TRANSPORT TECHNOLOGIES

Vol. 6, No. 1, 2025

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https://doi.org/10.23939/tt2025.01.095

TO DETERMINATION OF THE STABILITY OF AN ARTICULATED BUS WITH A SELF-ALIGNING TRAILER AXLE

Summary. *Recently, articulated buses with a capacity of 150-200 people have been increasingly used in passenger transportation. The overall length of such buses is limited to 18.5 meters. It is explained by the need to meet regulatory requirements for maneuverability.*

In Ukraine, the Road Traffic Rules allow the total length of a road train to be 22 meters. With this length, the passenger capacity of the articulated bus increases significantly, but the issues of maneuverability and stability of such buses remain open. Preliminary studies have shown that even a bus with a total length of 18.75 m with an unmanageable trailer axle does not meet the requirements of regulatory documents for maneuverability. Using a self-aligning trailer axle meets the requirements for maneuverability even with an articulated bus length of up to 20.0 m. Still, the question of the stability of such a bus remains open. Using a specified mathematical model of a two-link road train adapted for an articulated bus with a self-aligning trailer axle, the stability indicators of an articulated bus in different driving modes were determined. It is shown that the critical velocity of the articulated bus with the blocked wheels of the trailer's self-aligning axle was 31.87 m/s, which significantly exceeds the maximum velocity of the bus. In the presence of a perturbation, the pattern of changes in the lateral and angular velocities of the articulated bus during the transition process at a velocity of 6 m/s is damped by a logarithmic law, which indicates the stability of the articulated bus movement. When the velocity is increased to 12 m/s, the pattern of changes in lateral and angular velocities also dampens. Still, there are more intense oscillations, which at a velocity of 14 m/s become divergent, leading to a loss of stability of the articulated bus. Similar results were obtained when the rotation of the articulated bus was 90 degrees. Thus, the maximum velocity of the articulated bus with unlocked wheels of the trailer's self-aligning axle should not exceed 14 m/s. When this velocity is reached, the wheels of the self-aligning axis should lock.

It determines the field of application of the self-aligning trailer axle on articulated buses. For driving at higher velocities, fundamentally new control systems for the bus control and the articulated bus trailer are needed.

Key words: *articulated bus, self-aligning axle, velocity, perturbation, oscillations, steadiness.*

1. INTRODUCTION

Technological development and global urbanization have led to serious problems in large cities, namely: environmental, fuel and resource, an increase in diseases related to environmental pollution, and an oversaturation of cities with vehicles, which causes traffic jams, accidents, and reduces the mobility of the

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population. Addressing these issues is an urgent task. In many developed countries, various measures are being taken to achieve this. And the first thing that is being emphasized is the reduction of the number of vehicles with internal combustion engines in large cities through the introduction of energy-efficient public transport with a large passenger capacity, i.e., especially large class buses [1]. According to the analysis, extra-large buses include buses with lengths of 16.5 m and 18.5 m. Among the manufacturers of extra-large buses, the leading positions are occupied by companies from France, Belgium, Austria, Hungary, Switzerland, Germany, the United States, and Japan [2].

The capacity of extra-large buses ranges from 200 to 150 people. An analysis of trends in the change in gross vehicle weight of buses confirms their natural dependence on capacity and size. For extra-large articulated buses, the most numerous category is the one with a gross vehicle weight of 28 tons.

Buses of large and especially large capacity are usually articulated buses with motor-wheel drive; unconventional power systems, including hybrid energy sources, duobuses (trolleybuses) with two energy sources – a heat engine and an electric motor [3]. The overall length of such buses is limited to 18.5 m. It is due to the need to meet the requirements of DSTU UN/ECE R 36-03:2002 "Uniform Technical Requirements for the Approval of Extra-Large Passenger Vehicles in Relation to General Construction (UNECE Regulation No. 36-03:1993, IDT)": the internal turning radius should be 5.3 m, and the point of the bus protruding most from the center of the turn should describe an arc with a radius of 12.5 m.

In Ukraine, the Road Traffic Rules allow the total length of a road train to be 22.0 m [4]. With this length, the passenger capacity of the articulated bus increases significantly, but the issues of maneuverability and stability of such buses remain open. For example, study [4] shows that even for buses with a total length of 18.75 m, the trailer with an unmanageable axle does not meet the requirements of regulatory documents for maneuverability (B_{vce} = 7.349 m > [B_{vce} = 7.2 m]). Using self-aligning trailer axle wheels (self-aligning axle) meets the maneuverability requirements only for road train lengths up to 20.0 m. With an increase in the bus length to 22.0 m, such a bus with a steerable trailer axle with a direct drive to its wheels meets the requirements for maneuverability, but the issue of stability of such buses needs to be addressed.

2. ANALYSIS OF RECENT RESEARCH AND PUBLICATIONS

Recently, articulated vehicles have been increasingly used in passenger and cargo transportation. As a rule, these are large vehicles, and the safety of these vehicles and other vehicles is an essential and unresolved problem. To avoid accidents, the dynamic behavior of the vehicle must be stable, and the lateral characteristics that determine lateral stability must be satisfactory. Thus, in [5], a generalized analysis of the maneuverability and stability of combined vehicles, particularly two- and three-link semitrailer and trailed road trains, was performed as linear dynamic systems with two degrees of freedom for each link. For this purpose, the equation of unsteady motion of a road train was developed, from which the characteristic equation for motion at a constant velocity was subsequently obtained. In [6], a model of road train dynamics was built to study the stability of linear and nonlinear characteristics of a car tractor, trailer, and their combination. An essential result of this study is that the critical velocities of the nonlinear and linear models are almost identical. Paper [7] presents a stability analysis of a linearized model of a two-link road train, the links of which are connected by a universal joint. The developed differential equations make it possible to consider nonlinear elements (nonlinear viscoelastic characteristic functions of the connection) when determining indicators of system stability. The key parameters affecting the stability of a road train, particularly the coefficient of friction and stiffness in the joint, are determined to determine the criteria for its design. Paper [8] presents a single-track dynamic model of the articulated bus based on the assumptions of small lateral accelerations of the bus body and the angles of its links. In addition, friction and gaps in the hitch joint are not considered. The system is designed to accurately describe a vehicle's translational and rotational motion based on the differential equations of kinematic parameters representing articulated buses. Matlab-Simulink was used to solve the equations. The simulation results are kinematic and dynamic parameter functions that allow the trajectory and width of the articulated bus to be determined. The results are the basis for accurately evaluating the dynamic model and studying the dynamics of an articulated bus at a higher and more complex level.

In [9], the equations of dynamics with 6 degrees of freedom in the vertical and horizontal planes were developed to study the stability of a car. It was found that the developed methodology for studying the stability of a vehicle can also be used for two-link road trains. In [10], a model of a road train with 31 degrees of freedom was developed using the AutoSim software product. However, as noted in [11], model complication does not always have a positive effect, as it requires a large amount of input data, the errors of which can negate the model's complexity. In addition, such a complication of the model for analyzing the behavior of systems in unstable modes can be improved.

