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MATHEMATICAL MODELING OF HEAT AND MASS TRANSFER PROCESSES IN A RECIPROCATING ENGINE OF A VEHICLE WITH A HYBRID POWERTRAIN

Summary. *Automotive transport plays a crucial role in the functioning and development of any country's economy. In Ukraine, it accounts for over half of passenger transportation and three-quarters of freight transportation. A promising development direction is using electric and hybrid vehicles in transportation logistics. It also fosters advancements in battery production technologies, components of hybrid power units, recycling, and the country's transportation infrastructure. Modern vehicle hybridization combines the advantages of traditional internal combustion engines (ICE) and electric drives. The efficiency of hybrid power units can be considered from design and thermodynamic perspectives. The design approach requires the development of new materials and manufacturing technologies, necessitating significant resource expenditures. The thermodynamic approach involves modeling and optimizing thermal processes occurring in the ICE within hybrid power units. The aim of this study is to identify opportunities for improving heat and mass transfer processes in a reciprocating engine to ensure the energy efficiency of a vehicle's hybrid power unit. Heat and mass transfer processes in the ICE are described by a system of differential equations that account for heat transfer in various environments (working gas, cylinder walls, coolant), considering key parameters such as wall temperatures, gas temperatures, heat transfer coefficients, and combustion kinetics. Several scenarios were examined to study the overall heat and mass transfer process. The first scenario assumes constant temperatures of gases and ICE walls, resulting in a steady heat transfer coefficient. The second scenario involves overloading, leading to increased heat loss through the walls and elevated thermal stress on the cooling system. The third scenario considers a decrease in ambient temperature. This study modeled the dependence of engine wall temperatures over time for these three operating conditions, enabling control of thermal modes and prediction of ICE performance to enhance the efficiency of the vehicle's hybrid power unit. It was found that increasing the temperatures of gases and walls affects engine operation duration, the effectiveness of recovered heat utilization, and the optimization of hybrid power unit performance. The more heat recovered during engine operation, the longer it operates with minimal heat loss and maximum efficiency.*

Key words: *heat and mass transfer, reciprocating engine, hybrid power unit, energy efficiency, operating conditions.*

1. INTRODUCTION

Automotive transport plays a vital role in the functioning and development of any country's economy. In Ukraine, it accounts for over half of passenger transportation and three-quarters of freight transportation [1]. The use of automotive transport in Ukraine has its peculiarities:

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- rapid growth in motorization levels;
- technical and/or moral obsolescence of the vehicle fleet;
- high consumption of scarce resources;
- significant negative environmental impact, contributing to nearly 40 % of total anthropogenic environmental pollution, surpassing any other industrial sector [1].

The internal combustion engine (ICE), as the primary power unit of vehicles, consumes large amounts of automotive fuel and air during operation, exerts considerable chemical and thermal pressure on the environment, and operates very inefficiently under certain conditions [2].

The contradiction between the benefits of automotive transport and its harmful environmental impact amid resource scarcity is addressed through various means: improving vehicle designs, using high-quality automotive operational materials and new structural materials, and enhancing diagnostics, maintenance, and repair quality.

A promising direction for automotive transport development is the advancement of electric and hybrid vehicles.

The electrification of automotive transport, which began actively with the Nissan Leaf electric vehicles mass-produced by Nissan since 2010, addresses several automotive industry issues and reduces:

- the use of automotive fuels-gasoline and diesel;
- vehicle operating costs;
- the expenses for maintenance and current repairs;
- the environmental pollution during the operation of electric and hybrid vehicles.

Advancements in battery production technologies, government incentives, and infrastructure development facilitate the transition of automotive transport to electric and hybrid vehicles.

The use of hybrid power units in vehicles offers extensive opportunities to reduce fuel consumption, decrease harmful emissions into the environment, and improve overall vehicle efficiency [2].

