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Modeling of Wastewater Sludge Dewatering Kinetics Using the Method of Linear Proportionalities

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Abstract

Sewage sludge accumulating at treatment facilities is aqueous suspensions separated from wastewater during treatment processes. Untreated sludge has been discharged for decades into overloaded sludge beds, dumps, or quarries, leading to environmental safety violations and deteriorating people living conditions. Due to a high content of colloidal substances, sludge poorly releases water. An important step in sludge disposal is dewatering, which significantly reduces sludge volume. Factors such as moisture content, the ratio of free to bound water, the degree of dispersion of solid phase particles, chemical composition, structure, and viscosity significantly influence sludge dewatering. The compressibility of sewage sludge under external pressure is one of its characteristic properties. Modeling was carried out using the method of linear proportions, which allowed the derivation of a uniform functional dependence under different combinations of similarity numbers.

Keywords: dewater ability; water content; sewage sludge; dimensional analysis; linear proportionalities method; similarity numbers.

1. Definition of the problem to be solved

Today, many cities, towns and industrial enterprises face a very acute problem of treatment and disposal of sludge generated during wastewater treatment. Often, untreated sludge has been dumped into overloaded sludge sites, dumps, tailings ponds, and quarries, which has led to a violation of environmental safety and living conditions.

Wastewater sludge that accumulates at wastewater treatment plants is an aqueous suspension released from wastewater in the process of its mechanical, biological or physicochemical treatment, with a volume concentration of polydisperse solid phase from 0.5 to 10%. Sludge is classified as a hard-to-dewater polydisperse suspension. As in all suspensions, moisture in wastewater sludge is in chemical, physical-chemical and physical-mechanical connection with solid particles, as well as in a free state.

The main objective of sludge treatment is to produce a sanitary and transportable product that can be used in agriculture. Due to the large amount of colloidal substances, sludge does not release water well. The water release of sludge is greatly influenced by humidity, the ratio of free to bound water, the degree of dispersion of solid phase particles, chemical composition, structure, viscosity of the sludge, etc.

Sludge produced in wastewater treatment plants represents a small part (about 1%) of the wastewater volume treated, while sludge handling and disposal processes account for 20% to 60% of operating costs, including labor, energy and sludge disposal [1].

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2. Analysis of the recent publications and research works on the problem

Wastewater sludge disposal is a complex and expensive process; however, sludge is a source of carbon, nutrients and trace elements and can be disposed of effectively [2], [3].

The following sewage sludge treatment processes are used in wastewater treatment plants: compaction, stabilization, conditioning, dewatering, drying, thermal treatment, disposal of valuable products, or sludge disposal.

Densification and dewatering reduce the moisture content, volume, and weight of sludge and are usually an integral part of any sludge treatment process.

Although sludge densification provides the lowest percentage of moisture reduction, it leads to the greatest effect of water removal and maximum decrease of the primary volume of sludge.

Dewatering is carried out by natural drying of stabilized sludge at sludge drying beds to a moisture content of 80...85% or by mechanical dehydration of previously conditioned stabilized or raw sludge using vacuum filters, filter presses or centrifuges to a moisture content of 65...75%.

An important step in the disposal of sludge is its dewatering, which allows for a significant reduction in sludge volume [4], [5]. Depending on the sensitivity of sludge to mechanical dewatering, the water content in sludge can range from 95-99 to 65-85% [6], [7]. Difficulties in sludge dewatering are associated with the high content of organic and colloidal substances in the sludge. Sludge dewatering from wastewater treatment plants is a serious problem due to the large amount of sludge produced annually. In recent years, research and development has focused on improving the dewatering process to reduce the subsequent costs of sludge management and transportation [8].

The disposal and transportation of sludge is expensive due to its high-water content. Reducing the water content of sludge is the most effective strategy to reduce treatment costs. However, sludge contains a large amount of hydrophilic organic substance, which causes poor dewatering. Therefore, research on preconditioning and mechanical dewatering is of great importance for sludge dewatering [9].

To improve dewatering, sewage sludge is chemically conditioned. Chemicals such as aluminum sulfate, iron (III) chloride, iron (II) sulfate [10], [12] and polyelectrolytes are added [11]. Currently, polyelectrolytes are used for the initial treatment of sludge before dewatering processes. However, the use of polyelectrolytes increases the cost of sludge processing and can cause secondary environmental pollution. Therefore, various alternative methods of sewage sludge conditioning, including ultrasonic treatment [13], microwave treatment [14], thermal treatment [15]. Sludge conditioning process was modified to reduce the consumption of chemicals, and new methods of sludge pretreatment were proposed and investigated, such as various combined methods [16].

