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ABOUT THE STABILITY OF THE ALTITUDE BASE BENCHMARKS

The purpose of the study is to assess the stability of the height base benchmarks by comparing the approximate and strict methods of processing the results of repeated leveling cycles. The study involves the analysis of methods based on selecting the most stable benchmark, utilizing the principles of “relative” and “absolute” evaluation. The goal is also to identify the most efficient approach to the mathematical processing of leveling results within free geodetic networks, which pose a high risk of false identification of stable points. This issue mainly concerns the initial selection of the reference surface, which is essential for calculating the benchmark marks, their vertical displacements, and assessing the stability of the benchmarks themselves. Methods and results. The article considers various methods for assessing the stability of benchmarks, which can be classified into “relative” and “absolute”, depending on the accuracy measurement criteria. Mathematical calculations are presented for the measurement results of the elevations between benchmarks in several leveling cycles of the leveling network at the industrial site. The maximum errors for each benchmark are determined. Through this analysis, benchmarks that meet the stability requirements were identified, and criteria were established for assessing the relative stability and instability of benchmarks. A comparison of the approximate and strict methods for evaluating benchmark stability, based on the measurement results, showed that the approximate method developed by A. Kostekhel is one of the most efficient approaches for determining the benchmark stability. The strict method allows for the accurate and reliable determination of the benchmark displacement; however, it may require more computational resources. The studies showed that the two methods considered in the article agree on the results of determining the benchmark stability. Scientific novelty. The article contains valuable information for ensuring the accuracy of monitoring deformations and the stability of building structures. By utilizing the proposed universal algorithm and open-source software to analyze measurement results from repeated leveling cycles, we accurately assessed the stability of benchmark heights. This assessment is vital when performing repeated geodetic measurements on large construction sites, as it ensures the precise determination of building and structure settlement.

Key words: benchmark height stability, methods for analyzing the benchmark stability, building settlement, free height network.

Introduction

Studying the benchmark stability of the Altitude Base is an important aspect of ensuring the reliability and accuracy of monitoring deformations in engineering structures. This is particularly important in cases where these deformations are associated with vertical displacements that can lead to serious consequences for buildings and infrastructure. Increased attention to this issue highlights the need to improve the accuracy of observations and increase the benchmark stability, underpinning such measurements. Geodetic control networks are free networks that can be affected by data errors [Chen et al., 1990]. As a result, accurately identifying mutually stable points can be quite challenging, and, in some cases, impossible for many unstable points. Misidentification leads to incorrect determination of the basic data for the calculated deformations of the object and, as a result, to misinformation. Identifying mutually stable points is the only serious problem, and it is a subject of interest for geodesists. Therefore, increased attention is always paid to monitoring deformations of engineering structures. It is known that vertical displacements of structures mainly depend on the physical and mechanical properties of soils and random factors affecting them, including temperature, humidity, and changes in groundwater levels. All this leads to the need to solve the problem of assessing the stability benchmarks of the Altitude Base to determine the deformations of the upper soil layer and the foundations of engineering structures. Of course, the task is relevant due to the increasing demands for stability in both newly constructed and existing buildings and structures. There have been numerous incidents where buildings have entered an emergency state.

The spectrum of possible deformations is quite broad. Buildings and structures can have subsidence, buckling, tilting, torsional deformations, as well as local tension and compression. Therefore, there is a need to monitor various objects, especially hydroelectric power plants and hydro-storage power plants in the energy sector. At the same time, it is important that these observations can be organized under a wide range of conditions with the necessary accuracy and can operate autonomously. This involves creating observation networks that allow for a sufficient degree of freedom in mathematical processing, ensuring reliable and accurate results.

This article considers the issue of assessing the stability of reference benchmarks directly responsible for the accuracy of measurements of building settlements. The reliability of the results of the observations of the building foundations' subsidence largely depends on the invariability of the height position of the initial benchmarks. The stability of benchmarks is periodically checked by measuring the elevations $h_1, ..., h_i$. The change in elevation values between benchmarks in repeated measurement cycles is random and depends mainly on their stability.

During the mathematical processing of the results of repeated levelling, questions arise that have not been finally resolved in the geodetic literature. First of all, this is related to the problem of the initial choice of the reference surface used to calculate the benchmark marks and assess their vertical displacements, as well as the stability of the benchmarks themselves. Considering the complexity of the problem and the ambiguity of the results yielded by current methods for assessing benchmark stability, further research is needed to develop new, more accurate, and universal approaches for determining the stability of height benchmarks. This may include the development of automated data processing systems that reduce the risks of random factors influencing measurement accuracy and increase the efficiency of such systems in real-world conditions.