2. STATEMENT OF THE PROBLEM AND RELEVANCE OF THE STUDY

There is an ongoing hybridization of road transport, combining the advantages of a conventional internal combustion engine and an electric drive.

Hybrid power units comprise a combination of engines operating on different physical principles.

Hybrid vehicles typically include two types of engines: an internal combustion engine (e.g., gasoline or diesel) and an electric motor. Depending on various factors, such engines can work in tandem or separately [2].

The main component of hybrid systems is the reciprocating engine, where complex heat and mass transfer processes occur. The efficiency of these processes largely determines the energy performance of the entire system.

Engine performance can be evaluated from both constructive and thermodynamic perspectives.

Improvements in engine design require new materials and manufacturing technologies, which involve significant resource investments.

On the other hand, thermodynamic optimization offers more accessible ways to improve engine characteristics through modeling and thermal process control.

In addition to modeling and optimizing thermal behavior, attention must be given to the challenges arising in reciprocating engines that operate as part of hybrid systems.

One major problem is the non-uniform thermal load – uneven heat distribution across cylinder walls. Long-term operation under such conditions can cause localized overheating and premature material wear. Sudden temperature fluctuations due to variable operation modes may also cause material cracking, alter the chemical stability of the cylinder block, and reduce engine life.

Heat losses through the cylinder wall and cooling system are inevitable and can significantly reduce engine efficiency. Current research focuses on new materials with improved thermal insulation or novel design solutions to minimize such losses.

Optimizing the cooling system is crucial to ensure efficient ICE operation, especially in hybrid systems. Insufficient cooling may lead to overheating, reduced engine life, or gasket failure between the cylinder head and the engine block.

Thus, the issue lies in the need for in-depth study and mathematical modeling of heat and mass transfer processes in reciprocating engines operating within hybrid power units. It includes heat transfer through cylinder walls, mass exchange with cooling liquids, and heat loss accounting.

Solving this problem is essential to ensure the reliability, durability, and energy efficiency of hybrid installations.

Given the rising demand for environmentally friendly technologies and the need to reduce harmful emissions, transitioning to cleaner transport systems and employing hybrid power units is highly relevant.

In addition, the use of evolving materials and technologies in producing hybrid vehicles requires further research into achieving a balance between fuel economy and ecological performance.

The working process of a reciprocating engine involves complex energy transformations, combining physical evaporation, chemical fuel combustion, and heat exchange. Heat transfer within the cylinder and through its walls to cooling systems is crucial in energy loss mechanisms.

The degree of thermal energy utilization is a measure of ICE energy efficiency and an indicator of its environmental impact.

Studying ICE working cycles provides valuable insights into how design and operational factors affect engine performance. It is expected that the results of this research will aid in designing new engines and optimizing existing ones.

Therefore, research into heat transfer, as described in this paper, and its modeling for powertrain optimization is significant and timely.

The aim of this study is to investigate the energy efficiency of a hybrid power unit combining an internal combustion engine and an electric drive, as well as to identify opportunities to improve heat and mass transfer processes in the reciprocating engine.

Research tasks include:

– conducting a critical review of current research on heat transfer and conduction;
– performing modeling of heat and mass transfer in a reciprocating engine;
– studying the influence of operational conditions on overall heat and mass transfer;
– analyzing the primary challenges of thermal exchange;
– assessing engine operation duration under various modes and temperature conditions;
– forecasting engine efficiency as part of a hybrid power unit.

3. ANALYSIS OF RECENT RESEARCH AND PUBLICATIONS

Recent studies reveal a growing interest in enhancing the efficiency and environmental friendliness of vehicles. The challenges and prospects of developing clean transportation are drawing increasing attention from researchers and practitioners in Ukraine and globally.

One of the most relevant global trends in automotive transport is the shift towards green mobility, including electric and hybrid vehicles.

