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Computer simulation of dynamic processes in power skiving gear cutting

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The paper presents the results of simulation of the dynamic processes during gear cutting using the power skiving method. Oscillations in the elastic system of the machine tool have been studied using the Simulink Matlab system. The closed dynamic system is represented as a series-coupled connection of the elastic system of the cutting process and the elastic system of the machine tool, which is assumed to be two-mass, with masses applied to the table assembly with the workpiece and to the tool spindle. The input signals of the cutting process system – the thickness of a single tooth cut and the total chip thickness in continuous cutting – are modeled on the basis of a previously developed computer modeling system for this process. It is shown that the combination of these systems makes it possible to select the maximum permissible values of the cutting conditions that satisfy the condition of stable operation of the machine tool and maximum reduction of machining time.

Keywords: *power skiving; dynamic system; simulation; transfer function; chatter; instability cutting conditions; adjustment.*

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1. Introduction

Gears and transmissions will remain indispensable components of modern machines and drives for the foreseeable future. Due to their extensive application in mechanical engineering, considerable attention is devoted to their enhancement and manufacturing technologies. One of these technologies that is becoming increasingly widespread is gear turning, now known as power skiving. Known since the beginning of the twentieth century, this technology and its method of tooth formation have received a new impetus thanks to advances in machine tool engineering, the development of CNC drives and tooling. However, despite the attention paid to the subject by scientists and engineers and the numerous publications that have been produced, there are still a number of problems with its implementation and improvement, which is why the subject of this article is relevant.

2. Literature analysis

The analysis of recent research dedicated to the power skiving process has revealed the following trends. Much of the researchers' attention is focused on mathematical modeling and computer simulation of this process [1–3]. At the initial stage of describing the processes and phenomena accompanying the gear cutting process, models are developed to determine the three-dimensional structure of undeformed chips. It is produced by the skiving cutter and their parameters – cross-sectional area, thickness and width of the cut, chip length. In most cases, the mistake is made to equate the kinematics of power skiving with hobbing [3–5]. However, they are different processes. If in hobbing the angle between the cutting speed vector and the axis of the spur gear to be cut is 20° (Figure 1a), in power skiving this angle is 70°: in both cases the angle of engagement (profiling angle) is the same, equal to 20° (Figure 1b).

A typical error in the analysis of the kinematics of power skiving, which is repeated by researchers of this process, is described in the paper [6]. For a hob, the cutting motion is represented as the rotation of the cutter, whereas for power skiving the cutting motion is identified with the axial feed motion. In fact, in hobbing, the main motion is the rotation of the tool and the feed motion is an auxiliary motion. This is a fundamental position, any deviation from which distorts the actual kinematics of the power skiving process and leads to errors in chip modeling and subsequent calculations based on its parameters, in particular in the determination of cutting forces [4], kinematical angles [7], cutting heat and tool wear [2, 8]. The actual kinematic diagram of the two processes under comparison is shown in Figure 1 where the positions of the planes in which the cutting speed vectors lie in the form of their traces are indicated. Its use is of great practical importance for an adequate assessment of the dynamics of the power skiving process.

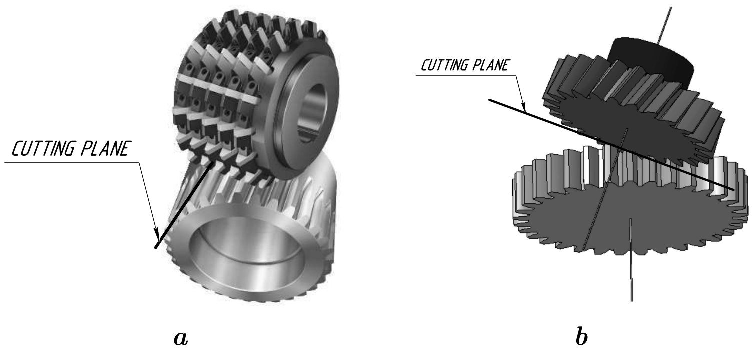


Fig. 1. Differences in kinematics between hobbing and power skiving.

One of the unsolved problems today is to establish a connection between the cutting conditions, which would ensure the minimum operating time and maximum process productivity, and the conditions of stable and vibration-free operation of the machine. Since there is no information on how to solve this problem at the theoretical level, it can be assumed that in practice it is solved only experimentally, by trial and error. Since power skiving technology is mainly used in the mass production of gears, the task of reducing the time required for gear machining is relevant, and the development of a methodology for solving it will reduce production costs.

