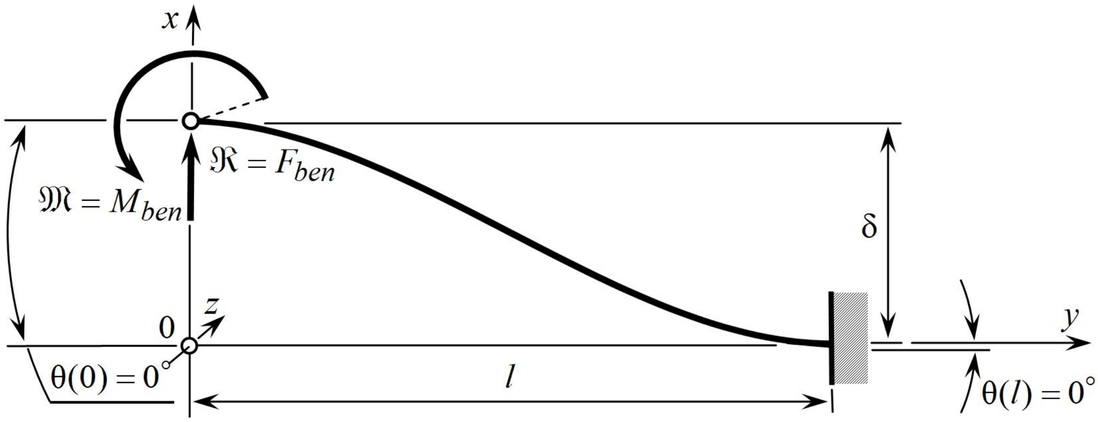

Fig. 3. Flat model of a loaded elastic element

For the case of deformation of an elastic element in Fig. 3, we are interested in expression (1), in which the linear displacement δ is related to the force F_{ben}, which, in fact, represents the forces that arise in the oscillatory system during the movement of masses. For this case, the stiffness coefficient c of one elastic element, when its ends are displaced relative to each other without rotation, using expression (1), is defined as follows [14]:

$$c = \frac{F_{ben}}{\delta} = \frac{12 E J}{l^3}. \quad (3)$$

If we use elastic elements of rectangular and circular cross-sections that bend along the axis y relative to the axis z, their moments of inertia of the cross-section will be defined as:

$$J = J_f = \frac{h b^3}{12}; \quad (4)$$

$$J = J_r = \frac{\pi d^4}{64}, \quad (5)$$

where h and b are the width and thickness of the elastic element of rectangular cross-section; d is the diameter of the elastic element of circular cross-section. Taking into account (4) and (5), expression (3) for rods of rectangular and circular cross-sections, respectively, can be written as:

$$c = \frac{h b^3 E}{l^3}; \quad (6)$$

$$c = \frac{3 \pi d^4 E}{16 l^3}. \quad (7)$$

To constructively form an elastic element with a rectangular cross-section with an already known value of stiffness c, it is most convenient l to find the thickness of the working section by specifying its length b and width h. Considering the elastic node immediately, instead of the parameter c we introduce c_{12} – the value of the stiffness of the elastic node, which is determined through the inertial-stiffness parameters of the system. So, using (6), we obtain

$$b = l \sqrt[3]{\frac{c_{12}}{E h n k_p}} \quad \text{or} \quad b = l \sqrt[3]{\frac{m_{agg} \Omega^2}{E h n k_p z^2}}, \quad (8)$$

where in expression (8) we additionally introduce

$$k_p \approx 0.7...0.9 \quad (9)$$

is the clamping coefficient of the elastic element, which takes into account the loss of stiffness during fastening. It is introduced into practice during calculations and is caused by the presence of micro-gaps, the flexibility of the clamping itself, the quality of bolt tightening; n is the number of working sections of the elastic unit. In Fig. 1 the elastic unit consists of eight elastic elements. There are two working sections in each (Fig. 2). Therefore, $n = 16$. Similarly, for a circular section element, constructively specifying only its length l, the diameter d of the working section, using (7)

$$d = 2\sqrt[4]{\frac{c_{12}\, l^3}{3\pi E n k_p}} \quad \text{or} \quad d = 2\sqrt[4]{\frac{m_{agg}\,\Omega^2 l^3}{3\pi E n k_p z^2}}. \tag{10}$$