In the study [2], an analysis of current innovations and trends in hybrid vehicle power units is presented. The concept of automotive powertrains and the issue of increasing their efficiency are discussed. The main operating modes of hybrid powertrains are electric , hybrid , and charging . The study also analyzes the challenges of improving fuel efficiency and reducing toxic gas emissions in modern automotive engineering.

Study [3] offers a critical and comprehensive comparison of studies and concepts related to various aspects of electric vehicle development, including market conditions, battery performance, electric motors, and electronic systems.

Authors of [4] conducted a comprehensive study of electric vehicle development and related infrastructure in Ukraine through the lens of global experience. The study analyzes the issues of ensuring sufficient electrical grid capacity, emphasizing the need for an extensive network of charging stations for electric vehicles.

Study [5] provides a comparative analysis of various engine types used in modern vehicles. It explores the design and construction features of internal combustion engines, electric vehicles, and hybrid vehicles and evaluates their future application prospects.

In [6], the authors analyze combinations of ICE fuel types-gasoline, diesel, and gaseous fuels-and propose a neural approach to optimizing fuel selection based on operating conditions. The neural method allows adaptive engine control, enhancing efficiency under varying loads.

The automated methodology for calculating the optimal number of vehicles in a company's fleet is developed in [7]. It allows analysis of cost changes depending on fleet composition and supports data-driven decision-making during transitions to electric mobility. Recommendations for enterprise development options help reduce vehicle operation costs and improve competitiveness under economic crisis conditions, energy resource shortages, and high environmental pollution.

Fundamentals of internal combustion engine theory, including definitions of their performance indicators and characteristics, are presented in [8]. The study reviews ICE classifications, thermodynamic cycles, fuel mixtures and combustion products, and the real-cycle processes of ICEs.

In [9], the main concepts of electric vehicles are described. The study reviews components and energy storage systems used in electric vehicles and analyzes batteries as key components ensuring environmental sustainability and economic efficiency. It also discusses hybrid combined technologies based on various energy storage combinations, which may have promising applications.

Study [10] explores different ICE operating strategies in hybrid vehicles and demonstrates potential fuel savings and emission reductions. An optimized hybrid model showed reductions of 47.2 % in CO_2 emissions and 20.7 % in NO_x emissions under real-world driving cycles.

According to [11], 99.8 % of the world's transport currently runs on internal combustion engines. It highlights the relevance of improving ICE efficiency and sustainability. The authors consider various alternatives, including alternative fuels, electric vehicles, and hybrids. They suggest that by 2040, 85–90 % of transport energy will still come from conventional liquid fuels. Therefore, ICE improvement and hybridization technologies offer the potential for reducing fuel consumption by up to 50 % compared to the average for passenger cars.

A study [12] emphasizes that heat transfer between combustion gases and cylinder walls is one of the most important but least understood phenomena in ICEs. The authors propose a new model based on one-dimensional heat conduction to characterize and predict heat transfer in ICEs. The model demonstrated a relatively low error of 10.2 %, offering greater predictive accuracy than traditional models.

In [13], the results of modeling and experimental investigations of heat exchange in engines with port fuel injection are presented. The study examined heat release in the cylinder and temperature distribution. Heat exchange with the piston, cylinder head, and cylinder was studied, and combustion was compared between diesel and port injection engines.

Modeling and thermal management of hybrid vehicles are covered in [14]. The authors explore thermal modeling capabilities and novel methods for creating a comprehensive vehicle model. Temperature analysis supports thermal optimization and energy distribution to enhance vehicle performance, improve efficiency, and reduce emissions.

Transitioning to electric and hybrid vehicles requires thermal problem analysis to improve vehicle efficiency [15]. The authors provide a systemic engineering perspective on thermal management, examining how each subsystem contributes to and interacts with the overall vehicle system. They address modeling, simulation, and optimization of thermal processes at both vehicle and component levels.

Study [16] presents a comprehensive analysis of design and control optimization levels for electrified vehicles. It addresses energy management and thermal control strategies in electric vehicles. The analysis demonstrates that energy and cost savings can be achieved through optimized vehicle design considering both energy and thermal domains.

