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Numerical solution for fractional differential equations by using Jacobi–Gauss–Radau collocation method

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This study proposes a novel numerical approach for addressing both linear and nonlinear initial fractional order differential equations (FDEs) through the implementation of the Jacobi–Gauss–Radau (JGR) integrated with Caputo fractional derivatives. The problem is effectively transformed into a simplified system of FDEs, encompassing the unknown coefficients, by employing shifted JGR points for the FDEs and their initial conditions. For the purpose of investigating the effectiveness and accuracy of the introduced method, some numerical illustrations are provided for various linear and nonlinear FDEs.

Keywords: *fractional differential equations; numerical solution; Jacobi–Gauss–Radau, collocation method; spectral methods; fractional calculus.*

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1. Introduction

In recent decades, fractional calculus has garnered the attention of researchers as an efficient tool in a range of fields, including engineering, biology, physics, mathematics, and many more [1–3]. Researchers, scientists, and engineers have shown interest in fractional calculus because it excels at modeling real-life problems and scientific phenomena with remarkable accuracy. Initially, when fractional calculus was first introduced, very little attentions was given to the area. However, in recent years, it has gained significant traction and is now extensively researched due to its practical applications in real life. Some applications include modeling biophysical behaviors, biological tissues, diffusion processes, gravitational forces and thermodynamics. It has also been applied to understanding properties like thermal conductivity, elasticity, viscosity, highlighting the capability and applicability of fractional calculus [4].

Due to increasing attention to fractional calculus, there has been a surge in the development and analysis of numerical techniques aimed at solving fractional differential equations (FDEs). These equations are inherently more complex than typical ordinary differential equations (ODEs), often requiring sophisticated numerical methods to approximate their solutions accurately [5]. Consequently, the literature is rich with numerical studies dedicated to solving FDEs using various methods, including the finite difference and finite element methods, Adams–Bashforth–Moulton predictor–corrector schemes, and splines method [6–10]. It has been a race to obtain numerical approximation as close as we could with the exact solutions.

In addition to these methods, spectral methods are often regarded as very efficient tools in solving a handful of types of differential equations that appear in numerous topics, especially in engineering, mathematics and science. The efficiency of spectral methods has motivated numerous researchers

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to adopt these approaches to handle different differential equations. In particular, collocation, tau, and Galerkin methods are some spectral methods recognized for their wide use and effectiveness [11]. Additionally, researchers in [12,13] studied the spectral tau method for dealing with FDEs. Meanwhile, in [14], Pedas and Tamme demonstrated the numerical resolution of FDEs through spline collocation methods. In some different works, spectral solutions for a distinct family of fractional-order initial value problems have been achieved where the pseudo-spectral method has been used [15–19].

Within this context, Abdelkawy, in 2018, applied a mixed approach of fractional order Jacobi orthogonal functions, which are used as a basis with shifted Jacobi polynomials, to approximate multi-dimensional fractional diffusion equations of distributed order [20]. Then, in 2020, he used the shifted fractional order JGR collocation and shifted Jacobi–Gauss–Lobatto collocation methods for solving fractional order partial differential equations of distributed order [21]. A Jacobi collocation method was developed and implemented to solve the Ginzburg–Landau equation of fractional order by Yang et al. in 2020 [22]. In 2022, Zaky et al. employed the Jacobi collocation method to handle fractional differential equations of Caputo–Hadamard type [23].

This paper introduces an optimized Jacobi–Gauss–Radau collocation method, specifically designed to solve nonlinear initial value problems (IVPs) in fractional-order settings. We aim to demonstrate the formulation of this method and to showcase its superiority through numerical testing. By comparing its performance against established methods, we highlight the significant improvements in accuracy and computational efficiency achieved by our proposed method.

The structure of this work is organized as follows. Section 2 presents essential preliminaries required for readers to understand fractional calculus. This section also discusses the characteristics of Jacobi polynomials and concludes with a description of the application of the collocation method to solve both linear and nonlinear initial value problems (IVPs) of FDEs. Section 3 outlines the numerical results to demonstrate the effectiveness of the proposed methodology. Section 4 provides the conclusion to summarize the findings.

2. Preliminaries

In this section, we provide fundamental concepts and properties related to fractional derivatives and integral operators [24–26]. Then, we discuss the shifted Jacobi polynomials, highlighting their properties. All of this will be used in the upcoming sections.

2.1. The fractional integral and derivative

We begin this subsection by introducing the Riemann–Liouville fractional integral operator. This serves as a basis for the reader to fully understand this paper.

