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Performance of geometric Brownian motion (GBM) with various volatility measurement models in forecasting market indices

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Many investors use market indices to manage their portfolios and keep track of the financial markets. Forecasting financial trends in a complex market is a critical factor for investors. Given how challenging and unpredictable future predictions can be, forecasting market indices cannot rely solely on regular patterns based on technical analysis. Therefore, this paper proposes a way to forecast future market indices of Financial Times Stock Exchange (FTSE) Bursa Malaysia Kuala Lumpur Stock Exchange Composite Index (KLCI) and MSCI All Country World Index (ACWI) by using geometric Brownian motion (GBM). Four different types of formulae are used to determine the suitable volatility measurement that may yield forecast values closer to the actual movements of stock market indices. The objective of this paper is to analyze the performance of geometric Brownian motion (GBM) and Simple Exponential Smoothing (SES) based on FBM KLCI and MSCI ACWI stock returns. Mean Squared Error (MSE), Mean Absolute Percentage Error (MAPE) and Mean Absolute Deviation (MAD) are chosen to be used in this paper to measure accuracy. The findings show that using high-low-close volatility yields forecast values closer to the actual movements of stock market indices. Since SES has the smallest error values, it can be considered the most effective method for forecasting.

Keywords: *forecast market index; geometric Brownian motion; volatility; simple exponential smoothing.*

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1. Introduction

A stock index, also known as a stock market index, is a statistical measure of market volatility and can identify changes in a particular type of stock. An index also works as a tool that enables investors to monitor the performance of a group of stocks within a specific market. The main index in Malaysia, Financial Times Stock Exchange (FTSE) Bursa Malaysia Kuala Lumpur Stock Exchange Composite Index (KLCI), usually referred to as the FTSE Bursa Malaysia KLCI or FBM KLCI, is a capitalization-weighted stock market index. FBM KLCI comprises 30 largest companies listed on the Main Board that meet the qualifying standards set out in the FTSE Bursa Malaysia Ground Rules. Morgan Stanley Capital International (MSCI) maintains a stock index known as the MSCI All Country World Index (ACWI), which was established to track the performance of the global equity market [1]. MSCI ACWI consists both of the MSCI World Index and the MSCI Emerging Market Index such that large-cap and mid-cap stocks from 23 developed countries with 26 emerging markets from 11 different sectors are included. Stock market indices are among the most important and frequently analyzed financial market characteristics. Many investors use market indices to manage their portfolios and keep track of the financial markets. In financial markets, forecasting stock prices is fundamental for asset allocation,

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portfolio management, risk evaluation and derivative valuation [2, 3]. Forecasting financial trends in a complex market is one of the crucial factors for investors. Precise predictions of stock market indices can result in significant returns with minimal investment risk for major organizations and investors who transact a substantial amount across various markets. Investors today strive to shield their assets from inflation-induced depreciation while remaining wary of the risks associated with stock market trading. Consequently, refining the forecast for the KLCI index through an analysis of its relationships with different variables would assist investors in making better-informed and more reliable decisions when selecting stocks to invest in [4]. Financial time series data, such as stock market data, are profoundly complicated and volatile. Financial market movements are non-linear, uncertain and constantly changing due to the impact of volatile and unforeseen market conditions on investors. Market indices are fluctuating significantly because of market turbulence, affecting both investor confidence and financial experts. This was strongly stated by AbdulKadir et al. [5], given the inherent trends and seasonality in trading patterns, the FTSE Bursa Malaysia KLCI index is regarded as highly chaotic, thereby complicating the process of analysis and forecasting. Given how challenging and unpredictable future predictions can be, forecasting the market index cannot be done only by using a regular pattern based on technical analysis. Time series analysis is a key element in many stock market prediction techniques as it makes forecasts by analyzing past data points in the series. A time series consists of one or more channels with measurable outputs but no measured inputs. In contrast, a time series model, also known as a signal model, is a dynamic system found to be well-suited for a particular signal or time series data. One of the classical forecasting models, the time series model, uses historical data to produce precise forecasts. Simple exponential smoothing (SES) is an example of a widely used time series forecasting model. Stochastic processes, a branch of mathematical methods, are used to analyze dynamical system with random variable that change over time. Geometric Brownian motion (GBM) is a type of stochastic process that occurs in continuous time. It is characterized by Brownian motion with a drift component and the logarithm of a random variable. An equation that includes a random variable or a stochastic process is generally known as a stochastic model. Stochastic models are applied in financial markets to represent the seemingly random behavior of various financial assets, including the changing value of one currency relative to another and the variability in interest rates. The most basic time series forecasting method, such as SES, does not attempt to identify the factors that affect the variable; instead, they simply use data on the variable itself to make predictions. Their forecasting performance is limited by the assumption of linear behavior, resulting in unsatisfactory results. Unlike classical models, stochastic models like GBM are used to estimate possible outcomes by incorporating uncertainties and randomness. Stochastic models provide transparency and precision, allowing us to assess our understanding of financial market trends by comparing model results with observed patterns. GBM is commonly used to model asset prices and assumes that volatility remains constant. Fama [6] presented compelling evidence supporting the notion that stock market prices are unpredictable and follow a random walk. His findings indicated that the random walk model, particularly in the context of GBM, outperformed other methods. Since its development, GBM has emerged as a widely respected model essential for analyzing a diverse range of financial assets [7]. Therefore, this paper proposes a way to forecast future market indices of FBM KLCI and MSCI ACWI by using GBM with suitable volatility measurement that may yield forecast value closer to the actual movement of stock market index. Furthermore, the objective of this paper is to analyze the performance of GBM and SES based on FBM KLCI and MSCI ACWI stock returns. SES method offers several significant advantages, including its simplicity, cost-effectiveness and relatively high accuracy over the short term. The process can be easily executed on a computer, requires minimal prior data and allows for easy generation of new projections [8]. Moreover, SES places greater emphasis on recent data points compared to the immediately preceding values, which makes this technique more practical than other forecasting approaches [9–11]. Although numerous studies have investigated forecasting methods for individual stock prices or general market indices, there has been limited research on SES and GBM. This work highlights the exploration of GBM in terms of volatility measurement and reviews the characteristics of both models in forecasting. In conclusion, the study's findings will provide advantages for future financial market measures. This research paper is structured into four more sections: 2. Literature Review, 3. Methodology, 4. Results and Discussions and 5. Conclusion.