Finally, in [17], an analysis of heat transfer and cylinder liner optimization using various materials is presented. Autodesk and Ansys software were used for modeling. The influence of temperature gradients on thermal stress, wear resistance, and corrosion resistance was studied for materials including cast iron, magnesium alloy, and titanium alloy, with 0.5 mm, 1 mm, and 1.5 mm liner thickness variations. The results showed that titanium alloy exhibited the highest heat transfer rates and thermal flux compared to cast iron and magnesium alloy.

4. MAIN CONTENT PRESENTATION

The processes of heat and mass transfer in a reciprocating engine are described by a system of differential equations that take into account heat transfer in different environments (working gas, cylinder walls, and coolant). The main physical parameters are wall temperature, gas temperature, heat transfer coefficient, and combustion kinetics.

Consider the differential equations for general heat and mass transfer in a reciprocating engine. We describe the general heat and mass transfer processes in a reciprocating engine using a system of differential equations that consider heat exchange between the gases in the engine cylinder and its walls [1].

The energy balance for the gases is described by the following formula [8]:

$$\frac{d}{dt}(m \cdot c_p \cdot T_g) = \dot{Q}_{intake} - \dot{Q}_{comb} - \dot{Q}_{loss}, \quad (1)$$

where m – mass of the working medium, i.e., fuel mixture, kg; c_p – specific heat capacity at constant pressure, J/(kg·°C); T_g – temperature of gases in the cylinder, °C; $\dot{Q}_{intake}$ – thermal power entering during intake, W; $\dot{Q}_{comb}$ – heat released during fuel combustion, W; $\dot{Q}_{loss}$ – power loss due to heat transfer through cylinder walls, *W*.

Then the heat transfer between the gases and the cylinder walls is described by the general heat transfer equation:

$$\dot{Q}_{loss} = h \cdot A(T_g - T_w), \quad (2)$$

where h – heat transfer coefficient, *W*/(m²·°C); A – surface area of the cylinder, m²; T_w – temperature of the cylinder walls, °C.

The next step in determining the equations is cooling the cylinder walls. The temperature of the cylinder walls changes as a result of heat exchange with the coolant:

$$\frac{dT_w}{dt} = \frac{\dot{Q}_{loss} - h_w \cdot A_w \cdot (T_w - T_{cool})}{m_w \cdot c_w}, \quad (3)$$

where h_w – heat transfer coefficient to the coolant, W/(m²·°C); A_w – surface area of walls in contact with the coolant, m²; T_{cool} – coolant temperature, °C; m_w, c_w – mass and specific heat capacity of the cylinder walls, respectively.

Several scenarios are considered to study the impact of operating conditions on the general heat and mass transfer process.

Scenario one is nominal ICE operation, where gas and wall temperatures remain constant, resulting in a stable heat transfer coefficient.

Scenario two is engine overload, where heat loss through the walls increases, and the thermal load on the cooling system rises.

Scenario three is low ambient temperature. The reduced ambient and coolant temperatures require analysis of the correction chart of heat transfer coefficient, which may lead to engine overcooling.

The relationships between operating conditions and engine wall temperatures can be represented graphically based on the research.

Let the dependence of engine wall temperature on time under three different operating conditions (three scenarios) be constructed.

Fig. 1 presents the wall temperature dependence on time for various operating conditions:

– under nominal ICE operation, the temperature oscillates around 100°C (coolant temperature), with minor fluctuations, reflecting stable engine performance;
– under overload conditions, with coolant temperature increased to 150°C, more pronounced fluctuations indicate high thermal stress on the engine;
– under reduced coolant temperature, lower values (around 80°C) with minimal fluctuations are observed, characteristic of operation in cold environments.