Definition 1. *The Riemann–Liouville fractional integral operator of order $\upsilon \in \mathbb{R}^+$ and defined on the usual Lebesgue space $L_1[a,b]$ is denoted by I_a^υ and defined as:*

$$I_a^\upsilon Z(x) = \frac{1}{\Gamma(\upsilon)} \int_a^x \frac{Z(\zeta)}{(x-\zeta)^{1-\upsilon}}\, d\zeta, \quad \upsilon > 0, \quad a \leqslant x \leqslant b,$$

$$I_a^0 Z(x) = Z(x).$$

The operator I_a^υ has the following properties:

$$I_a^\upsilon I_a^\mu = I_a^{\upsilon+\mu}, \quad I_a^\upsilon I_a^\mu = I_a^\mu I_a^\upsilon,$$

Next, we have the definition for Caputo differential operator.

Definition 2. *The fractional differential operator in Caputo sense is defined as following:*

$${}_cD_a^\upsilon f(x) = \frac{1}{\Gamma(s-\upsilon)} \int_a^x \frac{f(\zeta)}{(x-\zeta)^{\upsilon-s+1}}\, d\zeta, \quad a \prec x \leqslant b,$$

where $s-1 \prec \upsilon \prec s$, $s \in N$.

Some properties of Caputo fractional derivative operator are

$$I_a^\alpha D_a^\alpha f(x) = f(x) - \sum_{i=0}^{m-1} f^{(i)}(o^+)\frac{x^j}{i!},$$

$$D_a^\alpha x^\gamma = \begin{cases} 0, & \gamma \in N_0, \quad \gamma \prec [\alpha], \\ \dfrac{\Gamma(\gamma+1)}{\Gamma(\gamma-\alpha+1)} x^{\gamma-\alpha}, & \gamma \in N_0, \quad \gamma \succcurlyeq [\alpha]'. \end{cases}$$

2.2. Some results on Jacobi polynomials

The Jacobi polynomials, denoted by $P_r^{(\eta,\delta)}(x)$, $(r = 0, 1, 2, \ldots)$ of degree r, defined on the interval $[-1, 1]$. The formula presented here can be obtained by means of Rodrigue's formula as follows [11,27, 28]:

$$P_r^{(\lambda,\gamma)}(x) = \frac{(-1)^r}{2^r(1-x)^\lambda(1+x)^\gamma r!} \times \frac{d^r}{dx^r}\big[(1-x)^{\lambda+r}(1+x)^{\gamma+r}\big],$$

where $r = 0, 1, 2, \ldots$ and $\lambda, \gamma \succ -1$. The q-th derivative of $P_i^{(\eta,\delta)}(x)$ is given as follows

$$\frac{d^q}{dx^q} P_r^{(\lambda,\gamma)}(x) = \frac{\Gamma(\lambda+\gamma+r+1)}{2^q\,\Gamma(\lambda+\gamma+r-1)} P_{r-q}^{(\lambda+q,\gamma+q)}(x),$$

In addition to this, the Jacobi polynomial discussed satisfies the orthogonality relation given by

$$\big(P_k^{(\lambda,\gamma)}(x).P_L^{(\lambda,\gamma)}(x)\big)\omega^{(\lambda,\gamma)} = \int_{-1}^{1} P_k^{(\lambda,\gamma)}(x)\, P_L^{(\lambda,\gamma)}(x)\, \omega^{(\lambda,\gamma)} dx = h_k\, \sigma_{Lk},$$

where σ_{Lk} denotes the Kronecker delta function. Then, we have

$$\omega^{(\lambda,\gamma)}(x) = (1-x)^\lambda(1+x)^\gamma; \quad h_k = \frac{2^{\lambda+\gamma+1}\Gamma(k+\lambda+1)\Gamma(k+\gamma+1)}{(2k+\lambda+\gamma+1)k!\Gamma(k+\lambda+\gamma+1)}.$$

To utilize the Jacobi polynomials on the said interval $[0, l]$, we have to change the variable $\tau = \frac{2x-l}{l}$. Thus, we get polynomials termed the shifted Jacobi polynomials.