2. Literature review

2.1. Simple exponential smoothing (SES)

Exponential smoothing is a weighted moving average approach that is particularly beneficial when rapid forecasting and regular re-forecasting are required. Simple or single exponential smoothing (SES) is the most basic form of exponential smoothing, introduced by Robert Goodell Brown in 1956. This method works well for forecasting data that lacks both trend and seasonal patterns [12]. Nadhira et al. [13] conducted research utilizing single moving average (SMA) and SES methods to identify the most appropriate forecasting technique for predicting the demand for Softex 1400-M. However, the findings indicated that the SMA performs better than the SES with a smoothing constant of 0.6. Therefore, it can be asserted that the SMA will yield more accurate demand forecasts in the upcoming periods. Reference [14] uses SES in modelling and forecasting analysis of daily temperature of Faisalabad and Lahore districts of Punjab, Pakistan. The analysis revealed that SES is more effective for modelling temperature data than other methods, such as Holt's exponential and Holt–Winter's exponential methods. They concluded that the SES model provides more accurate forecasting due to its smaller range of error measures. As discussed in reference [15], the authors develop a new model using the combination of earned value management (EVM) and SES technology to estimate the final cost of a project. According to the results, the cost performance index (CPI) derived from both EVM and SES methods effectively estimates the project's final cost. They found that because SES technology is easy to use and implement, it helps simplify both the management and interpretation of the final project's cost forecast and programming. Autoregressive Integrated Moving Average (ARIMA) and SES are considered by Fatima et al. [16] to model and forecast CO_2 emissions of selected Asian countries, including India, Sri Lanka, Bangladesh, Nepal, Pakistan, Iran, China, Japan and Singapore. In this study, it was determined that the SES model is suitable for Sri Lanka and Pakistan, as evidenced by a comparison of the forecast mean absolute error (FMAE) between the two models. On the other hand, the ARIMA model was found to be appropriate for India, Iran, China, Japan and Singapore. Nonetheless, both models exhibited comparable performance for Bangladesh and Nepal.

2.2. Geometric Brownian motion (GBM)

In the nineteenth century, biologist Robert Brown discovered Brownian motion as he observed pollen particles suspended in water under a microscope [17]. In finance, the unpredictable (zigzag) fluctuations in stock prices are known as Brownian motion, and the Brownian motion model is used to measure the uncertainty of returns on riskier assets [18,19]. A geometric Brownian motion (GBM) model is a type of stochastic process where stock returns are presumed to be normally distributed and independent [20]. Wilmott [21] states that GBM is fundamental to mathematical modeling in financial processes. It is the continuous model, which is derived from the discrete model, that can be used to forecast short-term movements in stock prices. Reference [22] studied the price fluctuations of sugar in Malaysia and performs a predictive analysis of sugar prices using GBM. Within this research, the GBM model demonstrated its capability to effectively capture fluctuations in sugar prices, enabling forecasts for future periods. The authors concluded that the study provides strong evidence that GBM is a key method for both capturing and predicting fluctuations in Malaysian sugar prices. Reference [23] uses GBM in modelling and forecasting stochastic price movements of The Nigerian Stock Exchange. From the findings, the MAPE value is 50% or lower for holding periods of one to two years but exceeds 50% for three-year holding periods. This implies that although the GBM performs well as a predictor for one or two years, it is less reliable for three-year periods. Mestiri [24] conducts a study to evaluate the predictive capabilities of GBM, Prophet and ARIMA models for stock market returns during the coronavirus outbreak. A dataset containing daily stock prices of the Standard and Poor's 500 (S&P 500) index from 2017 to 2021 was gathered and utilized in this study. Throughout the investigation, the GBM model emerged as the most effective model for predicting stock market returns over short time intervals (weeks) due to its high accuracy. As discussed in reference [25], the authors examine the use of GBM for predicting the future closing prices of small-sized companies. The results revealed that GBM is highly accurate, with the MAPE value remaining below 10%. They found the model to be reliable because the predicted prices closely aligned with the actual prices over a maximum investment period of two weeks.