Paper [12] proposes a unified mathematical model of an articulated vehicle that includes both the dynamics of turning the entire vehicle and any axis of the tractor and trailer, and their combination. The model pays considerable attention to tire modeling. The interaction of the tires with the bearing surface is described by a formula that accounts for changes in the vertical load during driving. The model validation shows the proposed approach's feasibility, efficiency, and convenience, which can be customized for any articulated vehicle when determining the stability parameter in straight-line motion.

In contrast to articulated vehicles, the kinematic model of a trailered road train is more complex [13], due to the more complex trailer control system.

Lateral control systems based on a combination of braking and steering are proposed in the literature to improve lateral stability. In [14], an active feedback control system for semitrailer and trailed road trains was developed and studied. Paper [15] presents an active steering system based on fuzzy logic with a new method for calculating the desired folding angle. The proposed control system is tested for a low-velocity 90-degree turn and high-velocity lane change. In [16], two proportional-integral-derivative controllers are used to improve the lateral characteristics of a truck and trailer. Uncertainties in vehicle and trailer masses are taken into account. The results show better trailer stability in the controlled case than in the uncontrolled case. Paper [17] investigates the stability of a trailer based on neural network control. The results show that the neural network based on controlling the trajectory of the vehicle's center of mass improved vehicle stability, such as reduced side slip angle, turning velocity, longitudinal acceleration, etc.

Paper [18] addresses the issue of improving the maneuverability and stability of an articulated vehicle, investigating two problems: poor maneuverability at low velocities and loss of stability at high velocities. An optimal approach is proposed to control a linear quadratic controller at low velocity and modulate the coefficient of gain and roll at high velocity. In [19], a predictive control approach based on active steering was presented to minimize the increase in reverse turning velocity, particularly when changing lanes. A comprehensive review study was conducted in [20] to classify and compare schemes for improving road trains' lateral stability. This paper proposes an optimal control system based on active steering systems for controlling the turning velocity of a truck and trailer. The linearized model is used to obtain the desired value of the truck's turning velocity. The desired turning velocity of the trailer is calculated based on the desired turning velocity of the truck. The control system is tested using step and sinusoidal lane change maneuvers by calculating the nominal and unspecified weights for the truck and trailer. The values of the reverse velocity gain assess the transverse stability of the entire system. The main contribution of this research is that the turning angles of the car and trailer are activated. Thus, both units, the truck and the trailer, can be controlled separately in this work.

Much less attention has been paid to the issues of maneuverability and stability of road trains with a self-aligning trailer axle. In particular, in [21], mathematical models of varying degrees of complexity were developed for a road train with a self-aligning axle of a semitrailer, which allow predicting the indicators of maneuverability and stability of a road train at the stage of semitrailer components and systems design according to given design parameters. It has been shown that using a self-aligning semitrailer axle improves the fit of a road train into a curve and reduces its turning radius by an average of 20 %. At the same time, a road train with a self-aligning semitrailer axle has excessive maneuverability and a critical straight-line velocity. This velocity ranges from 12.1 to 15.6 m/s. It depends on the location of the first two axles, the elastic characteristics of the tires of these axles, the inertial characteristics of the frame and the self-aligning axle, and the normal load on the axles.

Summarizing the results of the analysis of the controllability and stability of cars and road trains, it can be noted that these issues are considered in two aspects [11]:

1. The study considers the characteristics of all elements of the driver-car-road system, which is considered as a closed automatic control system;
2. A study of the self-stability and controllability of a vehicle (road train), in which the driver's influence is excluded.

In the first group of studies, the concepts of controllability and stability are given the meaning used in the automatic control theory. In such a study, controllability considers specific characteristics of transient processes under the simplest typical control actions [7]. Stability characterizes the behavior of a system in a transition mode. It refers primarily to the system's motions generated by initial conditions (perturbations) and its internal properties, but not external influences. Sustainability is considered concerning any controlled and uncontrolled process [11].

In the second group of studies, the car is considered in isolation as an object of control, and the stability of the driver-vehicle system is determined by the vehicle's stability and the psychophysical capabilities of the driver, as well as the level and nature of the disturbances. According to the calculations performed for various types of vehicles, the presence of a closed-loop control scheme allows increasing their critical velocity by 1.5–2 times, including articulated buses.

Success in solving such problems depends on how well the mathematical model and its essential parameters describing the behavior of a dynamic system in different modes of motion are chosen. In [11], the differential equations of plane-parallel motion were developed to determine the maneuverability and stability of a road train with uncontrolled trailer varnishes. Their use to assess the stability of vehicles with steerable wheels (axles), in particular, self-leveling wheels (axles), can lead to significant errors. In this regard, the **aim of the study** is to determine the stability of an articulated bus with a self-aligning trailer axle in straight-line motion.

3. RESULTS OF THE STUDY

In the theory of controlled road train movement, the following basic assumptions are considered quite reasonable in simulation [11]:

– the road train is moving on a flat horizontal surface;
– the unsprung mass is considered to be non-tilting;
– the controlling influence on the parameters of the movement of the road train is carried out through the steering wheels of the traction vehicle, so the dynamics of steering is not taken into account;
– the presence of gaps in the coupling device is not taken into account;
– the longitudinal velocity of the road train is constant;
– the distance between the links of the road train does not change due to the small folding angles;
– the components of the road train are absolutely solid bodies;
– the cargo on the road train is arranged in such a way that the centers of mass of the traction vehicle and the trailer, as well as the coupling device connecting them, are located in the vertical plane of symmetry of the link;
– the trajectory of the road tractor's center of mass is taken as the main trajectory;
– gyroscopic moments and moments from unbalanced rotating masses can be neglected;
– the interaction of wheels with the bearing surface is expressed through the reaction of the roadway, which is a function of the angle of deflection, namely [11]:

$$Y_i = \frac{k_i \delta_i}{\sqrt{1 + k_i (\phi^2 G_i^2)^{-1} \delta_i^2}}, \tag{1}$$

where δ_i, Y_i – deflection angles and lateral reactions; ϕ – the coefficient of adhesion between the tire and the supporting surface in the transverse direction (we consider ϕ to be a constant value for given road conditions); k_i – coefficient of resistance to lateral deflection.

If we denote the lateral deflection resistance coefficient in the absence of longitudinal forces on the wheel by k_o, then the value of k is determined by the formula [11]:

$$k = k_0 \frac{\sqrt{1-(X/(\phi G)^2}}{1+0,375\,X/G}, \tag{2}$$

where G – vertical load on the wheel; X – the value of the longitudinal force given by the ratio:

$$X = \begin{cases} M/r, \text{ if } M/r < \phi G \\ \phi G, \text{ if } M/r \geq \phi G \end{cases}, \tag{3}$$

where M – traction or braking moments applied to the wheel;

– the moments of resistance in the joints between the links of the road train are as follows [11]:

$$M_{oi} = \frac{2}{3} Z_O \mu \frac{R^3 - r^3}{R^2 - r^2}, \tag{4}$$

where Z_{oi} – vertical load in the coupling device; μ – coefficient of friction (μ=0.15...0.20); R – outer radius of the rotary device, R=1500 mm; r – inner radius of the rotary device, r = 500 mm.