In practice, for a rectangular cross-section, it is convenient to specify the ratio of its sides

$$\wp = \frac{h}{b}, \tag{11}$$

and, therefore, expression (8) taking into account (11) can be written in the form

$$b = \sqrt[4]{\frac{c_{12}\, l^3}{\wp E n k_p}} \quad \text{or} \quad b = \sqrt[4]{\frac{m_{agg}\,\Omega^2\, l^3}{\wp E n k_p z^2}}. \tag{12}$$

Expressions (8), (10) and (12) require, after forming the geometric dimensions, checking the strength of the elastic elements, since their lengths l are chosen arbitrarily. However, it is this parameter that is responsible for the stress in the element. Let us justify the choice of lengths so that they already take into account the strength parameters.

It is known that the normal stress during bending that occurs in such elastic elements is defined as

$$\sigma = \frac{M_{ben}}{W}, \tag{13}$$

where M_{ben} and W are respectively the bending moment applied to the elastic element at a certain section and the moment of resistance of its working section. We can determine the bending moment according to (2). The moments of resistance of rectangular and circular cross-sections relative to the axis z are defined as follows:

$$W_f = \frac{h b^2}{6}; \tag{14}$$

$$W_r = \frac{\pi d^3}{32}. \tag{15}$$

We assume that the acting torque M_{ben} applied to one element of the elastic assembly is such that the stresses σ are equal to the permissible $[\sigma]$. More precisely, the stresses σ are equal to the permissible $[\sigma_{-1}]$ under symmetrical cyclic loading. Therefore, expression (13) can be written in the form

$$[\sigma_{-1}] = M_{ben} / W. \tag{16}$$

We use expressions (2) and (14), (15). For these expressions, we write down the values of the moments of inertia (expressions (4) and (5)) taking into account (11). Then expression (16) for elastic elements of rectangular and circular cross-sections can be respectively presented as follows:

$$[\sigma_{-1}] = \frac{3 k_\kappa E b \delta}{l^2}; \tag{17}$$

$$[\sigma_{-1}] = \frac{3 k_\kappa E d \delta}{l^2}, \tag{18}$$

where k_κ is the additionally introduced stress concentration coefficient in the elastic element. It mainly lies within the limits

$$k_K = 1.1...1.3 \tag{19}$$

and is selected from reference literature.

Taking into account (17) and (18) the lengths of the elastic elements:

– for rectangular cross-section

$$l = \sqrt{\frac{3k_K E b \delta}{[\sigma_{-1}]}}; \tag{20}$$

– for the round cross-section

$$l = \sqrt{\frac{3k_K E d \delta}{[\sigma_{-1}]}}. \tag{21}$$

Let us substitute the dependencies for b (expression (12)) and d (expression (10)) into expressions (20) and (21), respectively. Then, the minimum permissible length of the elastic element that can be accepted is determined as follows:

– for rectangular cross-section

$$l_{\min} = \sqrt[5]{\frac{81\, c_{12}\, E^3\, \delta^4\, k_K^4}{\wp\, n\, k_p [\sigma_{-1}]^4}} \quad \text{or} \quad l_{\min} = \sqrt[5]{\frac{81\, m_{agg}\, \Omega^2\, E^3\, \delta^4\, k_K^4}{\wp\, n\, k_p [\sigma_{-1}]^4\, z^2}}; \tag{22}$$

– for round cross-section

$$l_{\min} = \sqrt[5]{\frac{432\, c_{12}\, E^3\, \delta^4\, k_K^4}{\pi\, n\, k_p [\sigma_{-1}]^4}} \quad \text{or} \quad l_{\min} = \sqrt[5]{\frac{432\, m_{agg}\, \Omega^2\, E^3\, \delta^4\, k_K^4}{\pi\, n\, k_p [\sigma_{-1}]^4\, z^2}}. \tag{23}$$

Knowing the deformation δ of the elastic node, the calculated stiffness c_{12} of the elastic node and the modulus of elasticity E of the material, according to (22) or (23), we establish the minimum possible length of the working section of the elastic element. After that, choosing the length of the elastic element from the condition

$$l \geq l_{\min}, \tag{24}$$

According to (12) or (10), we determine its thickness b or diameter d.