3. Methodology

3.1. Data gathering procedure

This study uses secondary data sourced from `Investing.com` via its website (https://investing.com). The data comprises FBM KLCI and MSCI ACWI stock returns from August 2023 to May 2024. Furthermore, stock returns for both indices are collected daily for the last five months of 2023 and the first five months of 2024. SES and GBM methods are used to forecast future market indices of these selected markets using Excel formulas to determine their accuracy and effectiveness. SES requires minimal computation and is used when historical data shows neither cyclical variation nor trend [26,27]. This method creates a smoothing curve based on one actual past value and one predicted past value, as shown below [28]:

$$F_t = \alpha A_t + (1-\alpha)F_{t-1}, \tag{1}$$

where: F_t is new forecast value at time t, A_t is last actual value at time t, F_{t-1} is last forecast value at time $t-1$, α is smoothing constant where $0 \leqslant \alpha \leqslant 1$.

This paper calculates volatility (σ) using four different types of volatility measurement formulas to determine the most accurate GBM model for forecasting the market indices of FBM KLCI and MSCI ACWI.

1. Simple volatility (S):

$$\sigma = \sqrt{\frac{1}{(M-1)\Delta t}\sum_{i=1}^{M}(R_i - \bar{R})^2} \tag{2}$$

such that, σ is volatility, M is number of stock index return in the sample, Δt is time step, R_i is stock index return from day i to day $i+1$, $\bar{R}$ is returns distribution (mean) and

$$R_i = \frac{S_{i+1} - S_i}{S_i} \tag{3}$$

such that, S_i is stock index on the i^{th} day, $S_{(i+1)}$ is stock index on the $i+1^{th}$ day also given that

$$\bar{R} = \mu = \frac{1}{M}\sum_{i=1}^{M} R_i. \tag{4}$$

2. Log volatility (L):

$$\sigma = \sqrt{\frac{1}{(M-1)\Delta t}\sum_{i=1}^{M}\big(\log S(t_i) - \log S(t_{i-1})\big)^2} \tag{5}$$

such that, σ is volatility, M is number of stock index return in the sample, Δt is time step, $S(t_i)$ is stock index at time t_i, $S(t_{i-1})$ is stock index at time t_{i-1} where log is used.

3. Highs and lows volatility (HL):

$$\sigma = \sqrt{\frac{1}{(M-1)\Delta t\, 4\log 2}\sum_{i=1}^{M}\big(\log H(t_i) - \log L(t_i)\big)^2} \tag{6}$$

such that, σ is volatility, M is number of stock index return in the sample, Δt is time step, $H(t_i)$ is highest stock index on day t_i, $L(t_i)$ is lowest stock index on day t_i where log is used.

4. High-low-close volatility (HLC):

$$\sigma = \sqrt{\frac{1}{(M-1)\Delta t}\left(\sum_{i=1}^{M} 0.5\big(\log H(t_i) - \log L(t_i)\big)^2 - \sum_{i=1}^{M} 0.3\big(\log S(t_i) - \log S(t_{i-1})\big)^2\right)} \tag{7}$$

such that, σ = volatility, M = number of stock index return in the sample, Δt is time step, $H(t_i)$ is highest stock index on day t_i, $L(t_i)$ is lowest stock index on day t_i, $S(t_i)$ is stock index at time t_i, $S(t_{i-1})$ is stock index at time where log is used.

A stochastic process, $S(t)$ is considered to follow GBM if it meets the conditions of the following stochastic differential equation [29, 30]:

$$dS(t) = \mu S(t)dt + \sigma S(t)d\varepsilon(t) \tag{8}$$

such that, $S(t_i)$ is stock index at time t_i, μ is drift at time t, σ is volatility, $\varepsilon(t)$ is value from probability distribution at time t.

The random value drawn from the probability distribution, ε, is computed using the EXCEL function of NORM.S.INV(RAND()). This function generates a random value from the normal distribution table. After using the separation of variable technique, the equation is simplified to:

$$\frac{dS(t)}{S(t)} = \mu\, dt + \sigma\, d\varepsilon(t). \tag{9}$$

Taking integration on both sides

$$\int \frac{dS(t)}{S(t)} = \int \mu\, dt + \sigma\, d\varepsilon(t). \tag{10}$$

Since $\frac{dS(t)}{S(t)}$ relates to derivatives of $\ln(S(t))$, the Itô calculus is expressed as

$$\ln\frac{dS}{S} = \left(\mu - \frac{1}{2}\sigma^2\right) t + \sigma\, \varepsilon(t). \tag{11}$$

Taking the exponential in both sides

$$\begin{aligned} e^{\ln S} &= e^{\left(\mu-\frac{1}{2}\sigma^2\right)t+\sigma\,\varepsilon(t)+c}, \\ e^{\ln S} &= e^{\left(\mu-\frac{1}{2}\sigma^2\right)t+\sigma\,\varepsilon(t)} \cdot e^{c} \end{aligned} \tag{12}$$

such that $e^c = S(0)$. By substituting the initial condition $S(0)$, the analytical solution for GBM is obtained as

$$S(t) = S(0)\, e^{\left(\mu-\frac{1}{2}\sigma^2\right)t+\sigma\,\varepsilon(t)}. \tag{13}$$