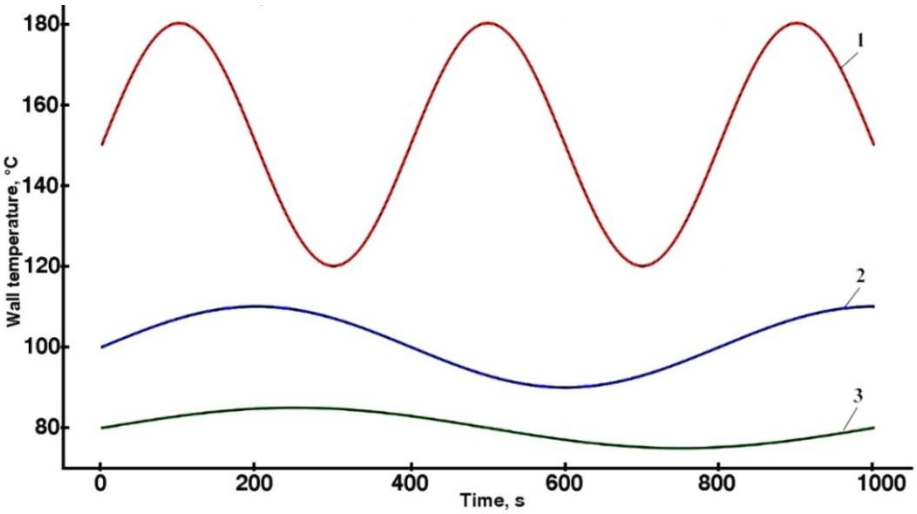

Fig. 1. Dependence of the temperature of the internal combustion engine walls on its operating time for different possible operating conditions: 1 – overload; 2 – nominal mode; 3 – reduced ambient temperature.

Assume the influence of three operating conditions on the thermal state of the engine, which will help to better understand the need to control temperature modes to improve the efficiency of a hybrid power plant.

More complex mathematical equations can be used to solve the problems of general heat and mass exchange in piston engines, which consider heat transfer through the walls and mass exchange with the environment. For example, the following equation can be used to model heat losses through the walls and optimize cooling [8, 18]:

$$\frac{dT_g}{dt} = \frac{1}{m \cdot c_p}\left[\dot{Q}_{comb} - h \cdot A \cdot (T_g - T_w) - k \cdot (T_g - T_{cool})\right], \quad (4)$$

where k – heat transfer coefficient from gases to the coolant, W/(m²·°C).

Modeling heat and mass exchange also allows predicting the efficiency of the piston engine operation as part of a hybrid power plant. It is important to consider the influence of external conditions on the thermal balance, such as ambient temperature, engine load level, and fuel-air mixture quality.

Heat and mass exchange models can help predict the duration of engine operation under different modes and temperatures. For this purpose, the calculation of heat losses depending on operating conditions can be used.

One of the promising directions for increasing the efficiency of hybrid power systems is waste heat recovery. Using heat pumps or thermoelectric generators makes it possible to convert some heat losses into additional energy.

It is important to consider heat losses through the cylinder walls, the operation of the cooling system, and the possibility of recovering the regenerated heat to predict the efficiency of a piston engine as part of a hybrid power plant.

Consider a mathematical model based on a matrix equation that allows us to predict engine runtime, taking into account these parameters.

The engine operation process can be described as a system of equations that takes into account the main energy flows in the system. Suppose the matrix equation is written as follows:

$$X(t+1) = AX(t) + BU(t), \tag{5}$$

where $X(t)$ – state vector of the system at time (t); A – system coefficient matrix characterizing thermal processes in the engine; B – external influence matrix accounting for recovered heat flows and other external parameters; $U(t)$ – input vector, such as coolant temperature, recovered heat power, and other external conditions.