3. Aim

Development of a methodology to determine the maximum allowable values of feed, depth of cut and number of passes and their rational combination to reduction gear machining time whilst ensuring stable machine operation..

4. Research results

4.1. Dynamic system of a skiving machine

The analysis of the dynamic processes taking place during cutting is based on the principle of the closed loop elastic system of the machine tool, whose components are the cutting process, which has its own elastic system (ES_CP), and the equivalent elastic system of the machine tool itself (ES_MT). Let us take a two-circuit machine system, where the masses are reduced to a tool spindle and a worktable with a gear.

The input to the cutting system is the thickness of the cut and the output is the cutting force. In turn, the cutting force is an input signal for the elastic system of the machine, at the output of which mechanical oscillations occur. Assuming linearity of both systems, the machine's chatter are equal to the sum of the maximum vibrations at each of its natural frequencies.

It is well known that the occurrence of cutting forces and transients in ES_CP is a consequence of the complex processes of plastic deformation of the cut layer as it is transformed into chips and contact friction, and this process itself has certain inertia. As a result, the cutting force lags behind the instantaneous change in thickness of the layer being cut, and this lag is a source of internal energy that causes self-oscillations in the machine's elastic system. The change in force by the thickness of the cut is characterized by an aperiodic link with the time of formation of the cutting force, and at the output of the SR system there is an oscillatory process in the form of a cutting force with a certain amplitude and frequency.

An instantaneous change in the thickness of the layer to be cut at the start of cutting is a single action that causes self-excited oscillations. Another cause of harmonic oscillations in the machine tool system is a periodic change in the thickness of the layer being cut, which is characterized by its own parameters – frequency and amplitude.

In a closed dynamic system of a machine tool, the elastic system of the cutting process plays a negative feedback role. Its essence is that as the cutting depth increases, the cutting force generated in this system increases and the elastic deformations increase. As a result of the elastic compression, the cutting depth decreases, leading to a decrease in the cutting force and elastic deformations and a new increase in the cutting force.

4.2. Transfer functions of dynamic subsystems

4.2.1. Dynamic system of the cutting process

Using the main provisions of the theory of automatic regulation, we present the links of the ES_CP and ES_MT systems in the operator form.

The transient process of forming the cutting force P along the cut thickness a is non-oscillating and is represented graphically by an exponent, the argument of which is the time constant T_p. The transfer function of the cutting process is described by Equation:

$$W = \frac{P}{a} = \frac{K}{1 + T_P\, p}, \tag{1}$$

where K is transmission coefficient if the link.

The transient time can be determined experimentally (Figure 2).

In this work, the transient time constant T_p, the cutting force, has been determined using the DEFORM-2D system for the conditions corresponding to this article: 325 MPa shear strength limit, 10° tool rake face angle and 100 m/min cutting speed.

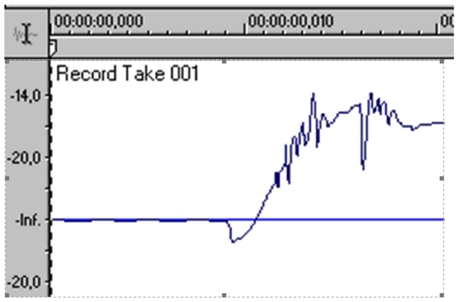


Fig. 2. Diagram of the experimentally obtained transient aperiodic process of the cutting force.

The second factor affecting the elastic system of the cutting process and the machine tool is the periodic change in the parameters of the cutting process, in particular the total chip thickness, which causes periodic fluctuations in the cutting force acting on the machine tool. The reaction of the elastic system of the machine tool to a change in such an input fluctuation of the cutting force is mechanical oscillations in this system, the parameters of which depend on its dynamic properties. The determination of the function of the change in the total thickness of the cross-section was solved on the basis of the system developed by the authors for modeling undeformed chips in the process of power skiving [9].

4.2.2. Dynamic system of the machine tool

The transfer function (dynamic characteristic) of the machine links in the operator form is described by Equation:

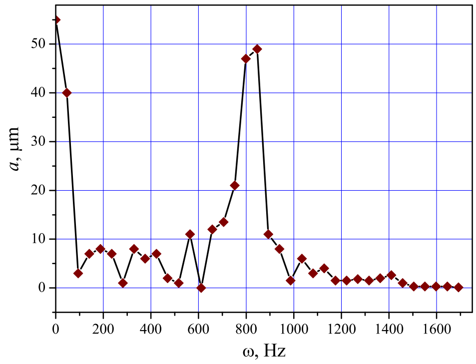
$$W_i = \frac{y}{P} = \frac{K}{T_m^2 p^2 + T_d p + 1}, \tag{2}$$

where K is the coefficient brings the pliability of the table and the spindle to the plane of the gear face. $K_1 = \frac{\cos\beta}{J_g}$ – for spindle; $K_2 = \frac{\cos\beta}{J_s}$ – for table, J_g, J_s are stiffness of the table and spindle, respectively (N/mm); β is the angle of intersection of the table and tool spindle axes.