3.2. Forecasting accuracy

Mean Squared Error (MSE), Mean Absolute Percentage Error (MAPE) and Mean Absolute Deviation (MAD) have been chosen to be used in this paper to measure accuracy. MSE represents the average of squared differences between actual and forecasted value as follows [31]:

$$\text{MSE} = \frac{\sum (Y_t - F_t)^2}{n}. \tag{14}$$

MAPE is determined by averaging the absolute differences between actual and forecasted values, expressed as a percentage of actual value. It can be calculated as below [31]:

$$\text{MAPE} = \frac{1}{n}\sum\left|\frac{Y_t - F_t}{Y_t}\right|. \tag{15}$$

In addition, MAD is calculated by dividing the sum of the absolute values of forecasting errors by the number of period forecast as follows [31]:

$$\text{MAD} = \frac{1}{n}\sum |Y_t - F_t|, \tag{16}$$

where for all MSE, MAPE and MAD, n is number of period forecast, Y_t is actual value in time period at time t, F_t is forecast value in time period at time t.

4. Results and discussion

In this paper, stock market of FBM KLCI and MSCI ACWI are selected for forecasting analysis based on their stock returns from 1st August, 2023 to 31st May, 2024.

4.1. FBM KLCI

In this section, FBM KLCI has been forecasted via SES method as well as GBM method with four different types of volatility measurement formulas. The aim is to compare the performance of all methods in forecasting the FBM KLCI. The results obtained can be found in the figures below.

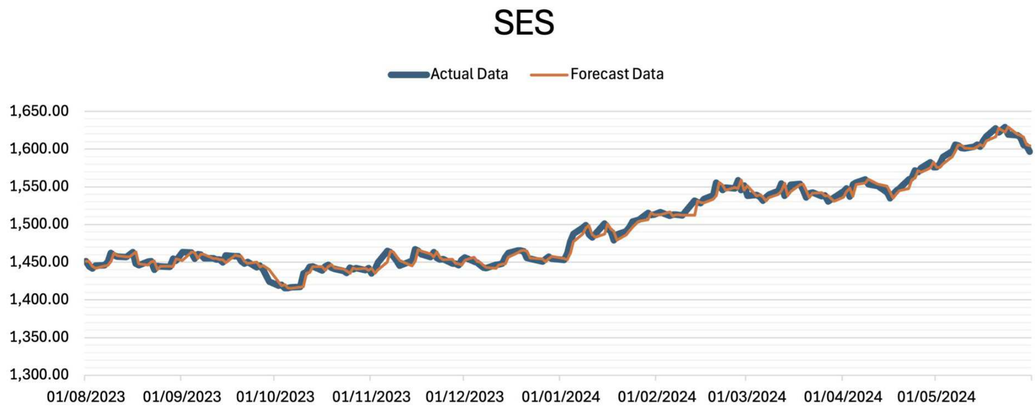

Fig. 1. SES for FBM KLCI.

Figure 1 compares the actual to the forecast values of FBM KLCI by using SES.

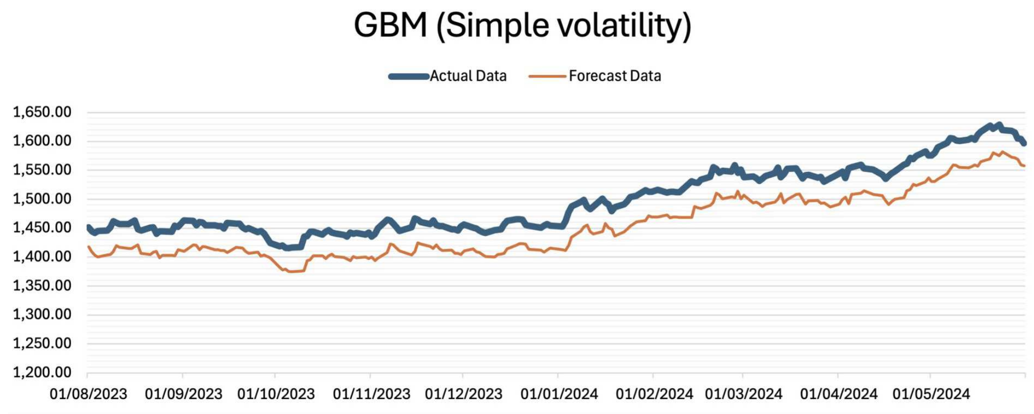

Fig. 2. GBM with simple volatility measurement for FBM KLCI.