3. Formulation of the goal of the paper

In order to model the processes occurring during the dewatering of sludge, it is necessary to derive functional dependencies that describe these processes. The aim of this article is to derive functional dependencies that describe the kinetics of mechanical dewatering of sewage sludge and to validate these dependencies through experimental investigation.

4. Presentation and discussion of the research results

The ability of sewage sludge to compress under external pressure is one of its characteristic properties [17]. Velocity, m/s, of sewage sludge filtration [18]:

$$q = f(G,\ e,\ W,\ \mu,\ t,\ p,\ h), \tag{1}$$

where G is sludge compression modulus, Pa; e is sludge porosity coefficient; W is moisture content of the sludge; μ is dynamic viscosity of the filtrate, Pa·s; t is duration of filtration, s; p is liquid pressure in the pores of the sludge, Pa; h is thickness of the sludge layer, m.

Moisture content of the sludge W depends on the sediment porosity coefficient e [18]. The functional relationship (1) was rewritten in the form:

$$\varphi(q,\ G,\ e,\ \mu,\ t,\ p,\ h) = 0. \tag{2}$$

The modeling was performed using the method of linear proportions. Without taking into account the dimensionless quantities, we will compose from dependence (2) linear proportions with linear dimensions, i.e., with the dimensions of length (L, m). The total number of linear proportions will be as follows [19]:

$$K = 0.5 \cdot (n - l - 2) \cdot (n - l - 1) + l, \tag{3}$$

where *n* is total number of variables included in dependence (2), *n* = 6; *l* is number of variables with linear differences in dependence (2), *l* = 1.

$$K = 0.5 \cdot (6 - 1 - 2) \cdot (6 - 1 - 1) + 1 = 7.$$

So,

$$\psi\left(\frac{q\cdot\mu}{G}, \frac{q\cdot\mu}{p}, \frac{G\cdot q\cdot t^2}{\mu}, \frac{p\cdot q\cdot t^2}{\mu}, \frac{G\cdot h\cdot t}{\mu}, \frac{p\cdot h\cdot t}{\mu}, q\cdot t, h, e\right) = 0. \tag{4}$$

Let's rewrite function (4) and use similarity numbers:

$$\psi\left(\frac{q\cdot\mu}{G\cdot h}, \frac{q\cdot\mu}{p\cdot h}, \frac{G\cdot q\cdot t^2}{\mu\cdot h}, \frac{p\cdot q\cdot t^2}{\mu\cdot h}, \frac{G\cdot t}{\mu}, \frac{p\cdot t}{\mu}, \frac{q\cdot t}{h}, e\right) = 0. \tag{5}$$

This equation contains redundant information. To get rid of the extra terms, we combine the similarity numbers:

1) The first and second numbers, the third and fourth numbers, and the fifth and sixth numbers differ only in the values of *G* and *p*. Therefore, we will keep the first $\frac{q\cdot\mu}{G\cdot h}$, third $\frac{G\cdot q\cdot t^2}{\mu\cdot h}$ and fifth $\frac{G\cdot t}{\mu}$ numbers, and write the relationship between *G* and p as a simplex $\frac{G}{p}$.

2) Divide $\frac{G\cdot q\cdot t^2}{\mu\cdot h}$ by $\frac{q\cdot t}{h}$, we get $\frac{G\cdot t}{\mu}$. But this number already exists, so you can remove $\frac{G\cdot q\cdot t^2}{\mu\cdot h}$ and not consider it further.

3) Multiply the numbers $\frac{G\cdot t}{\mu}$ and $\frac{q\cdot\mu}{G\cdot h}$ to get the number $\frac{q\cdot t}{h}$. Therefore, we remove the number $\frac{q\cdot\mu}{G\cdot h}$, but if necessary, it can be obtained by combining the numbers $\frac{q\cdot t}{h}$ and $\frac{G\cdot t}{\mu}$.

After the transformation,

$$\theta\left(\frac{G\cdot t}{\mu}, \frac{G}{p}, \frac{q\cdot t}{h}, e\right) = 0. \tag{6}$$

The similarity number $\frac{q\cdot\mu}{G\cdot h}$, which has been removed, is rewritten as $\frac{G\cdot h}{q\cdot\mu}$. Then it can be viewed as the ratio of the elastic force $[F_c] = E \cdot L^2$ and the viscosity force $[F_V] = \mu \cdot V \cdot L$ [20]:

$$\frac{[F_c]}{[F_V]} = \frac{E\cdot L}{\mu\cdot V}, \tag{7}$$

where *L* is characteristic length; *V* is velocity; *E* is compressive modulus.