The obtained dependencies and assumptions are considered when drawing up the equations of motion of a road train. The calculation scheme of the articulated bus is shown in Fig. 1.

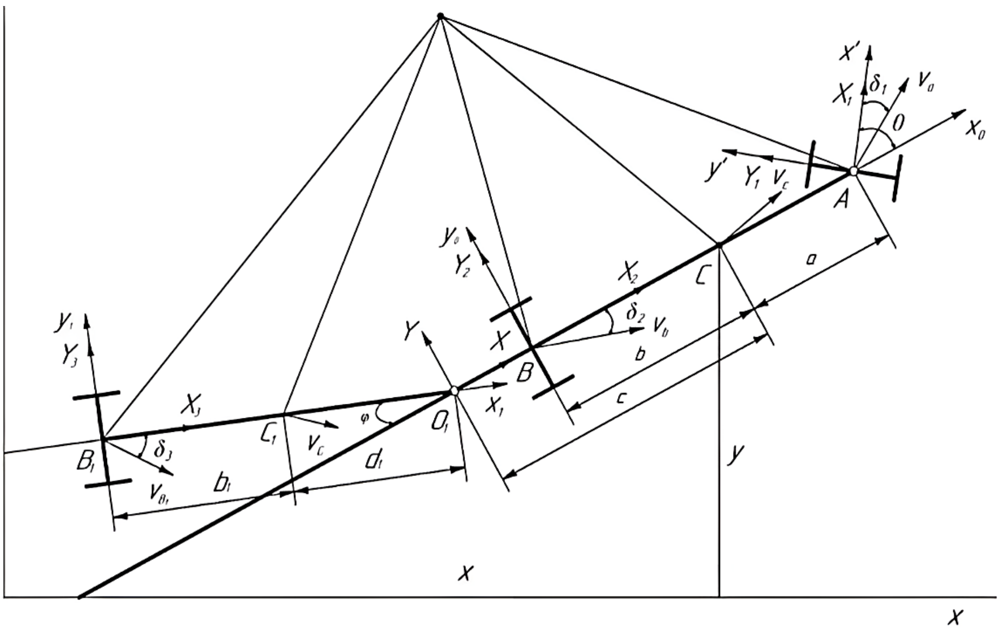


Fig. 1. Design diagram of the articulated bus

The bus is conventionally divided into two modules – the wheel control unit and the body. The wheel control unit includes the driven steering wheel, the steering wheel, and the steering elements located between them (steering gear, steering mechanism, etc.). The wheel control unit has one degree of freedom with respect to the car's carcass – turning around the driven tilted kingpin.

The steering angle is limited by the linkages imposed by the steering system, in particular, the elastic moments M_p and viscous friction moments M_h, which are proportional to the steering angle of the driven wheel [11]:

$$M_p = \chi \cdot \theta, \tag{5}$$

$$M_h = h_1 \cdot \dot{\theta}, \tag{6}$$

where χ – stiffness coefficient of the steering system; h_1 – coefficient of viscous friction in steering components.

A single-axle trailer is conventionally represented by a single module – a carcass with an unmanageable axle.

When finding the angles of the bus axles, we have:

$$\frac{\overline{V_{co}}_{\;Yo}}{\overline{V_{Co}}_{\;Xo}} = tg(\theta - \delta_1), \tag{7}$$

$$\delta_1 = \theta - arctg\frac{u + \omega(a - \lambda \cdot \cos\theta) - \lambda \cdot \dot{\theta} \cdot \cos\theta}{V + \lambda(\omega + \dot{\theta})\sin\theta}. \tag{8}$$

For the carcass of the bus:

$$\frac{\overline{V_{B2}}_{\;Yo}}{\overline{V_{B2}}_{\;Xo}} = tg\delta_2, \;\; \delta_2 = arctg\frac{-u + b \cdot \omega}{V}. \tag{9}$$

For a trailer carcass with self-aligning axle:

$$\frac{\overline{V_{B3}}_{\;Yo}}{\overline{V_{B3}}_{\;Xo}} = tg\delta_3, \;\; \delta_3 = -arctg\frac{-u_1 + \omega_1 b_1}{V_1} \to 0. \tag{10}$$

After determining the velocities of individual links of the road train and the angles of deflection of the wheels of its axles, the differential equations of the articulated bus, taking into account that the lateral reaction on the wheels of the trailer's self-aligning axle is 0, are written in the form:

$$\begin{gathered}
(m_0 + m + m_1)\dot{u} + [m_o(a - \lambda\cos\theta) - m_1(d_3\cos\varphi + d)]\dot{\varphi} - m_o\lambda\cos\theta \cdot \ddot{\theta} - \\
-\omega V(m_o + m + m_1) + m_o\lambda(\omega + \dot{\theta})^2\sin\theta = X_1\sin\theta + Y_1\cos\theta + Y_2\sin\phi; \\
J_1 \cdot \ddot{\theta} + (J_1 - m_0 \cdot a \cdot \lambda \cdot \cos\theta) \cdot \dot{\omega}_0 + m_0 \cdot a \cdot \dot{u} \cdot \cos\theta - m_0 \cdot a \cdot \dot{\mathrm{v}} \cdot \sin\theta - m_0 \cdot \lambda \cdot \mathrm{v} \cdot \omega_0 \cdot \cos\theta - \\
- m_0\lambda u\omega_0\sin\theta - m_0 a\lambda\omega^2\sin\theta + h\dot{\theta} + k\theta = -\lambda Y_1 + M_p + M_h; \\
[I + m_1(c + d_1\cos\varphi) + m_o a(a - c\cos\varphi)\dot{\omega} - \{[m_1 d_1 + (m_o a - m_1 d_1)]\dot{u} - m_o a\lambda\cos\theta\ddot{\theta} + \\
+ (m_o a - m_1 d_1)\omega V + m_o a\lambda(\omega + \dot{\theta})^2\sin\theta = X_1 a\sin\theta + Y_1 a\cos\theta - Y_2 b + M_o; \\
(I_2 + m_1 d_1^2)\ddot{\varphi} - (I_2 + m_1 d_1^2)\dot{\omega} + m_1 d_1 \cdot \dot{V} + m_1 d_1\dot{u} + m_1 d_1\omega V = Y_3(b_1 + d_1).
\end{gathered} \tag{11}$$

The resulting system of differential equations describes the model of the articulated bus movement as a three-mass system, taking into account the bus wheel control module. The system allows to investigate the influence of the design parameters of the bus and trailer on the stability indicators of the articulated bus.

In real-world vehicle designs, the difference in steering angles between the outer and inner steering wheels averages a fraction of a degree when driving in a straight line. Under such conditions, it is possible to calculate the average angle of rotation of the driven wheel θ with a fairly high degree of accuracy and neglect the redistribution of lateral forces.

To solve the problem of the stability of the straight-line motion of a road train, it is necessary to draw up a system of equations for its disturbed motion. This system makes it possible to determine the reactions of road train links in the event of a single disturbance (a sharp turn of the steering wheel of the road tractor), as well as the critical velocity of the road train.