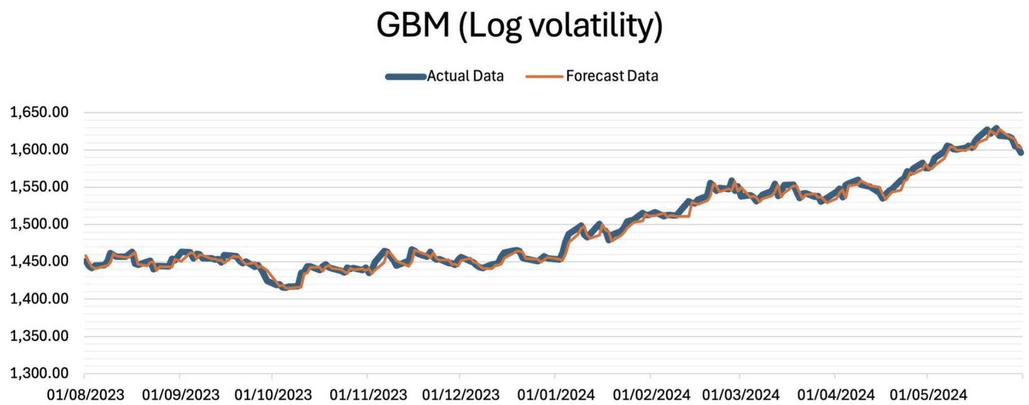

Fig. 3. GBM with log volatility measurement for FBM KLCI.

Figures 2–5 compare the actual to the forecast values of FBM KLCI by using GBM with four different types of volatility measurement formulas.

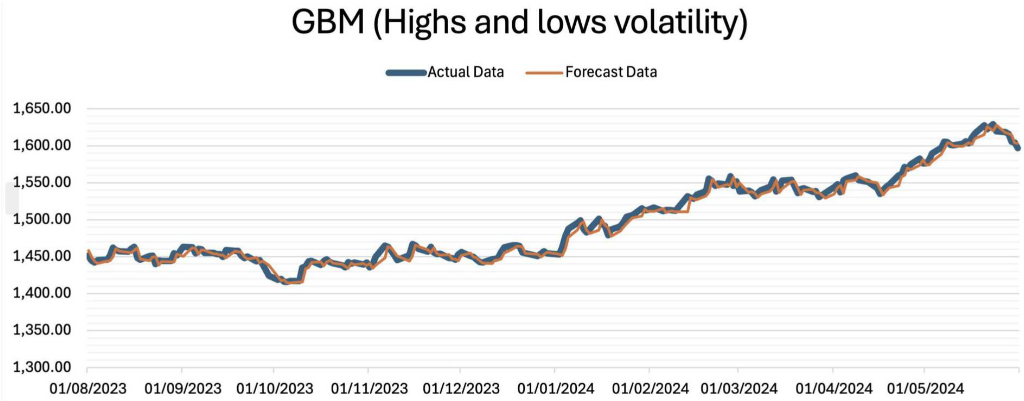

Fig. 4. GBM with highs and lows volatility measurement for FBM KLCI.

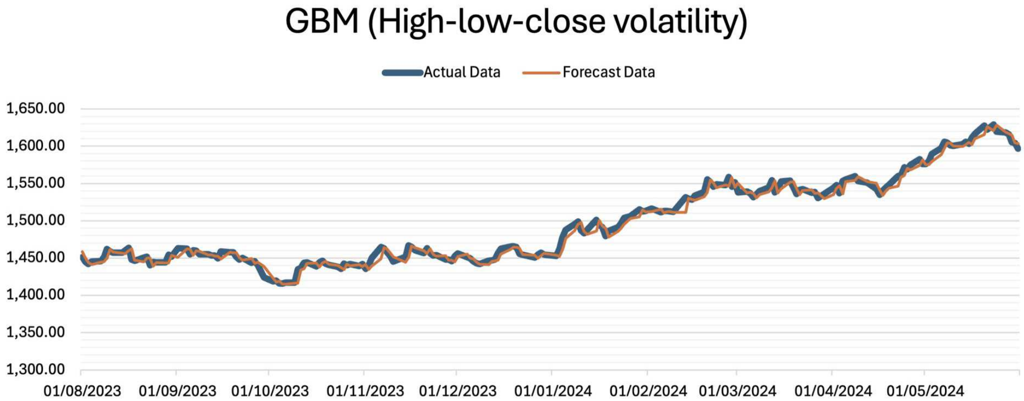

Fig. 5. GBM with high-low-close volatility measurement for FBM KLCI.

Table 1. The values of volatility measurement for each volatility measurement model in forecasting FBM KLCI.

Volatility measurement	Simple (S)	Log (L)	Highs and Lows (HL)	High-low-close (HLC)
Value	1.1060	0.0303	0.0386	0.0249

Table 1 shows the volatility values calculated using four different formulas from Eq. (2), (5)–(7). The high-low-close volatility has the smallest volatility value of 0.0249.

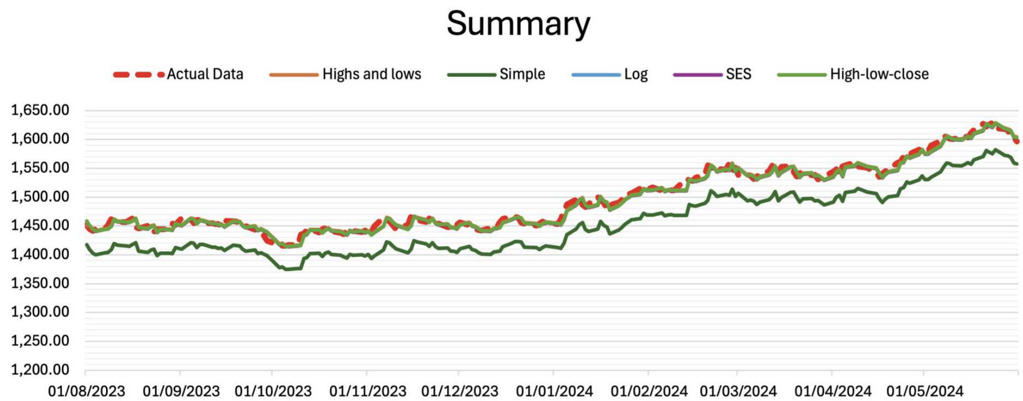

Fig. 6. Performance of SES, GBM with low, log, highs and lows and high-low-close volatility measurements in forecasting FBM KLCI.