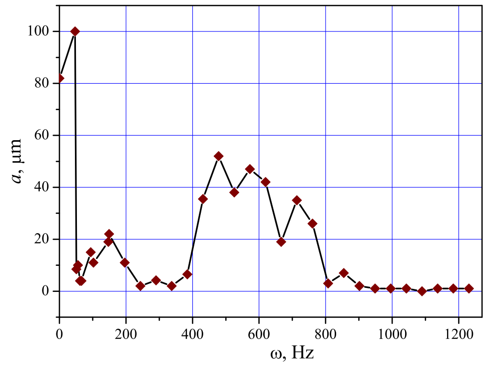
The arguments of both harmonic functions characterizing the oscillations of the table and spindle are inertial constants T_{m1} and T_{m2}. The time constants of dissipation (damping) are T_{d1} and T_{d2}.

This study aims to improve the machining accuracy by investigating the radial oscillations of the elastic system of the skiving machine. These oscillations cause periodic changes in the center distance between the tool and the gear, resulting in engagement errors.

Chatter in the elastic system of a machine tool occurs in the direction perpendicular to the force causing it, according to the basic principles of machine tool dynamics [10].



a (natural frequencies in the tool circuit)



b (natural frequencies in the table circuit)

Fig. 3. Amplitude-frequency response of a hobbing machine mod. 5E32.

It is possible to detect the machine's own vibrations by scanning with frequency variation over a wide range, or by passing a broadband signal of the "white noise" type through an elastic system. Considering that the maximum vibrations occur at low frequencies, which have the highest spectral density, the frequency range can be between 5 and 1000 Hz. Figure 3 shows the amplitude-phase response of an average hobbing machine 5E32, whose natural frequencies in the tool circuit are 3 Hz and 882 Hz. Natural frequencies are 96 Hz and 715 Hz in the table circuit. From the value of the natural frequency ω we can find the inertial constant of the equation of motion of the corresponding mass: $T_m = \frac{2\pi}{\omega}$, s. Let us assume that the natural frequencies of the power skiving machine correspond to these parameters of the gear milling machine. Then, for the given initial data, $T_{m1} = 0.0088$ s, $T_{m2} = 0.0071$ s.

The dissipation time constant T_{d1} can be calculated from Equation:

$$T_{d1} = \frac{\lambda}{\pi} \cdot T_m, \tag{3}$$

where λ is the logarithmic decrement of the vibration damping at the corresponding frequency, it characterizes the damping rate of free vibrations:

$$\lambda = \log \frac{a_1}{a_2}, \tag{4}$$

where a_1 and a_2 are the previous and next amplitudes.

Taking into account that the vibration stability of a skiving machine is much higher than that of a hobbing machine, we assume that the damping time constant in the table chain is 0.0003 s (logarithmic decrement of damping 0.107) and in the spindle chain is 0.0002 s (logarithmic decrement of damping 0.187).

Based on the above, the transfer function of the open-loop elastic system of the machine tool is given by the formula:

$$W = W_C \cdot W_M = \left\{ q \cdot b \cdot \frac{a}{1 + T_C p} \right\} \cdot \left\{ K_3 \cdot \left[\frac{1}{J_g} \cdot \frac{1}{T_{11}^2 p^2 + T_{21} p + 1} + \frac{1}{J_s} \cdot \frac{1}{T_{12}^2 p^2 + T_{22} p + 1} \right] \right\}, \tag{5}$$

where q is the specific cutting force, N/mm^2; b is average cutting width, mm; J_g is the static rigidity of the work table with a device for fixing the workpiece in the tangential direction, J_s is the static rigidity of the machine spindle, N/mm; K_3 is gear ratio that takes into account the angular position of the tooth, which eliminates maximum chips and brings the cutting force and chatter into the plane of the center line perpendicular to the gear face and the cutting speed vector.

The average specific cutting force is equal to the product of the average chip thickness ratio and the shear limit strength $[\tau]$, MPa: $q = \xi \cdot [\tau]$. Data for parameter ξ are given in [11].