The simplex *G/p* can be considered as the ratio of the elastic force $[F_c]$ and the pressure force $[F_p] = p \cdot L^2$ [20]:

$$\frac{[F_c]}{[F_p]} = \frac{E}{p}, \tag{8}$$

The similarity number $\frac{q\cdot t}{h}$ is similar to the homochrony criterion [21]

$$H_0 = \frac{V\cdot t}{L}, \tag{9}$$

which has a physical meaning as a characteristic of unsteady fluid flow [22]. This criterion can be viewed as the ratio of the additional (local) force caused by the unsteady nature of the flow to the force of inertia [21].

Let's rewrite (6) as

$$\frac{q\cdot t}{h} = \phi\left(\frac{G\cdot t}{\mu}, \frac{G}{p}, e\right). \quad (10)$$

The similarity numbers in (5) were combined in different way. The second $\frac{q\cdot\mu}{p\cdot h}$, fourth $\frac{p\cdot q\cdot t^2}{\mu\cdot h}$, and sixth $\frac{p\cdot t}{\mu}$ numbers leave and the relationship between *G* and *p* in the form of a simplex *G/p*. After the transformations:

$$\delta\left(\frac{p\cdot t}{\mu}, \frac{G}{p}, \frac{q\cdot t}{h}, e\right) = 0, \quad (11)$$

The removed similarity number $\frac{q\cdot\mu}{p\cdot h}$, is written as $\frac{p\cdot h}{q\cdot\mu} = \frac{p\cdot h^2}{q\cdot\mu\cdot h}$. The resulting expression is the ratio of the pressure force $[F_p]$ and the viscosity force $[F_V]$ [20]:

$$\frac{[F_p]}{[F_V]} = \frac{p\cdot L}{\mu\cdot V}. \quad (12)$$

This corresponds to the Lagrange's criterion La, which is a criterion for the similarity of pressure and velocity fields in the flow of fluid in straight channels [21]. For a given channel, it characterizes the relationship between the dimensionless pressure and velocity fields.

Let's multiply and divide expression (12) by the force of inertia $[F_i] = \rho \cdot V^2 \cdot L^2$, where ρ is specific mass [20]:

$$\frac{[F_p]}{[F_V]} \cdot \frac{[F_i]}{[F_i]} = \frac{[F_p]}{[F_i]} \cdot \frac{[F_i]}{[F_V]} = \mathrm{Eu} \cdot \mathrm{Re},$$

where Eu is Euler's criterion; Re is Reynolds criterion.

$$\mathrm{Eu} = \frac{[F_p]}{[F_i]} = \frac{p}{\rho\cdot V^2}. \quad (13)$$

$$\mathrm{Re} = \frac{[F_i]}{[F_V]} = \frac{p\cdot V\cdot L}{\mu}, \quad (14)$$

Therefore, the number $\frac{p\cdot h}{q\cdot\mu}$ is similar to the criterion La = Eu·Re. Accordingly, the combination of numbers $\frac{p\cdot t}{\mu}$ and $\frac{q\cdot t}{h}$ is similar to the combination of criteria Eu, Re, La.

We can rewrite (11) as follows

$$\frac{q\cdot t}{h} = \eta\left(\frac{p\cdot t}{\mu}, \frac{G}{p}, e\right). \quad (15)$$

Dependencies (10) and (15) differ in their first similarity numbers. However, taking into account the second number that connects them, we can consider these dependencies to be the same.

5. Comparison of the modeling results with experimental data

The experimental setup scheme and the methodology for studying the kinetics of sludge dewatering from the Lviv wastewater treatment facilities were presented in [18].

The simplex *G/p* in dependencies (10) and (15) is presented (Fig.1) as the dependence $G = f_1(p)$ for a mixture of wastewater sludge on primary and secondary sedimentation tanks in a 1:2 ratio. This relationship can be described by the equation ($R^2 = 0.729$):

$$G = 115.1 \cdot p + 0.76 \cdot 10^7. \quad (16)$$

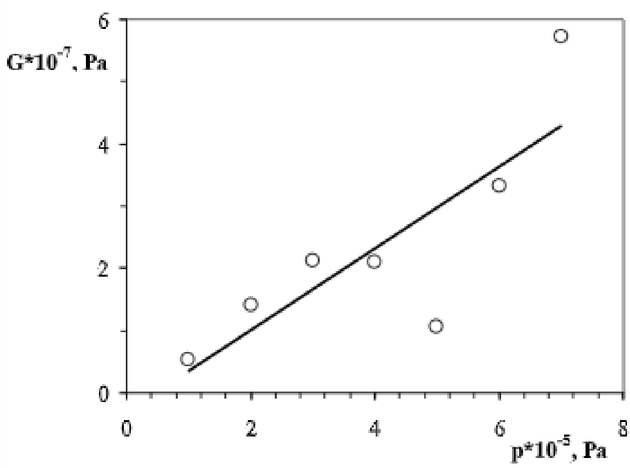

Fig.1. The dependence of the sludge compression modulus *G* on pressure p.