Figure 6 illustrates the performance of SES and GBM with four different types of volatility measurement formulas in forecasting FBM KLCI. It is evident that there is a significant gap between the actual data and the forecast data when using GBM with simple volatility measurement. However, Table 2 shows that all the MAPE values are lower than 10%, indicating high accuracy. Following that,

Table 2. Accuracy measures of all methods in forecasting FBM KLCI.

Forecasting Method	MSE	MAPE	MAD
SES	43.10	0.34%	5.11
GBM (S)	1964.03	2.92 %	43.80
GBM (L)	46.19	0.35 %	5.28
GBM (HL)	47.37	0.36 %	5.34
GBM (HLC)	45.53	0.35%	5.24

when comparing the values of MSE and MAD of each method, SES has the smallest error values of 43.10 and 5.11 respectively. Therefore, forecasting FBM KLCI using the SES method shows the most accurate results than other methods. Among different type of volatility in GBM, high-low-close volatility surpass in its performance as opposed to simple volatility, log volatility and highs and lows volatility.

4.2. MSCI ACWI

In this section, MSCI ACWI has been forecasted via SES method as well as GBM method with four different types of volatility measurement formulas. The aim is to compare the performance of all methods in forecasting the MSCI ACWI. The results obtained can be found in the figures below.

Figure 7 compares the actual to the forecast values of MSCI ACWI by using SES.

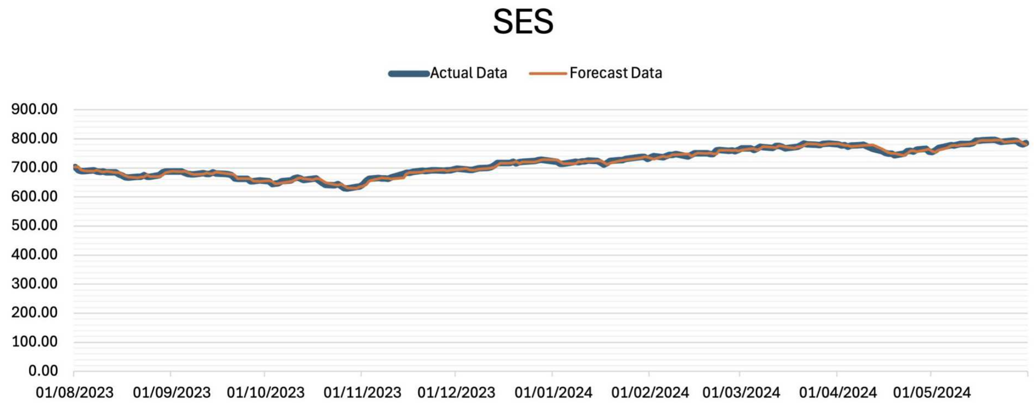

Fig. 7. SES for MSCI ACWI.

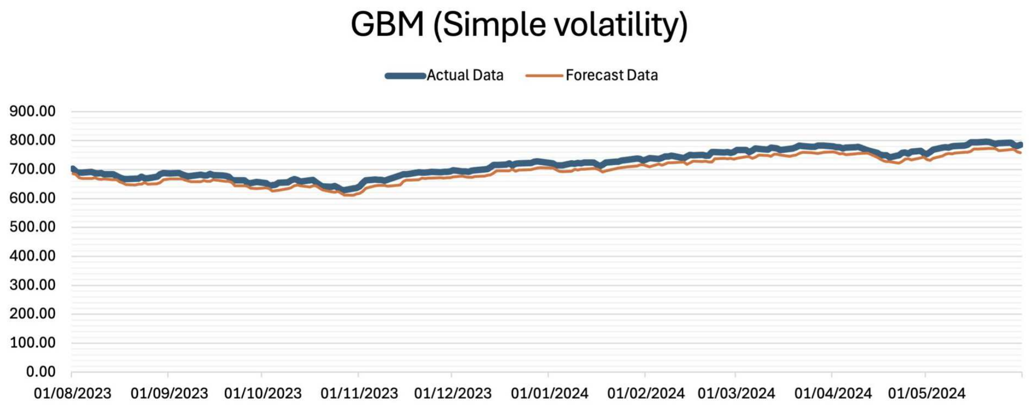

Fig. 8. GBM with simple volatility measurement for MSCI ACWI.

Figures 8–11 compare the actual to the forecast values of MSCI ACWI by using GBM with four different types of volatility measurement formulas.