The stiffness of a link with reduced mass m and natural frequency ω can be determined from Equation $T_m = 10^{-1} \cdot \sqrt{\frac{m}{J}}$. For the data given above, the stiffness of the table is 28 kN/mm and that of the spindle is 31 kN/mm.

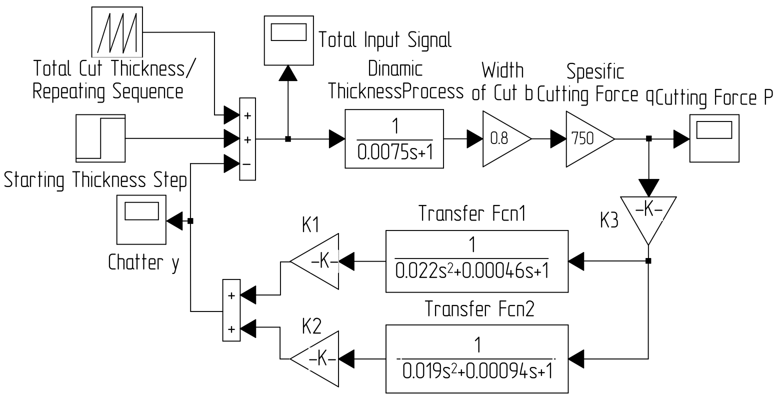


Fig. 4. Diagram of a closed dynamic system of a machine tool with a cutting process.

The diagram of the dynamic system of the machine with the cutting process in the Simulink Mathlab system is shown in Figure 4.

5. Modeling of dynamic process

We will use the results of the study of cutting parameters given in [12], where the process of cutting gears in three passes has simulated.

Initial data: it is a spur gear; module 2.5 mm; number of teeth: gear 33, cutter 24; axial feed 0.75 mm/rev; cutting speed 190 m/min; rotational frequency of tool 931 rpm; rotational frequency of gear 667 rpm; number of passes three; depth per pass: 1 mm, 1.5 mm and 2.5 mm; cutter insert material: titanium-tantalum carbide; tool outer diameter 66 mm; cutter tooth angle and axis crossing angle 25°; coefficient of friction on the face at this speed 0.63.

In accordance with the methodology for modeling 3D undeformed chips and cutting forces [1] and the initial data, Figure 5*a* shows the cut thickness plots for single tooth cutting for three passes and a plot characterizing the change in total thickness in continuous cutting in the second pass, where the highest cutting force occurs. The change in the thickness of the cut layer is characterized by a jump in the input variable of the cutting process, which causes self-oscillation; the cutting force time for these conditions is 0.005 s. The change in the total thickness of the cuts (Figure 5*b*) characterizes the patterns of periodic oscillation at the input of the elastic system of the cutting process, represented in the diagram by the "Repetitive Sequence" block. In the third pass, the maximum cut thickness for single tooth cutting is 0.653 mm and the maximum total chip thickness is 0.76 mm; the average cut width is 2.2 mm.

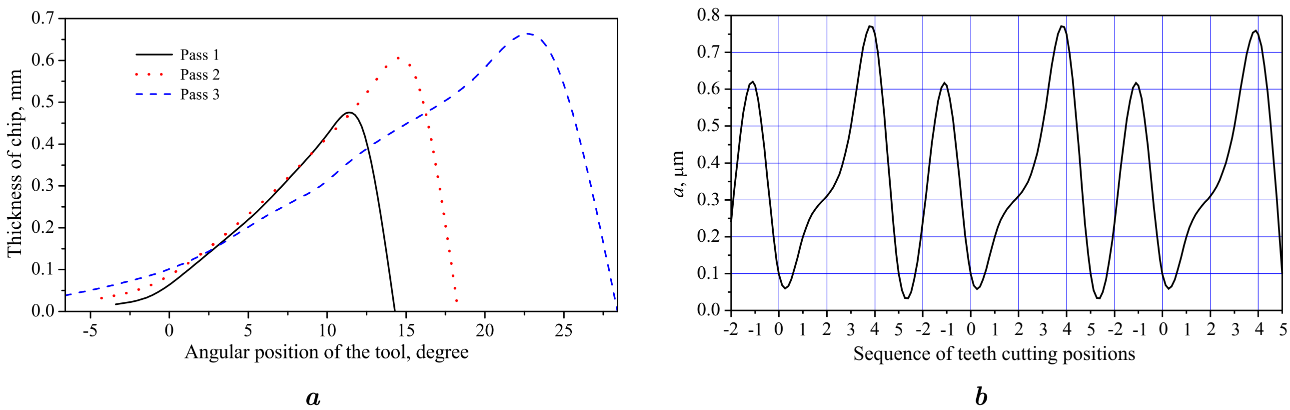


Fig. 5. Thickness of the cuts on the passes in single tooth cutting (***a***) and the change in total thickness in continuous cutting in the second pass (***b***).