Consider a conditionally selected two-mass oscillatory system of the vibration machine in Fig. 1. The oscillatory masses for this vibration machine are $m_1 = 146$ kg and $m_2 = 38$ kg. The system oscillates rectilinearly at a frequency $\Omega = 628$ rad/s ($\nu = 100$ Hz). It is necessary to calculate the resonant elastic node. When the resonant tuning is equal to $z = 0.97$, the stiffness coefficient is [2, 3]

$$c_{12} = m_{agg}\left(\frac{\Omega}{z}\right)^2 = \left(\frac{m_1 m_2}{m_1 + m_2}\right)\left(\frac{\Omega}{z}\right)^2 = \left(\frac{146 \cdot 38}{146 + 38}\right)\left(\frac{628}{0.97}\right)^2 = 1.264 \cdot 10^7\ \frac{\text{N}}{\text{m}}.$$

Eight elastic elements are used. In this case, the number of working sections that work on bending is $n = 16$. The material of the elastic elements is steel, the modulus of elasticity of which is $E = 2.1 \cdot 10^{11}$ Pa. The allowable stress under the action of symmetrical cyclic loading for such a material is $[\sigma_{-1}] = 170$ MPa.

Let us assume that the relative amplitude of the oscillations of masses m_1 and m_2 is $X_{rel} = X_1 + X_2 = 0.8$ mm. The elastic node is deformed by the stiffness to the specified value $\delta = X_{rel} = 0.8$ mm. According to (19), we assume that the stress concentration coefficient in the elastic elements is $k_K = 1.1$, and the clamping coefficient $k_p = 0.8$ (see expression (9)). We assume that the parameter $\wp$ (see expression (11)) is $\wp = 14$. If we use expression (22), the minimum length of the working section of the elastic element will be

$$l_{\min} = \sqrt[5]{\frac{81 c_{12} E^3 \delta^4 k_\kappa^4}{\wp n k_p [\sigma_{-1}]^4}} = \sqrt[5]{\frac{81 \cdot 1.264 \cdot 10^7 \cdot [2.1 \cdot 10^{11}]^3 \cdot 0.0008^4 \cdot 1.1^4}{14 \cdot 16 \cdot 0.8 \cdot [1.7 \cdot 10^8]^4}} = 0.131 \text{ m}.$$

If we accept $l = 135$ mm, then the thickness of the working section of the elastic element according to (12) will be equal to

$$b = \sqrt[4]{\frac{c_{12} l^3}{\wp E n k_p}} = \sqrt[4]{\frac{1.264 \cdot 10^7 \cdot 0.135^3}{14 \cdot 2.1 \cdot 10^{11} \cdot 16 \cdot 0.8}} = 5.36 \cdot 10^{-3} \text{ m} = 5.36 \text{ mm}.$$

The width of the elastic element, using expression (11), is

$$h = \wp b = 14 \cdot 5.36 = 75 \text{ mm}.$$

In an elastic element, the stresses do not exceed the permissible ones $[\sigma_{-1}] = 170$ MPa, because according to expression (17)

$$\sigma = \frac{3 k_\kappa E b \delta}{l^2} = \frac{3 \cdot 1.1 \cdot 2.1 \cdot 10^{11} \cdot 0.00536 \cdot 0.0008}{0.135^2} = 163 \text{ MPa} < [\sigma_{-1}] = 170 \text{ MPa}.$$

Analysis of elastic elements of circular cross-section. The cross-sectional shape of resonant elastic elements is predominantly rectangular, which is why these elements are often referred to as planar. The advantage of a rectangular cross-section over a circular one is that, in the case of providing directional vibrations, the stresses that arise in rectangular-section elements are lower than in circular-section elastic elements, since the radius of inertia of the latter is larger.

Suppose we need to implement a resonant elastic node of a given stiffness to provide directed oscillations to the masses. The elements of the node are deformed according to the scheme in Fig. 2. For this purpose, we will try to use rectangular and circular cross-sections of the elastic element.