The theory of stability of wheeled vehicles is based on the mathematical apparatus for studying differential equations, given in [11]. There, the properties of the perturbed state of the system, which consist in the tendency to restore the trajectory of undisturbed motion, characterized by the parameters that occurred before the perturbation were determined. To quantify this property, the time it takes for the motion parameters to return to the original one can be taken. During the oscillatory process of returning these parameters to their original values, it is possible to quantify the stability by decrement, i.e. the degree of decrease in the amplitude of oscillations.

The driver must turn the steering wheel alternately in both directions to keep the road train on a given trajectory during an oscillating transient. In this case, it is more difficult to control a road train than in an aperiodic transient. And if the period of the oscillatory process is close to the reaction time of the car-driver system, then the driver's actions can cause an unquenchable oscillatory process. It is necessary that the oscillation period exceeds the reaction time of the specified system by at least 3-4 times, i.e., it is more than 4...6 s [10].

A steady motion realized in a previously unknown region of initial perturbations, which is called the region of attraction of undisturbed motion. The task of determining the boundaries of this area arises. The critical velocity $v_{кр}$ is the velocity at which at least one of the links of a road train loses stability. Stability means the property of a road train link to maintain the direction of movement and orientation of the longitudinal and vertical axes within specified limits, regardless of the velocity of movement and the action of external forces, in the absence of controlling influences from the driver.

The system of equations of motion of the articulated bus (11) admits the solution $\theta=0$, $u=0$, $\omega=0$, $\varphi=0$ (φ – angle of folding of the road train in steady motion), to which on the road plane corresponds the movement of all points of the train with velocity v along a straight line $\theta=const$ [11]. Let's take this movement as undisturbed.

Let's investigate the stability of the stationary solution v^*, u^*, ω^*, φ^* (in the case of straightforward undisturbed motion, all these values except v are equal to zero) first without taking into account the oscillations of the control wheel module. At a constant velocity of motion ($v=const$), we put

$$v=v^*+v',\quad u=u^*+u',\quad \omega=\omega^*+\omega',\quad \varphi=\varphi^*{}_2+\varphi',\quad \phi=\phi^*+\phi'. \qquad (12)$$

Given that at $u^*=0,\ \ \omega^*=0,\ \ \varphi^*=0$ we obtain:

$$\sin\varphi_2=0,\quad \cos\varphi_2=1$$

In this case, the expressions for the longitudinal and lateral velocity are written as:

$$\begin{gathered} v_2=v; \\ u_2=v\phi+(u-\omega_0(b+c_0))-(\omega_0-\dot{\phi})(c+c_{ш})). \end{gathered} \qquad (13)$$

When driving in a straight line, lateral velocities are much lower than longitudinal velocities. In this case, the average steering angles of the δ_{si} axles of the road train will be written as:

$$\delta_1=-\frac{u+\omega_0 a}{v};\quad \delta_2=\frac{-u+\omega_0 b}{v};\quad \delta_{s3}=\frac{v\varphi+u-\omega_0(b+c_0)}{v}\to 0. \qquad (14)$$

For the variations u^*, ω^*, φ^*, we obtain, omitting the strokes, the equations solved with respect to the higher derivatives [11]:

$$\begin{gathered} (m+m_2)(\dot{u}+\omega_0 v)-m_2(B+C)\dot{\omega}_0+m_2C\ddot{\varphi}=2Y_1+2Y_2; \\ (-m_2B)\dot{u}+(m_2(B^2-3BC+I_2)\dot{\omega}-BCm_2\phi-m_2v\omega_0(B+C)+m_2v\omega C=2Y_1a-2Y_2b; \\ m_2C\dot{u}-(m_2C(B+C)+I_2)\dot{\omega}+(m_2C^2+I_2)\ddot{\phi}+m_2v\omega C=0. \end{gathered} \qquad (15)$$

where $B=в+c_0$, $C=c$ – geometric parameters of the road train.

Substituting the values of lateral forces in (15) by the dependence $Y_i = k_i\delta_i$, taking into account the notation, we obtain:

$$\left(\frac{A_2}{v}+(m+m_2)v\right)\omega+\frac{A_1u}{v}+A_3\varphi+\frac{A_4\dot{\varphi}}{v}+(m+m_2)\dot{u}-m_2(B+C)\dot{\omega}+m_2C\ddot{\varphi}=0;$$

$$\left(\frac{B_2}{v}+m_2(B+2C)v\right)\omega+\frac{B_1u}{v}+B_3\varphi+\frac{B_4\dot{\varphi}}{v}-m_2B\dot{u}+(m_2(B+C)+I))\dot{\omega}-m_2BC\ddot{\phi}=0; \quad (16)$$

$$\left(m_2vC+\frac{C_2}{v}\right)\omega+\frac{C_1u}{v}+C_1\varphi+\frac{C_3\dot{\varphi}}{v}+m_2C\dot{u}-(I_2+m_2(B+C)C)\dot{\omega}+(m_2C+I_2)\ddot{\varphi}=0,$$

where the expressions for the coefficients of the equation are written in the form [11]:

$$A_1=(k_1+k_2+k_3);\ A_2=2(k_1a-k_2b-k_3(B+C+c);\ A_3=2k_3;\ A_4=2k_3c_t;$$

$$B_1=2(k_1a-k_2b-k_3(c_t+\mathrm{B}));\ B_2=2(k_1a^2+k_2b^2+k_3(B+C-c)(B+c_t);\ B_3=-2k_3(c_t+B);$$

$$B_4=-2k_3c_t(c_t+B);$$

$$C_1=2k_3c_t;\ C_2=2k_3c_t(\text{c-B-C});\ C_3=2k_3c_t^2$$

After solving these equations with regard to the higher derivatives, we obtain:

$$\dot{u}=-\frac{B\cdot m_2\cdot I_2\cdot B_0-m_2\cdot C\cdot C_0\cdot I+A_0\cdot m_2\cdot C^2\cdot I+m_2\cdot I_2\cdot B^2\cdot A_0+A_0\cdot I_2\cdot I}{m_2\cdot C^2\cdot m\cdot I+m_2\cdot I_2\cdot B^2\cdot m+m_2\cdot I\cdot I_2+m\cdot I\cdot I_2}; \quad (17)$$

$$\dot{\omega}=-\frac{m\cdot m_2\cdot C^2\cdot B_0+m_2\cdot I_2\cdot B_0+m_2\cdot I_2\cdot B\cdot A_0+m_2\cdot C\cdot B\cdot m\cdot C_0+I_2\cdot m\cdot B_0}{m_2\cdot C^2\cdot m\cdot I+m_2\cdot I_2\cdot B^2\cdot m+m_2\cdot I\cdot I_2+m\cdot I\cdot I_2}; \quad (18)$$

$$\ddot{\varphi}=-\frac{\begin{array}{c}m\cdot m_2\cdot(C^2+I_2)\cdot B_0+Bmm_2(CB_0+BC_0)+m_2B(I_2A_0+CmC_0)+\\+I_2mB_0+mC_0I+m_2I(C_0-A_0C)\end{array}}{m_2\cdot C^2\cdot m\cdot I+m_2\cdot I_2\cdot B^2\cdot m+m_2\cdot I\cdot I_2+m\cdot I\cdot I_2}; \quad (19)$$

where:

$$A_0=\left(\frac{A_2}{v}+(m+m_2)v\right)\omega+\frac{A_1u}{v}+A_3\varphi+\frac{A_4\dot{\varphi}}{v};\quad B_0=\left(\frac{B_2}{v}-m_2(B+2C)v\right)\omega+\frac{B_1u}{v}+B_3\varphi+\frac{B_4\dot{\varphi}}{v};$$

$$C_0=\left(m_2vC+\frac{C_2}{v}\right)\omega+\frac{C_1u}{v}+C_1\varphi+\frac{C_3\dot{\varphi}}{v}.$$