We will use the Simulink Matlab system to verify the correctness of the cutting modes used in this work. Simulation of the dynamic processes for the above initial data shows that under such conditions instability will occur in the elastic system of the machine tool and the cutting force and elastic vibrations will increase (Figures 6, 7). This means that the previously selected parameters are overestimated and it is necessary to correct those that characterize the cutting process or the machine tool.

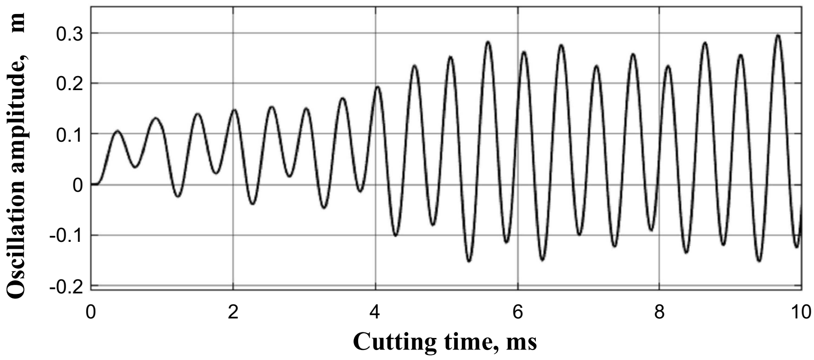


Fig. 6. Total chatter at the input of the elastic system of the cutting process.

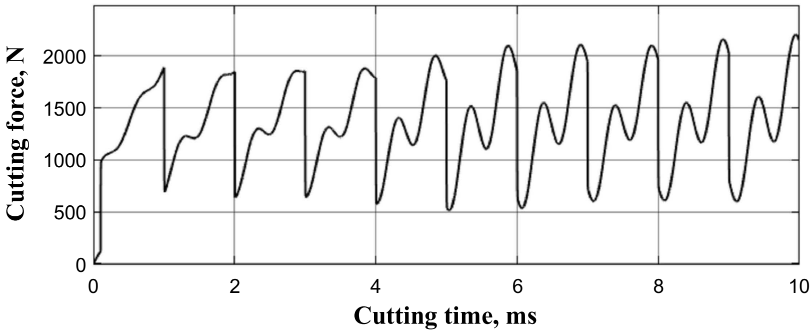


Fig. 7. Loss of stability in the dynamic system of the machine tool in terms of cutting force.

The solution to this problem can be found by selecting a machine with greater rigidity in the table and spindle chain with the cutting conditions unchanged, or by reducing the cutting conditions. In turn, the cutting conditions can be adjusted either by reducing the axial feed rate or by reducing the depth of cut by increasing the number of passes.

Simulation of the oscillations using the Simulink Mathlab system and a selection of options showed that the stability of the machine's elastic system is ensured at a maximum allowable feed rate of 0.24 mm/rev and three passes with an average depth of cut of 1.875 mm, or at a maximum feed rate of 0.31 mm/rev and four passes with an average depth of cut of 1.4 mm. The average transient time of the cutting force will be 0.0018 s. In both cases we will obtain chatter stabilization, the cutting force for the second option is shown in Figure 8.

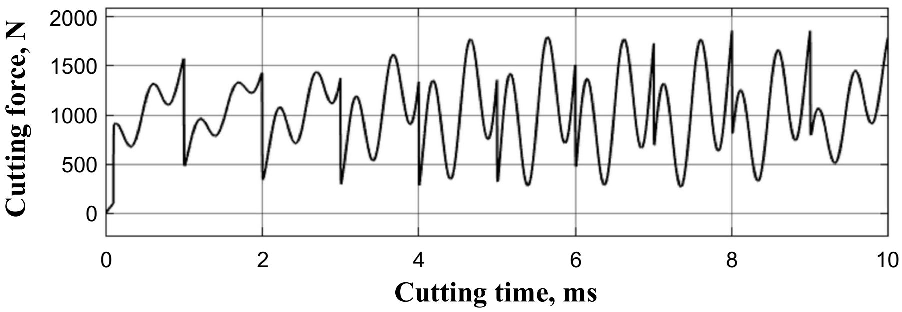


Fig. 8. Stabilization of cutting force after axial feed reduction.