To ensure a certain stiffness during bending, provided that the material and lengths of the elastic elements with rectangular and circular cross-sections are the same, it is sufficient for us that the moments of inertia J of both sections relative to the axis z are the same. Therefore, if we equate (4) and (5), taking into account (11), the ratio of the diameter of d the elastic element of circular cross-section to the thickness of b the elastic element of rectangular cross-section will be

$$\frac{d}{b} = 2\sqrt[4]{\frac{\wp}{3\pi}} \approx 1.14\sqrt[4]{\wp}. \tag{25}$$

That is, if we implement an elastic node with circular-section elements, their diameter will be greater than the thickness of rectangular-section elements by a factor of $\wp = 14$, $1.14\sqrt[4]{14} = 2.2$ times, provided that the lengths and stiffnesses of both elastic elements (nodes) are equal (Fig. 4). The force $F = 630$ N applied to one working section of an elastic element deforms it regardless of the shape of the cross-section by the value $\delta = X_{rel} = 0.8$ mm. This value of the force is established using expression (3) and taking into account the fact that the number of elastic sections $n = 16$. So

$$F = F_{ben} = \frac{c_{12}\delta}{n} = \frac{1.264 \cdot 10^7 \cdot 0.0008}{16} = 630 \text{ N}. \tag{26}$$

Due to the fact that the diameter of the element of circular section is greater than the thickness of the element of rectangular section, the normal stresses during bending that arise in these elements will differ proportionally. Since the bending moment is the same, substituting expressions (14) and (15) in turn into (13), taking into account (11) and (25), the ratio of normal stresses in elastic elements of circular and rectangular cross-sections, provided that they are made of the same material and their lengths are equal, will be defined similarly to (25), namely

$$\frac{\sigma_r}{\sigma_f} = \frac{d}{b} \approx 1.14\sqrt[4]{\wp}. \tag{27}$$

Therefore, the stresses arising in circular-section elements are several times higher, which is also confirmed using the finite element method (Fig. 5).

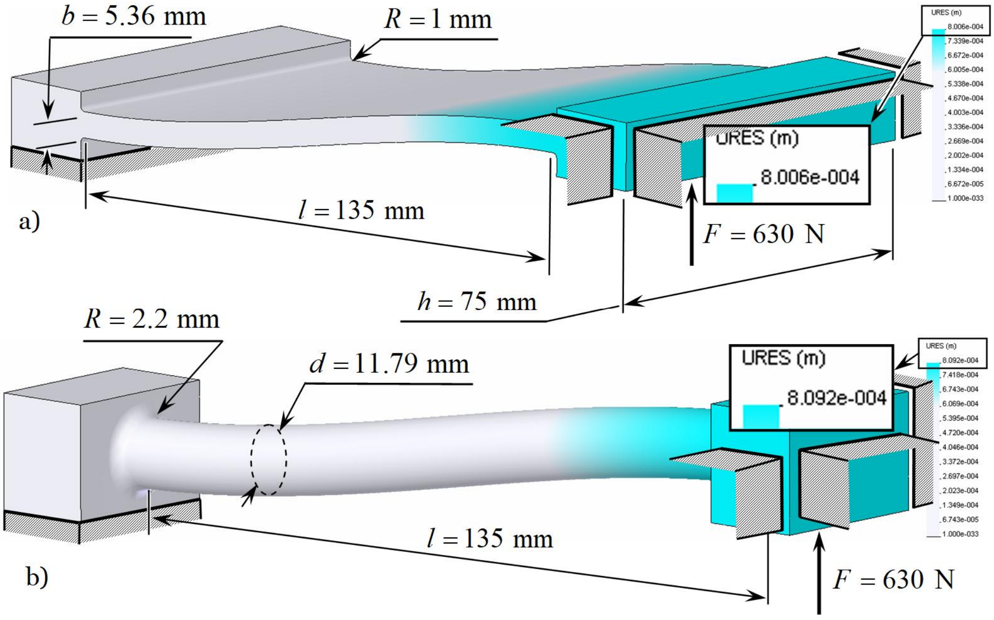

Fig. 4. Windows of the Cosmos Works module of the SolidWorks software product, which display the deformation of elastic elements of rectangular (a) and circular (b) cross-sections under the action of force $F = 630$ N (the deflection of the elements is scaled)