Table 3. The values of volatility measurement for each volatility measurement model in forecasting MSCI ACWI.

Volatility measurement	Simple (S)	Log (L)	Highs and Lows (HL)	High-Low-Close (HLC)
Value	1.1087	0.0448	0.0572	0.0370

Table 3 shows the volatility values calculated using four different formulas from Eq. (2), (5)–(7). The high-low-close volatility has the smallest volatility value of 0.0370.

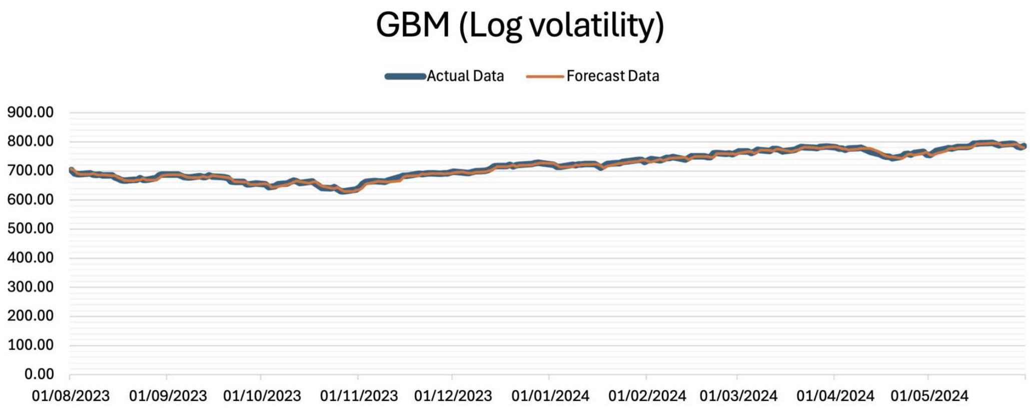

Fig. 9. GBM with log volatility measurement for MSCI ACWI.

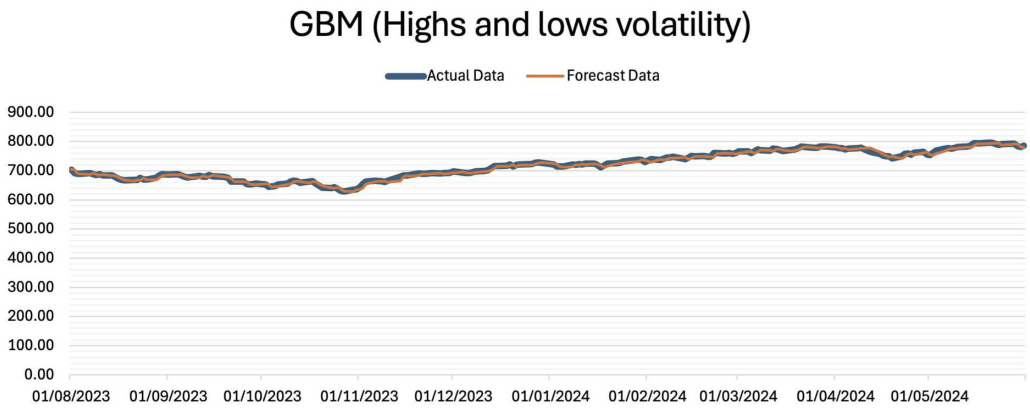

Fig. 10. GBM with highs and lows volatility measurement for MSCI ACWI.

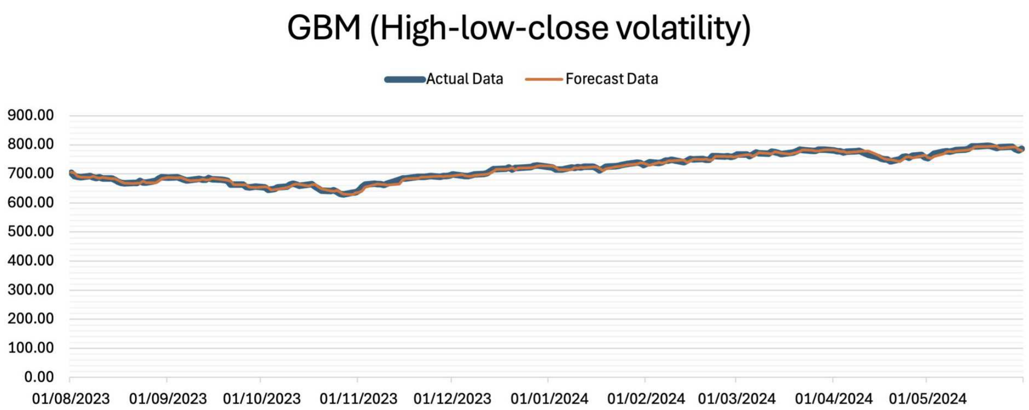

Fig. 11. GBM with high-low-close volatility measurement for MSCI ACWI for MSCI ACWI.

Table 4. Accuracy measures of all methods in forecasting MSCI ACWI.