Methods for assessing the stability of benchmarks of the Altitude Base

The stability of an elevation network can be assessed by various methods [Ganshin, Storozhenko, 1981; Martuszewicz, 1982; Dyakov,1992, 2009; Kostecka, et al., 2011; Rabynovych, 1977; Pylypiuk, & Ilkiv, 1986; Fedoseev, 1977; Tserklevych, & Khomyak, 1977; Velsink, 2015]. In contemporary geodetic literature, methods for establishing the most stable benchmark are typically divided into two main groups. In the first group, the benchmark considered the most stable serves as the reference plane. This approach encompasses the methods developed by A. Solovyov, A. Kostekhel, V. Martusevich, L. Sribnyakova, and I. Runov [Ganshin, Storozhenko, 1981; Martuszewicz, 1982]. In the second group, the initial reference plane is the average value of the marks of all the studied benchmarks. This method is attributed to P. Marchak, V. Chernikov [Dyakov, 2009]. Another classification distinguishes methods based on two principles, which can be labelled as "relative" and "absolute". The group of "relative" methods relies on the criteria that assess measurement accuracy based on the differences in subsidence. Notable methods in this category include those developed by V. Karpenko and Ya. Martusevich [Martuszewicz, 1982]. In contrast, the "absolute" evaluation methods use criteria based on the absolute calculation error in assessing measurement accuracy. This group includes methods developed by A. Kostekhel, V. Chernikov [Ganshin, Storozhenko, 1981; Rabynovych, 1977]. Considering these methods in detail reveals their specific aspects. Some methods are more widely used in the calculation assessment due to the algorithm's simplicity. Still, others may have a more universal nature and a complex algorithm to search for stable benchmarks. It should be noted that methods were compared in various studies. This enabled the continuation of research in this direction because of the ambiguity surrounding the assessment of benchmark stability [Fedoseev, 1977; Baselga et al., 2015]. This situation enabled the continuation of research in this area due to the ambiguity surrounding the assessment of benchmark stability [Fedoseev, 1977; Baselga et al., 2015]. If we examine the problem mathematically rather than qualitatively, it is important to note the degeneracy of the matrix of normal equations, which results in multiple possible solutions. The standard procedure, which allows obtaining a solution using a pseudo-inverse matrix, makes an implicit assumption regarding a given network: it assumes that the observed displacement is distributed between most points of the network [Nowel, 2019; Baselga et al., 2015; Zienkiewicz et al., 2017].

It should also be noted that due to the complexity of the assigned task and the ambiguity of the results using current methods for assessing benchmark stability, there is a need for further research. Specifically, we should develop new, more accurate, and universal approaches to determine the stability of height benchmarks. This may include creating automated data processing systems that minimize the impact of random factors on measurement accuracy and enhance the efficiency of these systems in real-world conditions. In the context of modern construction and operation of engineering structures, establishing autonomous networks of benchmark observations is becoming increasingly important. Such networks would enable the detection and assessment of deformations without constant human involvement. Such systems can be used for long-term monitoring of benchmark stability and maintaining constant control over the condition of buildings.

Comparative analysis of benchmark stability assessment using approximate and proposed strict methods

Let's consider the levelling network on an industrial site, as shown in Fig. 1. Table 1 presents the results of the determined elevations between the benchmarks over five monitoring measurement cycles for two variants of data sets. The height of the starting benchmark for the two variants is *Rp*1 = 10.4764 m and *Rp*1 =10.0537 m. Table 1 indicates the length of the traverse (L) and the number of tripods used in the traverse (n).

To determine the benchmark stability, it is best to use the method developed by A. Kostekhel [Ganshin, Storozhenko, 1981], which is one of the approximate methods. The theoretical basis of this method is to compare the equalized elevation values for the same traverse across previous and current cycles using the formula (1):

$$\Delta i = H_{jI} - H_{ji+1}, \qquad (1)$$

where j is the benchmark number, and i is the leveling cycle. It is believed that the most stable benchmark is the one for which the sum of the elevation differences is minimal, and its height obtained in the first or previous levelling cycle should be taken as the initial one.

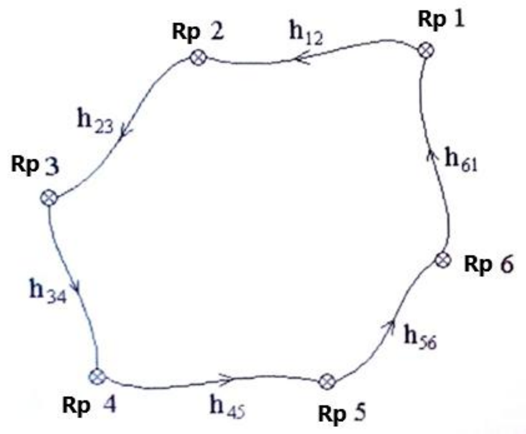

Fig. 1. Scheme of the leveling network

The benchmark stability or instability is determined by comparing Δi with the marginal error Δ_{lim}, calculated by the formula (2)

$$\Delta_{\lim} = k\sqrt{n}, \qquad (2)$$

where K is the accuracy of determining the elevation during the traverse, and n is the number of tripods.