System of equations in vector-matrix form:

$$\left\|a_{ij}\right\|_1^3\times\begin{vmatrix}\dot{u}\\ \dot{\omega}\\ \ddot{\phi}\end{vmatrix}+\left\|b_{ij}\right\|_{3,4}\times\begin{vmatrix}u\\ \omega\\ \phi\\ \dot{\phi}\end{vmatrix}=0. \quad (20)$$

A set of functions $u,\omega,\varphi=(a_1,\ a_2,\ a_3)\exp(\lambda\, t)$ forms a partial solution of the system if and only if λ is a root of the characteristic equation:

$$D(\lambda)=A_0\lambda^4+A_1\lambda^3+A_2\lambda^2+A_3\lambda+A_4=0. \quad (21)$$

Matrix of the characteristic equation:

$$\begin{vmatrix}a_{11}\lambda+b_{11} & a_{12}\lambda+b_{12} & a_{13}\lambda^2+b_{13}\lambda+b_{14}\\ a_{21}\lambda+b_{21} & a_{22}\lambda+b_{22} & a_{23}\lambda^2+b_{23}\lambda+b_{24}\\ a_{31}\lambda+b_{31} & a_{32}\lambda+b_{32} & a_{33}\lambda^2+b_{33}\lambda+b_{34}\end{vmatrix}=\sum_{i=0}^{n=4}A_i\lambda^{n-i}=0; \quad (22)$$

where:

$$a_{11} = m + m_2;\ b_{11} = \frac{A_1}{v};\ a_{12} = -m_2(B+C);;\ b_{12} = \frac{A_2}{v} + \ m + m_2\ \ v;\ a_{12} = m_2C;$$

$$b_{13}\lambda + b_{14} = A_3 + \frac{\lambda}{v}A_4;\ a_{21} = -m_2B;\ b_{21} = \frac{B_1}{v};\ b_{31} = \frac{C_1}{v};\ a_{22} = m_2(B^2 - 3BC) + I_2 + In;$$

$$b_{22} = -vm_2\ \ B + 2C\ \ + \frac{B_2}{v};\ a_{23} = -m_2BC;\ a_{33} = I_2 - m_2C^2;\ b_{23}\lambda + b_{24} = B_3 + \frac{\lambda}{v}B_4;\ a_{31} = m_2C;$$

$$b_{31} = I_2 + m_2(B+C)C;\ b_{32} = m_2vC - \frac{C_2}{v};\ b_{33}\lambda + b_{34} = C_1 + \frac{\lambda}{v}C_3.$$

According to Rausch, a necessary but not sufficient condition for stability is that all the coefficients of A_i are positive. The system will be stable if the determinant and its minors are positive. Analysis of the roots of the characteristic equation can characterize the state of the system.

In general, the following values of the roots of the characteristic equation are possible: λ is a real and positive value – the system is unstable, the motion will be unstable; λ is a real and negative value – the system eventually returns to a stable position. If the coefficient λ is a complex number, then its positive real part indicates the presence of increasing oscillations, and the negative part indicates the presence of decreasing oscillations.

Hurwitz determinants of the characteristic equation (21): the first Δ_1 is responsible for the presence of positive real roots, and the third Δ_3 – for the presence of the positive real part of imaginary complex roots. From equation (21), we obtain the factors on which the critical velocity depends:

$$v_{кр} = f\ \ m_0, m, m_2, a, L, c_0, c_ш, c, L_2, k_1, k_2, k_3\ \ . \qquad (23)$$

Using formula (23), we calculate the critical straight-line velocity, which is an evaluative indicator of the stability of a road train, a brief technical characteristic of which is given in Table 1.

Table 1

Brief technical characteristics of the articulated bus

Name of the indicator	Evaluative indicators
Passenger capacity, persons	250
Total weight, kg	29000
Bus weight, kg	18500
Trailer weight, kg	10500
Total length, mm	22000
Bus base, La, mm	6500
Trailer base, Lt, mm	6500
Distance from the center of mass of the bus to the front axle, *a*, mm	4000
Distance from the center of mass of the bus to the rear axle, *b*, mm	2500
Distance from the center of mass of the bus to the point of coupling with the trailer, *s*, mm	4800
Distance from the center of mass of the trailer to the axle, b_1, mm	285
Distance from the center of mass of the trailer to the point of coupling with the bus, d_1, mm	6215
Offset of the bus steering wheel, λ, m	0.0017
Outer radius of the rotary device, R, mm	R=1500
Internal radius of the rotary device, r, mm	R=500
Height of the center of mass of the bus and trailer, h, mm	1150
Friction coefficient in a rotary device, μ	μ=0.15
Bus track, Bb, mm	1800
Trailer track, Bt, mm	1800
Bus height, H, mm	3200

Continuation of Table 1

Name of the indicator	Evaluative indicators
Type and size of tires	275/70 R 22.5
Coefficient of resistance to wheel steering of the bus front axle, kN/rad	160000
Coefficient of resistance to wheel retraction of the rear axle of the bus, kN/rad	250000
Coefficient of resistance to wheel steering of the bus front axle, kN/rad	180000

The initial system of equations includes the layout, mass, and inertial parameters of the bus and trailer, as well as the moment of resistance to turning of the trailer. Let's determine these parameters.

For the reactions of the bearing surface on the bus axis and the trailer axis, we obtain [11]:

$$Z_1 = g \times m - (m \cdot g \cdot b - g \cdot m_2 \cdot b_1 \cdot \frac{c-b}{L_n}) \cdot \frac{1}{L} = 104.903 \text{ kN};$$

$$Z_2 = (m \cdot g \cdot a + m_2 \cdot g \cdot b_1 \cdot \frac{a+c}{L_n}) \cdot \frac{1}{L} = 179.587 \text{ kN}; \tag{24}$$

$$Z_3 = m_2 \cdot g \cdot \frac{d_1}{L_n} = 98.489 \text{ kN}.$$

We will determine the moments of inertia of the links of the road train in accordance with the methodology proposed in [21].

This methodology is based on two assumptions: first, the moment of inertia of a car depends on the law of distribution of its mass within the track, base, and height; second, the density of the moment of inertia distribution follows a normal distribution law.