Then, the matrix description of the general heat and mass exchange system (state vector $X(t)$) includes the following main parameters [18]:

$$X(t) = \begin{bmatrix} T_g(t) \\ T_w(t) \\ Q_{loss}(t) \\ E_{rec}(t) \end{bmatrix}, \tag{6}$$

where $T_g(t)$ – gas temperature in the cylinder at time t, °C; $T_w(t)$ – cylinder wall temperature at time t, °C; $Q_{loss}(t)$ – amount of heat loss through the walls, W; $E_{rec}(t)$ – amount of recovered heat that can be used for additional system power supply, J.

The matrix equation for predicting temperature and engine runtime is written as:

$$\begin{bmatrix} T_g(t+1) \\ T_w(t+1) \\ Q_{loss}(t+1) \\ E_{rec}(t+1) \end{bmatrix} = \begin{bmatrix} 1-h\dfrac{A}{m\cdot c_p} & h\dfrac{A}{m\cdot c_p}1 & 0 & 0 \\ h\dfrac{A_w}{m_w\cdot c_w} & 1-h_w\dfrac{A_w}{m_w\cdot c_w} & 0 & 0 \\ 0 & 0 & 1 & -\eta \\ 0 & 0 & \eta & 1 \end{bmatrix} \cdot \begin{bmatrix} T_g(t) \\ T_w(t) \\ Q_{loss}(t) \\ E_{rec}(t) \end{bmatrix} + \begin{bmatrix} \dfrac{Q_{comb}}{m\cdot c_p} \\ 0 \\ 0 \\ \dot{Q}_{use} \end{bmatrix}, \tag{7}$$

where h, h_w – heat transfer coefficients for gases and cylinder walls, respectively; A, A_w – surface areas for heat exchange; m, c_p – mass of gases and their specific heat at constant pressure; m_w, c_w – mass and specific heat of the cylinder walls; η – efficiency of heat recovery; Q_{comb} – heat released during fuel combustion; $\dot{Q}_{use}$ – amount of recovered heat used for additional power supply.

From equation (7), it is possible to predict the period during which the engine will operate with minimal heat losses and maximum efficiency.

For example, if the amount of heat $E_{rec}(t)$ increases, this contributes to a reduction in energy losses, and accordingly, the engine can operate longer without needing external recharging. The recovered heat can be used to improve the overall engine efficiency by powering auxiliary systems or even the electrical part of the hybrid power unit. It extends engine runtime and reduces the load on the primary fuel source.

Thus, using a matrix-based approach allows effective modeling of thermal processes in the engine, predicting its runtime, and optimizing the use of recovered heat to improve the efficiency of the hybrid power plant.

We will build a graph to predict the engine runtime based on the parameters: T_g – gas temperature in the cylinder; T_w – cylinder wall temperature; $E_{rec}(t)$ – amount of recovered heat, where this approach will allow visualization of how changes in these parameters affect the engine's runtime (Fig. 2).

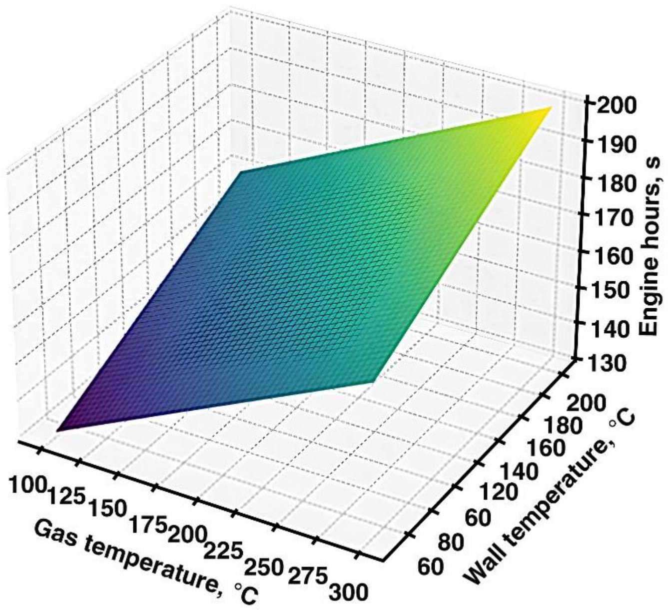

Fig. 2. Forecast of the duration of operation
of an internal combustion engine using recovered heat