Forecasting method	MSE	MAPE	MAD
SES	21.11	0.50%	3.59
GBM (S)	469.13	2.94%	21.13
GBM (L)	22.40	0.53%	3.76
GBM (HL)	22.96	0.53%	3.82
GBM (HLC)	22.10	0.52%	3.72

Figure 12 illustrates the performance of SES and GBM with four different types of volatility measurement formulas in forecasting MSCI ACWI. Table 4 shows that all the MAPE values are lower than 10%, indicating high accuracy. Following that, when comparing the values of MSE and MAD of each method, SES has the smallest error values of 21.11 and 3.59 respectively. Therefore, forecasting MSCI ACWI using the SES method shows the most accurate results than other methods. Among different type of volatility in GBM, high-low-close volatility surpass in its performance as opposed to simple volatility, log volatility and highs and lows volatility. According to the results shown in Tables 2 and 4,

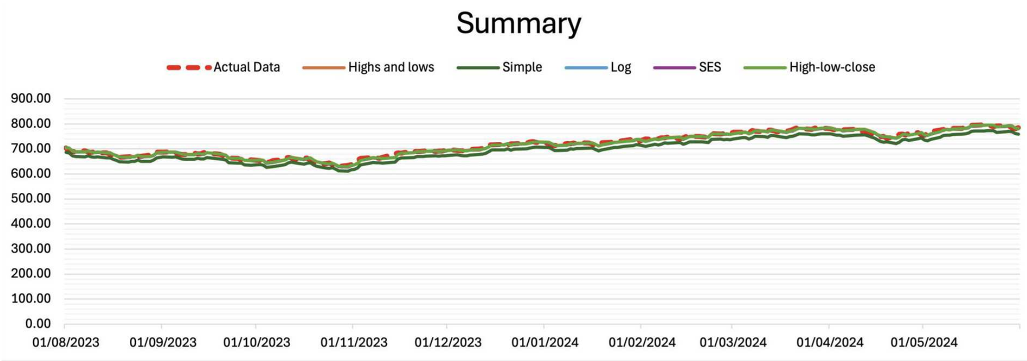

Fig. 12. Performance of SES, GBM with low, log, highs and lows and high-low-close volatility measurements in forecasting MSCI ACWI.

SES outperforms GBM with the smallest error values for MSE, MAPE and MAD. Brownian motion is commonly used in GBM to model stock prices. However, a previous study conducted by Singhal [32] stated that the Brownian motion process has the independent increments property. This indicates that current prices should not affect future prices. Indeed, the current stock price may impact its price at a later time. Thus, the Brownian motion process may not necessarily be effective in explaining stock prices. Apart from this, GBM also relies on a constant volatility assumption, which may not align with real market behavior. In fact, stock market volatility can fluctuate over time, particularly during periods of market turmoil or major news events.

5. Conclusion

This paper proposes a method for forecasting future market indices of FBM KLCI and MSCI ACWI using GBM with four different types of volatility measurement formulas. Daily data from August 2023 to May 2024 for both indices were collected. The study revealed that the high-low-close volatility is the most suitable volatility measurement to be used proven by lowest MSE, MAPE and MAD values. Furthermore, the objective of this paper is to analyze the performance of GBM and SES based on FBM KLCI and MSCI ACWI stock returns. Given the SES has the smallest error values for MSE, MAPE and MAD in both FBM KLCI and MSCI ACWI forecasts, it can be considered the most effective method for forecasting. Supposedly, GBM can more closely resemble the actual physical system as the model incorporates uncertainties and randomness. However, the results in this study differed due to limitations notably the volatility measurement used not being suitable for the data. Further research could develop an embedded volatility measurement in GBM to investigate whether GBM could be a more effective method for prediction.

Acknowledgement

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Ефективність геометричного броунівського руху (GBM) з різними моделями вимірювання волатильності в прогнозуванні ринкового індексу

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Багато інвесторів використовують ринкові індекси для керування своїми портфелями та відстеження фінансових ринків. Прогнозування фінансових тенденцій на складному ринку є ключовим завданням для інвесторів. Зважаючи на те, наскільки складними та непередбачуваними можуть бути прогнози на майбутнє, прогнозування ринкових індексів неможливо здійснити лише за допомогою звичайної моделі, заснованої на технічному аналізі. Тому в цій статті пропонується спосіб прогнозування майбутніх ринкових індексів FTSE Bursa Malaysia Kuala Lumpur Stock Exchange Composite Index (KLCI) та MSCI All Country World Index (ACWI) за допомогою геометричного броунівського руху (GBM). Чотири різні типи формул використовуються для визначення відповідного вимірювання волатильності, яке може дати прогнозовані значення, ближчі до фактичного руху індексів фондового ринку. Метою цієї статті є аналіз ефективності геометричного броунівського руху (GBM) і простого експоненціального згладжування (SES) на основі прибутковості акцій FBM KLCI та MSCI ACWI. Середня квадратична помилка (MSE), середня абсолютна відсоткова помилка (MAPE) і середнє абсолютне відхилення (MAD) були обрані для визначення точності. Висновки показують, що використовуючи волатильність high-low-close, прогнозовані значення наближаються до фактичного руху індексів фондового ринку. Оскільки SES має найменші значення похибок, його можна вважати найефективнішим методом прогнозування.

Ключові слова: *прогнозний індекс ринку; геометричний броунівський рух; волатильність; просте експоненціальне згладжування.*