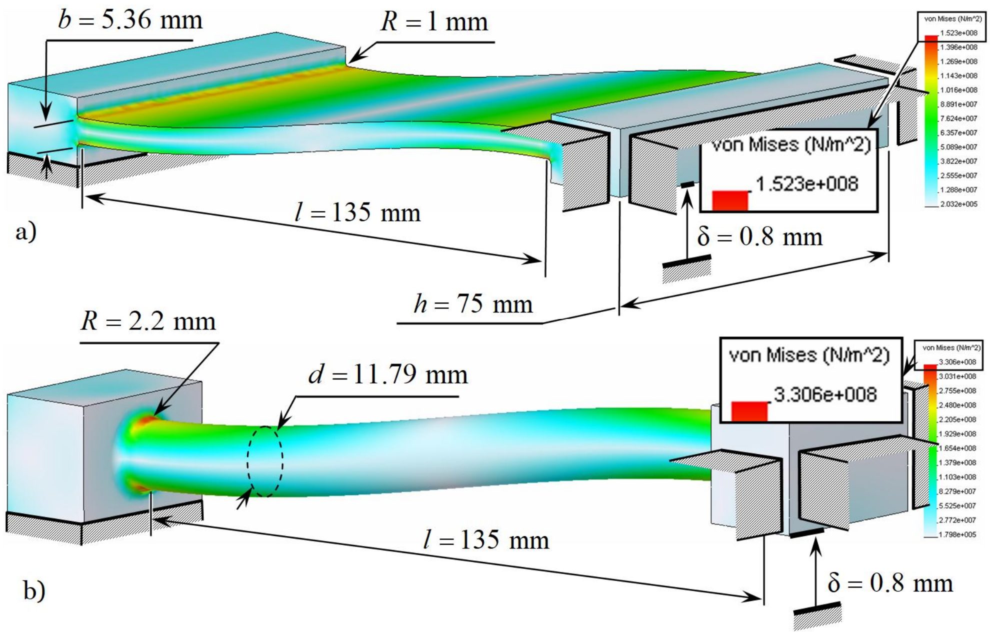

Fig. 5. Windows of the Cosmos Works module of the SolidWorks software product, which display the stresses in elastic elements of rectangular (a) and circular (b) cross-sections with the same bending stiffness and the same lengths of the working area that deforms (the deflection of the elements is scaled)

The stress in a circular cross-section element can be reduced by increasing its length. Therefore, if we need to ensure the same service life of elastic nodes with the same number of elastic elements, regardless of whether they have circular or rectangular cross-sections, the former will always be longer: $l_r > l_f$. Let's prove this.

It is clear that the stiffnesses of the elastic elements with both cross-sections must be the same. Therefore, using (3), we can write

$$\frac{12 E J_f}{l_f^3} = \frac{12 E J_r}{l_r^3}. \quad (28)$$

On the other hand, since the force F_{ben} is the same in both cases, and the bending moment M_{ben}, if we solve (1) and (2) together, is

$$M_{ben} = F_{ben}\, l/2, \quad (29)$$

using (16), we can write

$$\frac{F_{ben}\, l_f}{2W_f} = \frac{F_{ben}\, l_r}{2W_r}. \quad (30)$$

Solving (28) and (30) together, taking into account expressions (4), (5), (11), (14), (15) and introducing the notation

$$\wp_l = l_r / l_f \quad (31)$$

– the ratio of the lengths of elastic elements of circular and rectangular cross-sections, in which the same stresses are provided, through the parameters of the nodes, we write as

$$\wp_l = \frac{1}{3}\sqrt[5]{\frac{1296\wp}{\pi}} \approx 1.11\sqrt[5]{\wp}, \quad (32)$$

and the diameter of the circular cross-section through the thickness of the rectangular one will be

$$d = \frac{2b}{3\pi}\sqrt[5]{216\pi^3\wp^2} \approx 1.236 b\sqrt[5]{\wp^2}. \quad (33)$$