Suppose the shifted Jacobi polynomials $p_i^{(\lambda,\gamma)}\left(\frac{2x-l}{l}\right)$ are introduced by $p_{l,i}^{(\lambda,\gamma)}(\tau)$, which can be produced using the recurrence formula:

$$p_{l,r+1}^{(\lambda,\gamma)}(\tau) = \big(a_r^{(\lambda,\gamma)}(t) - b_r^{(\lambda,\gamma)}(t)\big)p_{l,r}^{(\lambda,\gamma)}(\tau) - c_r^{(\lambda,\gamma)}(\tau)\, p_{l,r-1}^{(\lambda,\gamma)}(\tau), \quad r = 1, 2, \ldots,$$

where

$$a_r^{(\lambda,\gamma)} = \frac{(2r+\lambda+\gamma+1)(2r+\lambda+\gamma+2)}{2(r+1)(r+\lambda+\gamma+1)},$$

$$b_r^{(\lambda,\gamma)} = \frac{(\gamma^2+\lambda^2)(2r+\lambda+\gamma+1)}{2(r+1)(r+\lambda+\gamma+1)(2r+\lambda+\gamma)},$$

$$c_r^{(\lambda,\gamma)} = \frac{(r+\lambda)(r+\gamma)(2r+\lambda+\gamma+2)}{(r+1)(r+\lambda+\gamma+1)(2r+\lambda+\gamma)}.$$

The shifted Jacobi polynomials $P_{l,r}^{(\lambda,\gamma)}(\tau)$ of degree r is written with the following explicit analytical form:

$$P_{l,r}^{(\lambda,\gamma)}(\tau) = \sum_{s=0}^{r}(-1)^{r+s}\frac{\Gamma(r+\gamma+1)\Gamma(r+s+\lambda+\gamma+1)}{\Gamma(s+\gamma+1)\Gamma(r+\lambda+\gamma+1)(r-s)!s!l^s}\tau^s,$$

and the orthogonality condition is

$$\int_0^l P_{l,r}^{(\lambda,\gamma)}(\tau)\, P_{l,z}^{(\lambda,\gamma)}(\tau)\, \omega_{l,z}^{(\lambda,\gamma)}(\tau)\, dt = h_{l,z}^{(\lambda,\gamma)}\delta_{r,z},$$

where $\omega_{l,z}^{(\lambda,\gamma)}(\tau) = \tau^\gamma(l-\tau)^\lambda$, and

$$h_{l,z}^{(\lambda,\gamma)} = \frac{l^{\lambda+\gamma+1}\,\Gamma(z+\lambda+1)\,\Gamma(z+\gamma+1)}{(2z+\lambda+\gamma+1)\,z!\,\Gamma(z+\lambda+\gamma+1)}.$$

Then, the function $y(t)$ is assumed to be square-integrable over the interval $[0, l]$ and they may be represented in the form of shifted Jacobi polynomials as follows:

$$y(t) = \sum_{k=0}^{\infty} c_k \, P_{l,k}^{(\lambda,\gamma)}(t),$$

and the coefficients c_k are defined as follows:

$$c_k = \frac{1}{h_{l,k}^{(\lambda,\gamma)}} \int_0^l y(t) \, P_{l,k}^{(\lambda,\gamma)}(t) \, \omega_{l,k}^{(\lambda,\gamma)}(t) \, dt, \quad k = 0, 1, \ldots.$$

Next, $y(t)$ is approximated by the first $(q+1)$-terms,

$$y_q(t) = \sum_{k=0}^{q} c_k \, P_{l,k}^{(\lambda,\gamma)}(t).$$

2.3. The collocation method

To further illustrate the proposed methodology, following fractional order linear differential equation is considered,

$$\sum_{k=0}^{n} D^{v_k} y(x) = f(x), \quad k - v_k \leqslant k + 1, \tag{1}$$

with the following initial conditions,

$$y^{(i)} = \beta_i, \quad i = 0, 1, \ldots, s. \tag{2}$$

By using approximation on the interval $[0, l]$ that has been partitioned using a truncated series of shifted Jacobi polynomials, the solution of $y(x)$ in (1) can be obtained:

$$y_q(x) = \sum_{z=0}^{q} c_z \, P_{l,z}^{(\lambda,\gamma)}(x), \tag{3}$$

and c_z denotes the unknown coefficients.