Figure 9 shows the total continuous cutting force modeled for the same data using the method described in [12]. The comparison of the simulation results of the two methods shows a coincidence in the form of oscillatory processes and in the value of the cutting force.

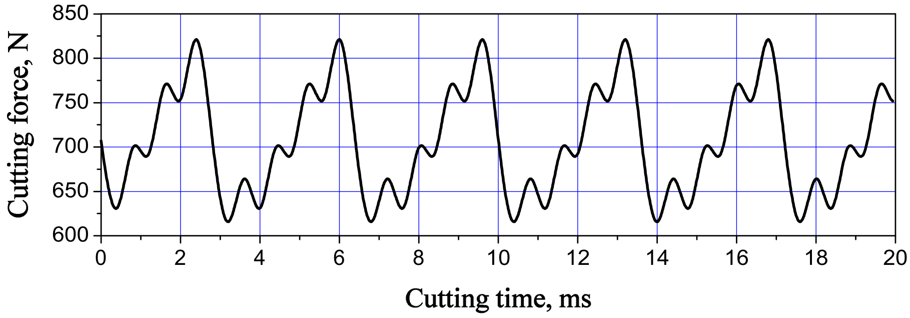


Fig. 9. Total continuous cutting force.

Choose the most efficient option for the operation. To choose the best of two options, you need to compare their performance. The gear machining time is equal to

$$t = \frac{L_{cut}}{f_{\min}} \cdot i = \frac{L_{cut}}{f_{ax} \cdot n} \cdot i = 10^{-3} \cdot \frac{\pi \cdot D \cdot L_{cut}}{V \cdot f_{ax}} \cdot i, \tag{6}$$

where f_{ax} is rate of axial feed, mm/rev.; V is cutting speed, m/min; D is gear outer diameter, mm; L_{cut} is total cutting length, mm; i is number of passes; n is rotational frequency of tool.

If we assume a rim width of 22 mm and an axis crossing angle of 200, the total cutting path, including the cut-in lenticule and the tool exit lenticule, will be 70 mm. At a cutting speed of 100 m/min, a feed rate of 0.24 mm/rev and a number of passes of 3, the time to cut one gear will be 1.79 minutes. For the second variant, with a feed rate of 0.31 and four passes, the cutting time will be 1.84 minutes. This difference in machining time will be significant in mass production. If the annual programme of gears is 3000 units, the time reduction for the first variant of the operation will be 150 minutes, or 2.5 hours.

6. Conclusions

A comprehensive machine tool stability assessment system has been developed for the selection of rational cutting modes in the process of cutting gears by the power skiving method. The system is based on a previously developed model of a 3D undeformed chip, the determination of its parameters and cutting forces. On the basis of this model, the parameters that cause self- and forced oscillations in the elastic system of the power skiving machine tool are set – the thickness of the single tooth cut and the total thickness during continuous cutting.

On the basis of these dependencies, the dynamic processes in the elastic system of the cutting process and in the elastic system of the machine tool were modeled using the Simulink Matlab system and the limits of feed, depth of cut and number of passes were found at which the machine tool remains stable and at which the minimum machining time and maximum process productivity are achieved.

It was found that at high cutting speeds in excess of 100 m/min, cutting with a lower feed and fewer passes at a greater depth of cut is more efficient.

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Комп'ютерне симулювання динамічних процесів при нарізанні зубчастих коліс методом Power Skiving

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Наведено результати моделювання динамічних процесів при нарізанні зубчастих коліс методом power skiving. Коливання в пружній системі верстата досліджували з допомогою системи Simulink Matlab. Замкнену динамічну систему представлено як послідовно зв'язані ланки пружної системи процесу різання і пружної системи верстата, яка прийнята двохмасовою, з приведення мас до вузла стола з заготовкою і до інструментального шпинделя. Вхідні сигнали системи процесу різання — товщина однозубого різання зрізу і сумарна товщина стружки в неперервному різанні змодельовані на основі розробленої раніше системи комп'ютерного моделювання даного процесу. Показано, що поєднання даних систем дає можливість вибирати гранично допустимі значення режимів, які задовольняють умову стабільної роботи верстата і максимального скорочення часу оброблення.

Ключові слова: *зуботочіння, динамічна система, симуляція, перехідна функція, вібраційна нестабільність, нестійкі умови різання, корекція.*