An expression identical to (32) can be obtained by dividing (23) by (22)

$$\wp_l = \sqrt[5]{\frac{432\wp}{81\pi}} \approx 1.11\sqrt[5]{\wp}, \quad (34)$$

and expression (33), if we divide (23) by (22) and (20) by (21) and equate, respectively, it will take the form

$$d = b\sqrt[5]{\left(\frac{432\wp}{81\pi}\right)^2} \approx 1.236 b\sqrt[5]{\wp^2}. \quad (35)$$

Let us assume that to ensure the required rigidity of the elastic unit (Fig. 1) it is necessary to use $n = 16$ elastic sections of rectangular cross-section with length $l = l_f = 135$ mm, thickness $b = 5.36$ mm and width $h = \wp b = 14 \cdot 5.36 = 75$ mm (Fig. 5, *a*). A similar elastic unit in terms of rigidity and strength characteristics on elements of circular cross-section will have working sections, the diameter of which, according to (33)

$$d = \frac{2 \cdot 5.36}{3\pi} \cdot \sqrt[5]{216 \cdot \pi^3 \cdot 14^2} = 19 \text{ mm},$$

and the length, using (31) and (32), is

$$l_r = \frac{l_f}{3}\sqrt[5]{\frac{1296\wp}{\pi}} = \frac{135}{3}\cdot\sqrt[5]{\frac{1296 \cdot 14}{\pi}} = 254.4 \text{ mm},$$

which is confirmed using the finite element method (Fig. 6).

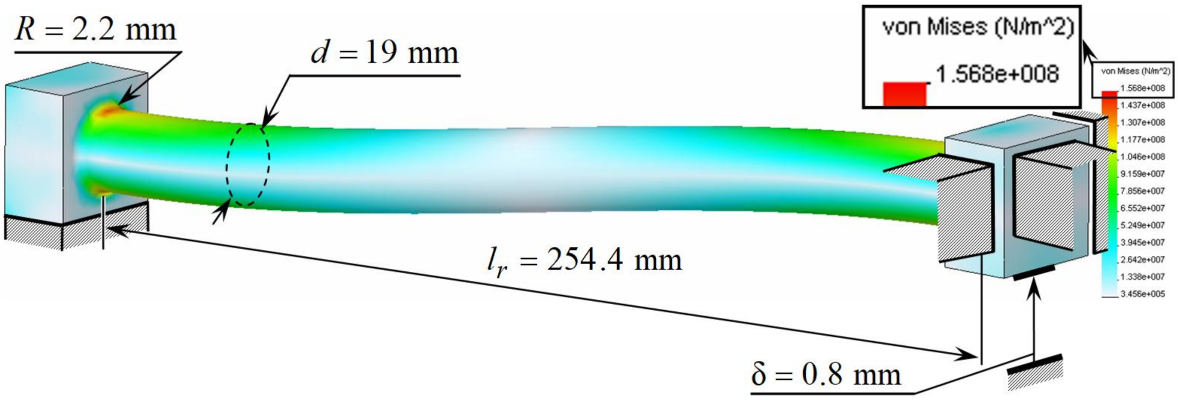

Fig. 6. Window of the CosmosWorks module of the SolidWorks software, showing the stress in a circular cross-section elastic element, identical in strength and stiffness characteristics to the element in Fig. 5.

Conclusions

The authors demonstrate that, under identical stiffness and material conditions, circular cross-section elastic elements exhibit significantly higher bending stresses than rectangular ones, due to their larger radius of inertia. Analytical derivations demonstrate that achieving the same operational stress level for a round element requires substantially greater length, making such solutions less compact and less structurally efficient. To validate these findings, finite-element simulations in SolidWorks (CosmosWorks) were performed to compare deflections and stress fields for rectangular and circular elements with equal bending stiffness. The numerical results fully confirm the analytical predictions: round elements experience higher stresses than flat ones and require lengthening to reach equivalent durability.

The study concludes that flat elastic elements provide superior structural efficiency, lower operating stresses, simpler mounting, and greater geometric adaptability within resonant vibration systems. The research thus provides a rigorous basis for selecting flat cross-section elastic components in the design of high-performance resonant vibration tables.

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