Table 1

Elevation measurement results in the leveling network

No. cycle	elevations, mm											
	h_{12}		h_{23}		h_{34}		h_{45}		h_{56}		h_{61}	
variant	1	2	1	2	1	2	1	2	1	2	1	2
1	118	522	750	94	0	–454	–3	131	–131	–128	–734	–165
2	117	519	745	93	0	–454	–2	128	–134	–122	–726	–164
3	112	520	749	92	–1	–456	1	124	–138	–125	–723	–155
4	123	524	747	96	0	–458	5	127	–137	–124	–738	–165
5	120	523	748	91	0	–450	–3	124	–134	–126	–731	–162
L, km	0.82	0.72	0.84	0.91	0.04	0.78	0.59	0.84	0.64	0.87	0.47	0.69
n	5	9	8	6	4	8	8	10	7	7	10	8

To establish a stable benchmark, we should follow the following steps:

– Determine the most stable benchmark in the running cycle.

– Calculate the heights of the benchmarks in the network based on the initial height of the benchmark and the corresponding elevation.

– Assess the degree of relative stability or instability for each benchmark in the network.

– Balance the polygon for each cycle.

– Determine the differences Δ*i* by using each benchmark (1, 2, 3, 4, 5) as the initial reference. This should be done according to formula (1) for various segments of the network (e. g., 1-2,1-3,1-4,1-5,1-6) and across different cycles I-II, I-III, I-IV, I-V (see Table 2).

– Calculate ΔΔ and [ΔΔ]. The benchmark that yields the minimum value for [ΔΔ] will be designated as the initial benchmark.

Table 2

Results of determining the differences in elevation between cycles

Starting benchmark	Part of the network	Cycle I–II		Cycle I–III		Cycle I–IV		Cycle I–V		ΣΔΔ
		Δ	ΔΔ	Δ	ΔΔ	Δ	ΔΔ	Δ	ΔΔ	
1	2	3	4	5	6	7	8	9	10	11
1	1–2	–1	1	–6	36	5	25	2	4	
	1–3	–6	36	–7	49	2	4	0	0	
	1–4	–6	36	–8	64	2	4	0	0	
	1–5	–5	25	–4	16	10	100	0	0	
	1–6	–8	64	–11	121	4	16	–3	9	
variant 1			162		286		149		13	**610**
variant 2			91		325		36		43	**495**
2	2–3	–5	25	–1	1	–3	9	–2	4	
	2–4	–5	25	–2	4	–3	9	–2	4	
	2–5	–4	16	2	4	5	25	–2	4	
	2–6	–7	49	–5	25	–1	1	–5	25	
	2–1	1	1	6	36	–5	25	–2	4	
variant 1			116		70		69		41	**296**
variant 2			31		209		52		63	**355**
3	3–4	0	0	–1	1	0	0	0	0	
	3–5	1	1	3	9	8	64	0	0	
	3–6	–2	4	–4	16	2	4	–3	9	
	3–1	6	36	7	49	–2	4	0	0	
	3–2	5	25	1	1	3	9	2	4	
variant 1			66		76		81		13	**236**
variant 2			35		141		116		39	**331**
4	4–5	1	1	4	16	8	64	0	0	
	4–6	–2	4	–3	9	2	4	–3	9	
	4–1	6	36	8	64	–2	4	–3	9	
	4–2	5	25	2	4	3	9	–1	1	
	4–3	0	0	1	1	0	0	–3	9	
variant 1			66		94		81		28	**269**
variant 2			35		121		36		164	**356**
5	5–6	–3	9	–7	49	–6	36	–3	9	

Continuation of Table 2

1	2	3	4	5	6	7	8	9	10	11
	5–1	5	25	4	16	–10	100	0	0	
	5–2	4	16	–2	4	–5	25	2	4	
	5–3	–1	1	–3	9	–8	64	0	0	
	5–4	–1	1	–4	16	–8	64	0	0	
variant 1			52		94		289		13	**448**
variant 2			119		429		148		123	**819**
6	6–1	8	64	11	121	–4	16	3	9	
	6–2	7	49	5	25	1	1	5	25	
	6–3	2	4	4	16	–2	4	3	9	
	6–4	2	4	3	9	–2	4	3	9	
	6–5	3	9	7	49	6	36	3	9	
variant 1			130		220		61		61	**472**
variant 2			59		225		36		55	**375**