The most probable values of the radius of inertia relative to the transverse axis:

$$\rho_y = \sqrt{\frac{1}{2}ab + \frac{1}{3}\ H - h\ \ h \pm \frac{1}{6}ab}. \tag{25}$$

So, for the bus and trailer, the radii of inertia were:

$$\rho_y = 2.731 \text{ m}^2; \qquad \rho_{y2} = 1.498 \text{ m}^2.$$

The moment of inertia of each of the links of the road train is determined by the known formula:

$$I_i = m_i \rho_i^2.$$

Then

$$I_{ya} = 79198.5 \text{kg} \cdot \text{m}^2; \qquad I_{yn} = 15724.3 \text{kg} \cdot \text{m}^2.$$

Let's integrate the system of equations using Maple 15 software with the following initial data g:=9.81; a:=4.000; b:=2.500; d_1:=6.215; b_1:=0.285; c:=4,800; lambda:=-0.0017; m_0:=250; m:=18500; m_1:=10500; k_1:=160000; k_2:=250000; k_3:=180000; k:=1200; h:=30; φ=0.8; θ: = 0,0; φ:=0; B:=1.80; Z_0:=500; R:=1,5; r:=0,5; μ=0,15.

With such initial data, the critical velocity of the articulated bus with the locked wheels of the trailer's self-aligning axle was 31.87 m/s, which is significantly higher than the maximum velocity of the bus.

Now let's investigate the velocity of the bus with the trailer axle wheels unlocked. This velocity can be determined by the roots of the characteristic equation (21). As an example, Table 2 shows the values of the roots of the characteristic equation, which can be used to determine the type of stability or instability of a road train.

Table 2

The roots of the characteristic equation

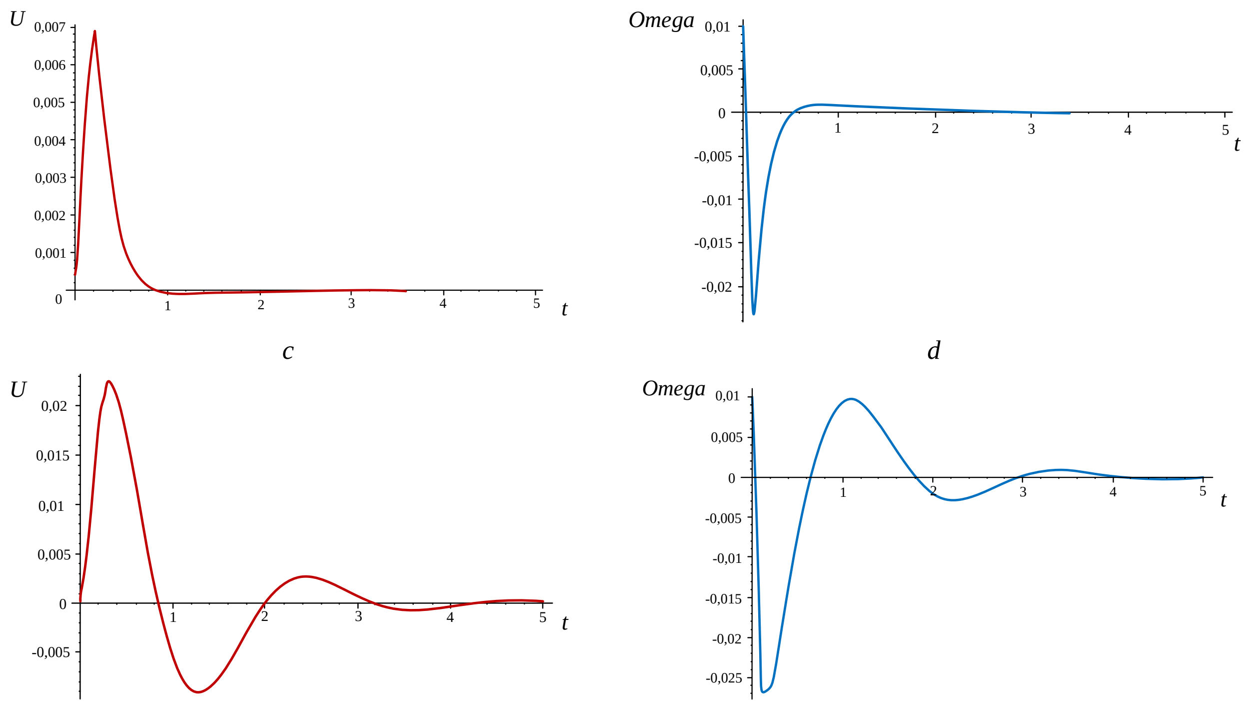
v_a, m/s	λ_1	λ_2	λ_3	λ_4
6	–18.5682764	–10.5815750	–1.60151079–2.30013558·I	–1.60151079+2.30013558·I
8	–16.2876286	–9.49905190	–0.972176748–2.70437281·I	–0.972176748+2.70437281·I
10	–14.5767626	–8.73623641	–0.475827856–2.90746335·I	–0.475827856+2.90746335·I
12	–12.1529473	–7.78625743	–0.0063740487–3.07708746·I	–0.0063740487+3.07708746·I
14	–10.8756911	–7.23344156	0.005314523–3.614524332·I	0.005314523+3.614524332·I

As follows from Table 1, up to a velocity of 14 m/s, the motion of the articulated bus is stable, all real parts of the roots of the characteristic equation are negative, but already at a velocity of 12 m/s, one of the real roots approaches zero, which may indicate the beginning of the loss of motion stability.

Let us determine the lateral and angular velocities of the center of mass of the articulated bus for velocities of 6, 12, and 14 m/s.

Fig. 2 shows the results of calculating the lateral and angular velocity of the bus center of mass depending on the velocity of the articulated bus. The analysis of the calculations shows that in the presence of a perturbation, the pattern of change in the lateral and angular velocities of the articulated bus during the transition process at a velocity of 6 m/s has a damped logarithmic nature (Fig. 2, *a*). Under the same conditions, when the velocity is increased to 12 m/s, the pattern of changes in lateral and angular velocities also decreases, but there are fluctuations (Fig. 2, *b*). At a velocity of 14 m/s, the oscillations of the motion parameters will be divergent, which will lead to a loss of stability of the articulated bus and the impossibility of further movement (Fig. 2, *c*). Thus, the maximum velocity of the articulated bus with a self-aligning trailer axis under the action of a disturbance in straight-line motion is 14 m/s. When this velocity is reached, the wheels of the self-aligning axle should lock.

a *b*



e *f*

Fig. 2. Changes in the lateral and angular velocity of the center of mass of the bus from the velocity of the articulated