Next, by taking (3) into (1) and (2), we have

$$P_{l,z}^{(\lambda,\gamma,v_k)}(x) = D^{v_k} P_{l,z}^{(\lambda,\gamma)}(x), \quad P_{l,z}^{(\lambda,\gamma,z)}(0) = \frac{d^i}{dx^j} P_{l,z}^{(\lambda,\gamma)}(0),$$

$$\sum_{z=0}^{q} c_z \sum_{k=0}^{n} P_{l,z}^{(\lambda,\gamma,v_k)}(x) = f(x),$$

$$\sum_{z=0}^{q} P_{l,z}^{(\lambda,\gamma,i)}(0) = B_i, \quad i = 0, \ldots, s,$$

that can be used for solving a system of equations with $s+1$ equations together with $q+1$ unknowns. For us to obtain the remaining $q - s$ equations, one of the types of collocation points is used, that is, the JGR points x_i $(i = 1, \ldots, q - s)$. Here, the method has successfully reduced the solution of (1) to the solution of $EC = B$ where E, C and B are defined as follows:

$$E = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix},$$

where

$$e_{1(q-s)(q+1)} = \sum_{k=0}^{s} P_{l,z}^{(\lambda,\gamma,v_k)}(x_i), \quad i = 1, \ldots, q - s, \quad z = 0, 1, \ldots, q,$$

$$e_{2(s+1)(q+1)} = P_{l,z}^{(\lambda,\gamma,i)}(0), \quad i = 1, \ldots, s, \quad z = 0, 1, \ldots, q,$$

$$C = \begin{pmatrix} c_0 \\ c_1 \\ \vdots \\ c_n \end{pmatrix},$$

and vector

$$B = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

where $b_{1(s+1)\times 1} = \beta_i$, $i = 0, 1, \ldots, s$ and $b_{2(q-s)\times 1} = f(x_i)$, $i = 1, \ldots q-s$. More details can be read in the following studies [11, 29, 30].

The JGR quadrate nodes are the zeros of $P_N^{(\lambda,\gamma+1)}(x)$ and we have the weights to be

$$\bar{\omega}_{N,0}^{(\lambda,\gamma)} = \frac{2^{\lambda+\gamma+1}\Gamma(\gamma+1)\,\Gamma(N+1)\,\Gamma(N+1+\lambda)}{\Gamma(N+\gamma+2)\,\Gamma(N+\lambda+\gamma+2)},$$

$$\bar{\omega}_{N,j}^{(\lambda,\gamma)} = \frac{1}{1+t_{N,j}^{(\lambda,\gamma)}} \frac{C_{N-1}^{\sim(\lambda+\gamma+1)}}{\left[\partial_t P_N^{(\lambda,\gamma+1)}(t_{N,j})^{(\lambda,\gamma)}\right]^2},$$

where

$$C_{N-1}^{\sim(\lambda+\gamma+1)} = \frac{2^{\lambda+\gamma+1}\,\Gamma(N+\lambda+2)\,\Gamma(N+\gamma+2)}{(N+1)!\,\Gamma(2N+\lambda+\gamma+2)}.$$

Note: the nodes of the shifted JGR on the said interval $[0, l]$ are $x_{l,i}^{(\lambda,\gamma)} = \frac{l}{2}(x_i^{(\lambda,\gamma)} + 1)$, where $x_i^{(\lambda,\gamma)}$ are the JGR quadrate nodes.

3. Examples and comparisons

Within this particular section, we consider the infinity norm to illustrate the efficiency and accuracy of the method proposed in this work for some numerical examples. Maple Software (Version 2017) has been used in this paper to obtain the results. For this purpose, 32-digit precision is considered.

Example 1. Suppose we have an IVP of fractional order linear differential equation given by

$$D^2 y + \sin(x) D^{\frac{1}{2}} y + xy = x(x^8 - x^7) + 56x^6 - 42x^5 + \frac{2048\sin(x)x^{\frac{13}{2}}(16x-15)}{6435i\sqrt{\Pi}},$$

$$x \in [0,1], \quad y(0) = y'(0) = 0.$$

The exact solution is $y(x) = x^8 - x^7$ [18].

Table 1. Comparison of our method and Q-SJT method [18] at different values of q using maximum absolute error Example 1.

q	Q-SJT ($\lambda = \gamma = 0$) [9]	Proposed method($\lambda = 0$, $\gamma = 0.0001$)
4	3.3×10^{-2}	6.3×10^{-2}
8	9.0×10^{-10}	1.3×10^{-29}
12	1.8×10^{-13}	7.1×10^{-30}
16	8.8×10^{-16}	3.8×10^{-30}

We can observe from Table 1 that the error for the proposed method is extremely small compared to the quadrature shifted Jacobi tau (Q-SJT) method in [18], indicating that the approximated solution obtained using the JGR collocation method is significantly similar to the exact solution. In addition, Figure 1 gives the graphical illustration for the approximate solution together with the exact solution.