Fig. 2 illustrates the dependence of engine operation duration on two key parameters: the temperature of the gases within the cylinder and the temperature of the cylinder walls. The amount of recovered heat, which influences the engine's operating time, is also considered. The figure demonstrates that as the temperature of the gases and cylinder walls increases, the engine operation time also increases due to more efficient utilization of recovered heat. This observation is valuable for further analysis and optimization of hybrid power systems.

Fig. 3 presents the forecasted engine operating time as a function of recovered heat utilization.

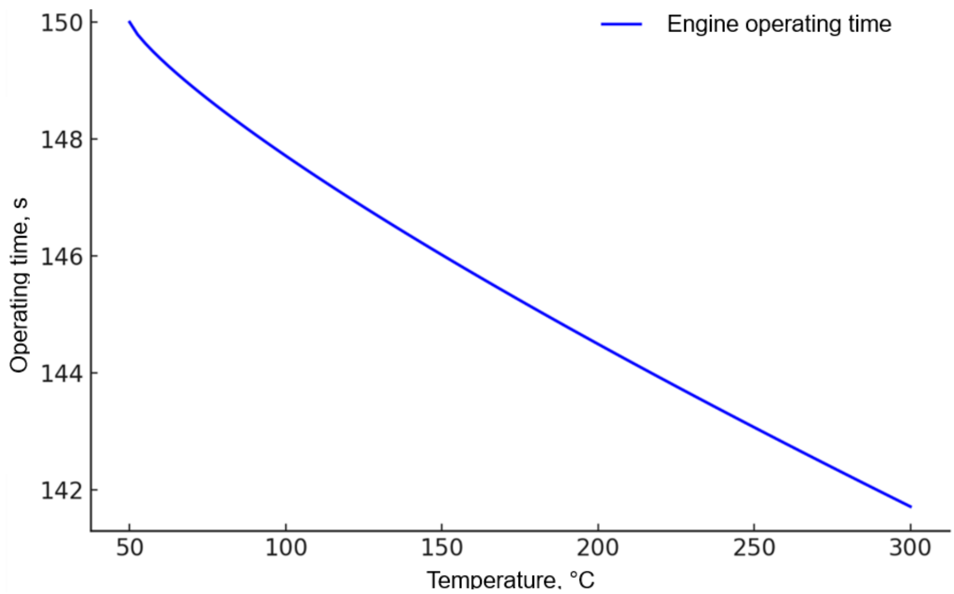

Fig. 3. Forecasting the duration of operation
of an internal combustion engine using recovered heat

The greater the amount of heat recovered during engine operation, the longer it can operate with minimal heat losses and maximum efficiency.

5. CONCLUSIONS AND PROSPECTS FOR FURTHER RESEARCH

The automotive industry is actively transitioning from internal combustion engine (ICE) vehicles to electric and hybrid vehicles, reflecting global trends in adopting environmentally friendly technologies driven by objective environmental, energy, and economic factors.

A key feature of hybrid vehicles is the combination of two different types of engines – an internal combustion engine and an electric motor, which allows for increased fuel efficiency, reduced emissions of harmful substances into the environment, and improved ecological performance. The combined operation of ICE and electric motors in hybrid vehicles enables the achievement of an optimal balance between fuel efficiency and environmental friendliness while also ensuring the reliability and longevity of hybrid systems.

The continuous development of automotive technologies and materials keeps the research and development of efficient hybrid power units relevant and critical. Solving this challenge offers technical, economic, and ecological benefits.

The overall heat and mass transfer in piston engines of hybrid power systems are complex and require careful research and modeling to improve their efficiency and reliability. Differential equations enable the study of how the thermal parameters of engines depend on operating conditions and allow for identifying ways to minimize heat losses.