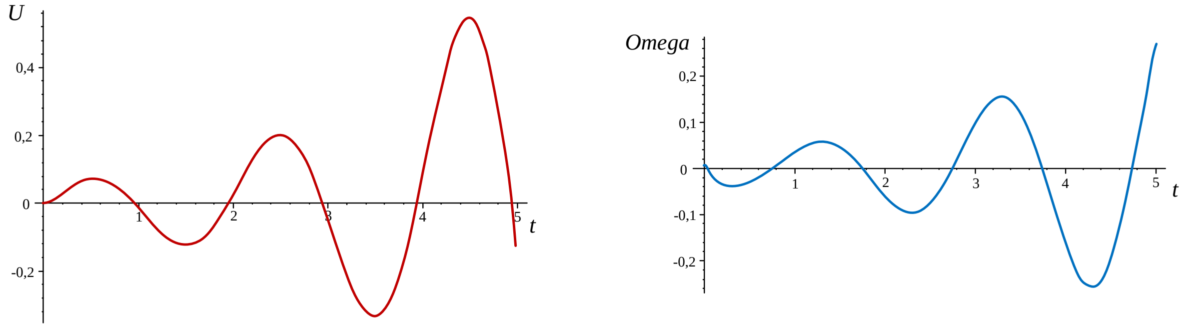


Fig. 2. (Continuation). Changes in the lateral and angular velocity of the center of mass of the bus from the velocity of the articulated

Now let's determine the velocity of the articulated bus in transitional modes of motion. For computer modeling of the most typical 90° turn of an articulated bus that had previously been moving in a straight line, the control law for the steering wheels of the bus is given as [11]:

$$\theta = \begin{cases} 0\_if\_0 \le t \le t_0 \\ \beta \times t\_if\_t_0 < t \le t_1 \\ \beta \times t_1\_if\_t_1 \le t \le t_2 \\ -\beta \times t\_if\_t_2 < t \le t_3 \\ 0\_if\_t > t_2 \end{cases}. \quad (26)$$

where [0; t_0] and [t_3; t_k] – the time of movement of the vehicle in a straight line in accordance with the entrance to the turn and after exiting the turn; [t_0; t_1] – time interval for entering a turn, the steered wheels of the bus turn evenly at a velocity β=0.05 c^{-1}; [t_1; t_2] – time interval of the articulated bus movement in a circle; [t_2; t_3] – the time interval for the articulated bus to exit the turn (the steering wheels of the bus return to the neutral position evenly).

Figs.3–5 show the changes in the angle of rotation of the bus steered wheels and the trailer self-leveling wheels, the angular and lateral velocity of the bus and trailer, as well as the lateral acceleration of the trailer when the track turns by 90^0.

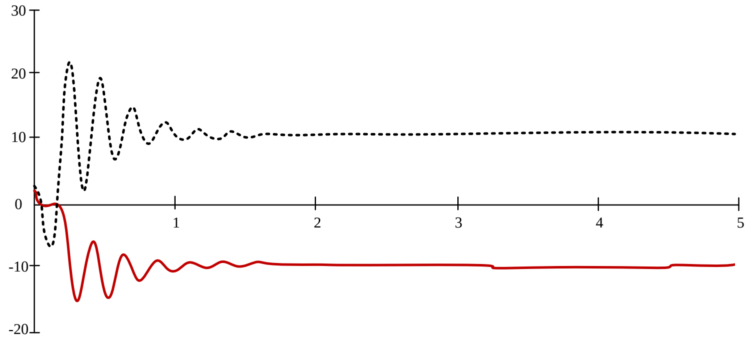


Fig. 3. Changing the angles of rotation of the bus steered wheels and trailer self-aligning wheels when turning 90^0 at a velocity of 10 m/s

Analysis of Fig. 3 shows that at the entrance to the curve there are different directions of rotation of the steered wheels of the bus and the self-aligning wheels of the trailer axle, and the angle of rotation of the steered wheels of the bus is almost 40 % larger than the angle of rotation of the self-aligning wheels of the trailer axle. It ensures the required indicators of vehicle stability at the entrance to the turn at a velocity of 10 m/s. The stability of the articulated bus at this velocity is also evidenced by Figs. 4–5. Thus, the maximum angular velocity of the trailer is only 27.5 % higher than the angular velocity of the bus (Fig. 4), and the lateral acceleration of the trailer does not exceed 2.0 m/s^2. When the velocity is increased to 12 m/s, the lateral acceleration of the trailer increases to 4.3 m/s^2 and the articulated bus is at the limit of driving stability, i.e., both in the case of straightforward movement under the action of a disturbance and during a turn of 90^0, the maximum velocity under driving stability must not exceed 14 m/s.

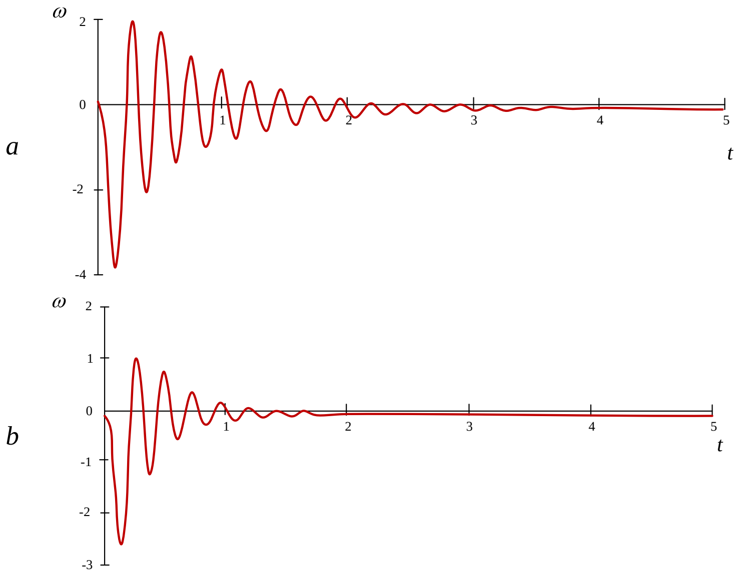


Fig. 4. Changes in the angular velocity of the bus (a) and trailer (b) during the transition process at a velocity of articulated bus of 10 m/s

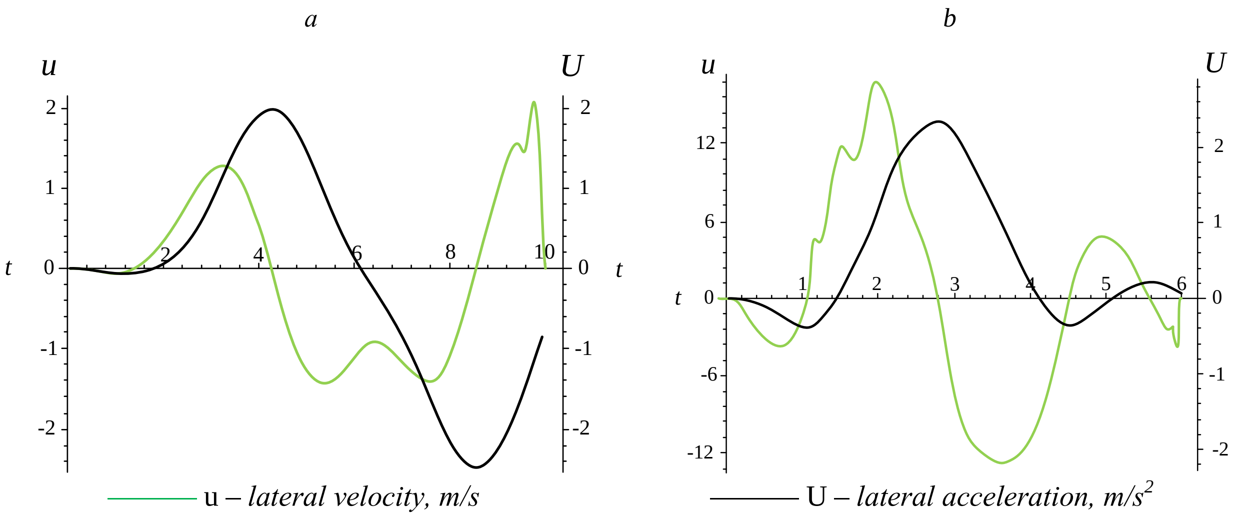


Fig. 5. Changes in lateral velocity u and lateral acceleration U of the trailer's center of mass when the articulated bus enters a turn at velocities of 10 m/s (a) and 12 m/s (b)