Example 2. Next, let us consider an IVP of the nonlinear FDE given by

$$D^{\frac{3}{2}} y + D^{\frac{1}{2}} y + Dy + y + e^y = \frac{4\sqrt{x}}{\sqrt{\pi}} + \frac{3x^{\frac{3}{2}}}{8\sqrt{\pi}} + 2x + e^{x^2}, \quad x \in [0,1], \quad y(0) = y'(0) = 0.$$

From [31], we know that the exact solution is $y(x) = x^2$.

We see from Table 2 that our method is superior to the Spectral Adomian Decomposition method (SADM) using the Legendre polynomial [31]. Figure 2 illustrates the comparison done between the approximate solution using the proposed method and the exact solution.

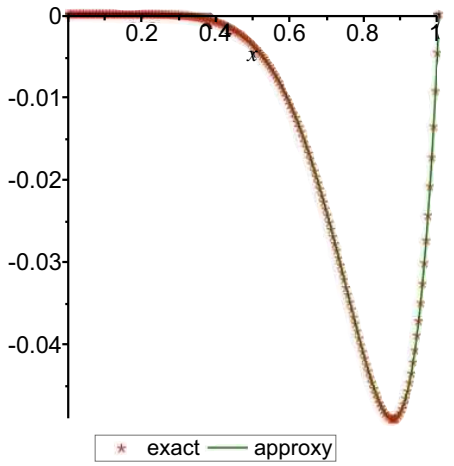

Fig. 1. The exact and approximate solutions for Example 1.

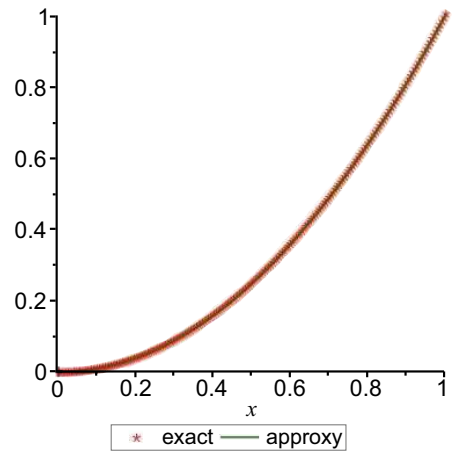

Fig. 2. A comparison between the exact and the approximate solutions for Example 2.

Table 2. A comparison between SADM using Legendre polynomials and our proposed method for Example 2.

SADM by Legendre Polynomial [31]		Proposed Method ($\lambda = \gamma = 0$)	
K	L_∞	q	L_∞
10	1.0×10^{-5}	4	1.3×10^{-31}
20	1.6×10^{-9}	7	1.2×10^{-31}
30	3.0×10^{-13}	14	2.4×10^{-32}

4. Conclusion

In this research, we introduced a direct approach for solving linear and nonlinear IVPs involving fractional-order differential equations, formulated using the Caputo–type fractional derivative. Our algorithm leverages the Jacobi–Gauss–Radau (JGR) collocation method, providing highly accurate numerical solutions. The precision and effectiveness of our approach are demonstrated through comparative analysis, where the results closely align with the exact solutions, indicating the effectiveness of the introduced method in handling fractional-order differential equations.

The convergence analysis of the current approach is based on operator theory, polynomial approximation theory for orthogonal polynomials, and the Lebesgue constants, which correspond to the Lagrange interpolation polynomials. These methods provide spectral accuracy and exponential rates of convergence for problems in simple geometries.

Given the efficiency and accuracy demonstrated, this method holds significant potential for future applications, including solving one- and two-dimensional partial integro-differential equations and fractional partial integro-differential equations.

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Чисельне розв'язування дробових диференціальних рівнянь методом колокації Якобі–Гаусса–Радау

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У цьому дослідженні пропонується новий чисельний підхід для розв'язування як лінійних, так і нелінійних диференціальних рівнянь дробового порядку (FDE) через реалізацію методу Якобі–Гаусса–Радау (JGR), інтегрованого з дробовими похідними Капуто. Задача фактично перетворюється на спрощену систему FDE, що містить невідомі коефіцієнти, шляхом використання зміщених точок JGR для FDE та їх початкових умов. З метою дослідження ефективності та точності запропонованого методу наведено числові ілюстрації для різних лінійних і нелінійних ФДУ.

Ключові слова: *дробові диференціальні рівняння; чисельний розв'язок; Якобі–Гаусс–Радау, метод колокації; спектральні методи; дробове числення.*