This study includes modeling heat and mass transfer in a piston engine. It also examines the impact of different operating conditions (nominal ICE operation mode, engine operation under overload, and operation at reduced coolant temperatures) on the overall heat transfer process in the piston engine.

The dependence of engine operating time on the temperature of the gases in the cylinder and the temperature of the cylinder walls was investigated, taking into account the amount of recovered heat. The analysis shows that increasing gas and wall temperatures increases the engine's operating time due to more efficient use of recovered heat. This finding is beneficial for further analysis and optimization of hybrid power systems.

The proposed equation makes it possible to assess the impact of heat recovery on the thermal balance and efficiency of engine operation, which can be used to optimize the energy systems of hybrid vehicles further. Recovered heat can be utilized to enhance overall engine efficiency by powering auxiliary systems or supplying energy to the electric component of the hybrid power unit.

Further research may focus on optimizing cooling system designs, introducing new materials with improved thermal insulation properties, and efficiently utilizing waste heat to improve the overall efficiency of hybrid systems.

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МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ПРОЦЕСІВ ТЕПЛОМАСООБМІНУ У ПОРШНЕВОМУ ДВИГУНІ АВТОМОБІЛЯ З ГІБРИДНОЮ ЕНЕРГОУСТАНОВКОЮ

Анотація. *Автомобільний транспорт виконує важливу роль у функціонуванні та розвитку економіки будь-якої країни. В Україні автомобільний транспорт забезпечує більше ніж половину обсягу пасажирських перевезень і три чверті вантажних перевезень. Перспективним напрямом розвитку є використання електромобілів і гібридних автомобілів у транспортній логістиці. Крім того, це сприяє розвитку технологій виробництва акумуляторних батарей, складових частин гібридних енергоустановок, рециклінгу та транспортної* *інфраструктури країни загалом. Сучасна гібридизація транспортних засобів поєднує переваги традиційного двигуна внутрішнього згорання (ДВЗ) та електричного приводу, а ефективність гібридних енергоустановок можна розглядати у конструктивному та термодинамічному напрямах. Конструктивний напрям потребує розроблення нових матеріалів та технологій виготовлення, тобто значних витрат ресурсів. Термодинамічний напрям – моделювання та оптимізація теплових процесів, що виникають у ДВЗ у складі гібридних енергоустановок. Мета цього дослідження – виявити можливості удосконалення процесів тепломасопередачі в поршневому двигуні для забезпечення енергетичної ефективності гібридної енергоустановки автомобіля. Процеси тепломасообміну в ДВЗ описано системою диференціальних рівнянь, які враховують передавання тепла в різних середовищах (робочий газ, стінки циліндра, охолоджувальна рідина), для яких взято основні параметри – температуру стінок, температуру газів, коефіцієнт теплопередачі та кінетику горіння. Для дослідження процесу загального тепломасообміну розглянуто декілька сценаріїв. Перший сценарій – постійна температура газів і стінок ДВЗ, коефіцієнт тепловіддачі сталий. Другий – перевантаження, коли відбувається збільшення втрат тепла через стінки, підвищення теплового навантаження на охолоджувальну систему. Третій – зниження температури навколишнього середовища. У цій роботі змодельовано залежність температури стінок двигуна від часу для трьох умов експлуатації (трьох сценаріїв), що дало змогу контролювати температурні режими та прогнозувати ефективність роботи ДВЗ задля підвищення ефективності роботи гібридної енергоустановки автомобіля. Встановлено, що збільшення температури газів і стінок впливає на тривалість роботи двигуна, ефективність використання відновленого тепла та оптимізацію роботи гібридних енергоустановок. Що більше тепла вдалося відновити протягом експлуатації двигуна, то довше він працює із мінімальними втратами тепла та максимальною ефективністю.*

Ключові слова: *тепломасообмін, поршневий двигун, гібридна енергоустановка, енергоефективність, умови експлуатації.*