To evaluate the filtration properties of wastewater sludge, the specific filtration resistance r_o is used. The similarity number $\frac{p\cdot t}{\mu}$, used as the argument in equation (15), can be transformed into the following form:

$$\frac{p\cdot t}{\mu} = \frac{p\cdot t}{\mu\cdot r_o\cdot h^2}. \tag{17}$$

Therefore, dependence (15) can be represented as $\frac{q\cdot t}{h} = f_2\left(\frac{p\cdot t}{\mu\cdot r_o\cdot h^2}\right)$ that is an analogue of the Darcy equation for laminar filtration [18] (Fig. 2). This relationship is described by the equation ($R^2 = 0.9395$):

$$\frac{q\cdot t}{h} = 0.9954 \cdot \frac{p\cdot t}{\mu\cdot r_o\cdot h^2}. \tag{18}$$

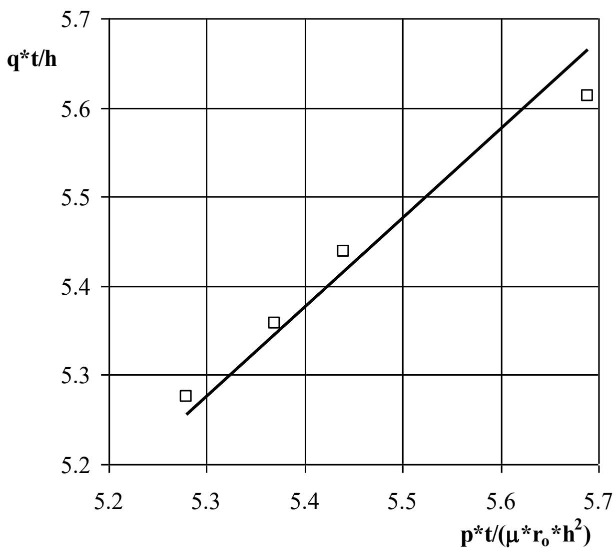

Fig.2. An analogue of the Darcy equation for laminar filtration.

6. Conclusion

Modeling by the method of linear proportions allowed us to obtain the same functional dependence for different combinations of similarity numbers. The similarity numbers have physical meaning as ratios of forces and serve as analogs to similarity criteria.

The physical modeling of the dewatering process of sewage sludge mixture from the Lviv municipal wastewater treatment facilities confirmed the validity of dependencies (10) and (15) derived using the method of linear proportions. An analog of Darcy's equation for laminar filtration was obtained.

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Моделювання кінетики зневоднення осадів стічних вод методом лінійних пропорційностей

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Анотація

Осади стічних вод, які скупчуються на очисних спорудах, є водними суспензіями, що виділяють із стічних вод в процесі їх механічного, біологічного або фізико-хімічного очищення. Часто осади в необробленому вигляді протягом десятків років зливалися на переобтяжені мулові площадки, у відвали, кар'єри, що привело до порушення екологічної безпеки й умов життя населення. Через велику кількість колоїдних речовин осади погано віддають воду. На водовіддачу осадів мають великий вплив вологість, співвідношення вільної і зв'язаної води, ступінь дисперсності частинок твердої фази, хімічний склад, структура, в'язкість осаду тощо. На очисних спорудах застосовують наступні процеси обробки осадів стічних вод: ущільнення, стабілізацію, кондиціонування, зневоднення, сушіння, термічну обробку, утилізацію цінних цінних продуктів або ліквідацію осадів. Важливим етапом в утилізації осадів є їх зневоднення, який дає змогу значного зменшення обсягів осадів. Здатність осадів стічних вод стискуватися під дією зовнішнього тиску є однією з характерних його властивостей. Проведено моделювання методом лінійних пропорційностей, що дозволило отримати однакову функціональну залежність за різних комбінацій чисел подібності.

Ключові слова: зневоднення; вміст води; осади стічних вод; метод аналізу розмірностей; метод лінійних пропорційностей; числа подібності.