4. DISCUSSION

The trailer's self-aligning axle ensures smooth steering of the articulated bus thanks to a special steering device, which significantly improves its steerability, reduces tire wear, increases operational efficiency and safety, and facilitates the operation of the articulated bus in difficult road conditions. The self-aligning axle is a passive control system for the articulated bus, but it ensures synchronous movement of the bus and trailer, reducing the resistance that occurs during asynchronous control and thus increasing the efficiency of the vehicle. The passive system uses a "command steering" control strategy, which is designed to ensure satisfactory operation of the articulated bus during various maneuvers at low velocities. Still, it does not ensure the vehicle's controllability and stability at high velocity.

The calculations of the lateral and angular velocities of the center of mass of the bus revealed that at a speed of 6 m/s, their oscillations have a logarithmic damped nature. Under the same conditions, when the velocity is increased to 12 m/s, the pattern of changes in lateral and angular velocities also dampens, but there are more intense oscillations, which at a velocity of 14 m/s become divergent, leading to a loss of stability of the articulated bus. The same results were obtained at the entrance of the articulated bus. Thus, during this maneuver at a velocity of 10 m/s, the stability of the articulated bus was ensured, since the maximum lateral accelerations were 2.0 m/s^2, which is much less than the permissible ones. At the same time, when the velocity is increased to 12 m/s, the lateral acceleration of the trailer increases to 4.3 m/s^2 and the articulated bus is at the limit of motion stability, i.e., both in the case of straight-line movement under the action of a disturbance and when turning 90^0, the maximum velocity under the condition of motion stability should not exceed 14 m/s. When this velocity is reached, the wheels of the self-aligning axle must lock. It determines the field of application of the self-aligning trailer axle on articulated buses.

For driving at higher speeds, fundamentally new control systems for the bus and the articulated bus trailer are needed.

5. CONCLUSIONS

Based on a specified mathematical model of a two-link road train adapted for an articulated bus with a self-aligning trailer axle, the stability indicators of an articulated bus in different driving modes were determined. In particular, it was found that:

– the critical velocity of the articulated bus with the locked wheels of the trailer's self-aligning axle was 31.87 m/s, which significantly exceeds the maximum speed of the bus;
– in the presence of a perturbation, the pattern of changes in the lateral and angular velocities of the articulated bus during the transition process at a speed of 6 m/s has a damped nature by a logarithmic law, which indicates the stability of the articulated bus motion. When the velocity increases to 12 m/s, the character of the change in lateral and angular velocities also fades away, but there are more intense oscillations, which at a velocity of 14 m/s become divergent, which leads to a loss of stability of the articulated bus movement;
– at a turn of 900 at a velocity of 10 m/s, the stability of the articulated bus was ensured, since the maximum lateral accelerations amounted to 2.0 m/s^2, which is much less than the permissible ones. At the same time, when the velocity increases to 12 m/s, the lateral acceleration of the trailer increases to 4.3 m/s^2, and the articulated is on the verge of driving stability;
– the maximum velocity of the articulated bus both in a straight line movement under the action of a disturbance and when turning through 900, the maximum velocity under motion stability should not exceed 14 m/s. When this velocity is reached, the wheels of the self-aligning axle should lock.

It determines the field of application of the self-aligning trailer axle on articulated buses. For driving at higher velocities, fundamentally new control systems for the bus and the articulated bus trailer are required.

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Received date 03.04.2025; Accepted date: 23.05.2025; Published date: 06.06.2025.

ДО ВИЗНАЧЕННЯ СТІЙКОСТІ РУХУ ШАРНІРНО-З'ЄДНАНОГО АВТОБУСА ІЗ САМОУСТАНОВЛЮВАЛЬНОЮ ВІССЮ ПРИЧЕПА

Анотація. *Останнім часом шарнірно-з'єднані автобуси (ШЗА), місткість яких 150–200 осіб, все ширше застосовують у пасажирських перевезеннях. Габаритна довжина таких автобусів обмежена на рівні 18,5 м, що пояснюється необхідністю дотримання нормативних вимог щодо маневреності.*

В Україні Правилами дорожнього руху дозволена загальна довжина автопоїзда на рівні 22 м. За такої довжини пасажиромісткість ШЗА істотно збільшується, проте відкритими залишаються питання маневреності та стійкості руху таких автобусів. Попередніми дослідженнями показано, що навіть автобус загальною довжиною 18,75 м з некерованою віссю причепа не задовольняє вимоги нормативних документів щодо маневреності. Застосування самоустановлювальної осі причепа задовольняє вимоги щодо маневреності навіть за довжини ШЗА до 20,0 м, проте залишається відкритим питання щодо стійкості такого автобуса. За уточненою математичною моделлю дволанкового автопоїзда, адаптованою для ШЗА з самоустановлювальною віссю причепа, визначено показники стійкості шарнірно-з'єднаного автобуса у різних режимах руху. Показано, що критична швидкість ШЗА із заблокованими колесами самоустановлювальної осі причепа досягає 31,87 м/с, що істотно перевищує максимальну швидкість руху автобуса. За наявності збурення зміна бічної та кутової швидкостей ШЗА у часі перехідного процесу за швидкості 6 м/с згасає за логарифмічним законом, що свідчить про стійкість руху ШЗА. У разі збільшення швидкості до 12 м/с зміни бічної та кутової швидкостей також згасають, але спостерігаються інтенсивніші коливання, які за швидкості 14 м/с стають розбіжними, що призводить до втрати стійкості руху ШЗА. Аналогічні результати отримані й у разі повороту ШЗА на 90^0*. Отже, гранична швидкість руху ШЗА із розблокованими колесами самоустановлювальної осі причепа не повинна перевищувати 14 м/с. Після досягнення цієї швидкості колеса самоустановлювальної осі повинні блокуватися.*

Це зумовлює галузь застосування самоустановлювальної осі причепа на шарнірно-зчленованих автобусах. Для руху із вищими швидкостями необхідні принципово нові системи управління автобусом і причепом ШЗА.

Ключові слова: *шарнірно-з'єднаний автобус, причіп, самоустановлювальна вісь, швидкість, збурення, коливання, стійкість.*