Table 3

Results of calculating the heights of the benchmarks and the deviation limits

Parameter	Cycle of levelling									
	I		II		III		IV		V	
Variants	1	2	1	2	1	2	1	2	1	2
H Rp 1	9.6084	9.4377	9.6144	9.4417	9.6154	9.4417	9.6064	9.4337	9.6084	9.4397
Δ_1			6.0	4.0	7.0	4.0	–2.0	–4.0	0.0	2.0
Δ_{lim}			2.8	2.5	2.8	2.5	2.8	2.5	2.8	2.5
H Rp 2	9.7264	9.9597	9.7314	9.9607	9.7274	9.9617	9.7294	9.9577	9.7284	9.9627
Δ_2			5.0	1.0	1.0	2.0	3.0	–2.0	2.0	3.0
Δ_{lim}			2.0	2.7	2.0	2.7	2.0	2.7	2.0	2.7
H Rp 3	10.4764	10.0537	10.4764	10.0537	10.4764	10.0537	10.4764	10.0537	10.4764	10.0537
Δ_3			0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Δ_{lim}			2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
H Rp 4	10.4764	9.5997	10.4764	9.5997	10.4754	9.5977	10.4764	9.5957	10.4764	9.6037
Δ_4			0.0	0.0	–1.0	–2.0	0.0	–4.0	0.0	4.0
Δ_{lim}			1.8	2.5	1.8	2.5	1.8	2.5	1.8	2.5
H Rp 5	10.4734	9.7307	10.4744	9,7277	10,4764	9,7217	10.4814	9.7227	10.4734	9.7277
Δ_5			1.0	–3.0	3.0	–9.0	8.0	–8.0	0.0	–3.0
Δ_{lim}			2.5	2.8	2.5	2.8	2.5	2.8	2.5	2.8
H Rp 6	10.3424	9.6027	10.3404	9.6057	10.3384	9.5967	10.3444	9.5987	10.3394	9.6017
Δ_6			–2.0	3.0	–4.0	–6.0	2.0	–4.0	–3.0	–1.0
Δ_{lim}			2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.4

– Hereinafter, we calculate the heights of the benchmarks relative to the initial benchmark No. 3. (see Table 3).

– Calculate the gap between cycles and the limiting gap (see Table 3).

$\Delta = Ni - N_I$, where Ni is the height of the benchmark in the current cycle (i = II, …, V), and N_I is the height of the benchmark in the first cycle. $\Delta_{lim} = 0.9$ mm·$\sqrt{n}$.

Let's form Table 4 and analyze the benchmark stability in all cycles. The formula determines the limit of the benchmark stability:

$$\boldsymbol{N} = |\Delta_{lim}/\Delta_i| > 1. \quad (3)$$

Relative benchmark instability

$$\boldsymbol{J} = |\Delta_{lim}/\Delta_i| < 1. \quad (4)$$

We calculate the stability of the benchmarks in all cycles I–II, I–III, I–IV and I–V relative to the original benchmark.

Table 4

The results of measuring the benchmark stability or instability

Number of the bench-mark	Reversed weight of the traverse		Change of the benchmark height $\varDelta = H_i - H_I$		$\Delta_{permissible}$ = 0.9 мм·$\sqrt{n}$, mm		Limit of stability N $N = \|\Delta_{lim}/\Delta_i\|$			
Variants	1	2	1	2	1	2	1	2	1	2
Cycles *I–II with six benchmarks (starting benchmark Rp 3)*										
4	4	8	0.0	0.0	1.8	2.5	infinite	infinite	stability	stability
5	8	10	1.0	–3.0	2.5	2.8	2.5	0.9	stability	instability
6	7	7	–2.0	3.0	2.4	2.4	1.2	0.8	stability	instability
1	10	8	6.0	4.0	2.8	2.5	0.5	0.6	instability	instability
2	5	9	5.0	1.0	2.0	2.7	0.4	2.7	instability	stability
Cycle *I with six benchmarks (starting benchmark Rp 3)*										
4	4	8	–1.0	–2.0	1.8	2.5	1.8	1.3	stability	stability
5	8	10	3.0	–9.0	2.5	2.8	0.8	0.3	instability	instability
6	7	7	–4.0	–6.0	2.4	2.4	0.6	0.4	instability	instability
1	10	8	7.0	4.0	2.8	2.5	0.4	0.6	instability	instability
2	5	9	1.0	2.0	2.0	2.7	2.0	1.3	stability	stability
Cycles *I–IV with six benchmarks (starting benchmark Rp 3)*										
4	4	8	0.0	–4.0	1.8	2.5	infinite	0.6	stability	stability
5	8	10	8.0	–8.0	2.5	2.8	0.3	0.4	instability	instability
6	7	7	2.0	–4.0	2.4	2.4	1.2	0.6	stability	instability
1	10	8	–2.0	–4.0	2.8	2.5	1.4	0.6	stability	instability
2	5	9	3.0	–2.0	2.0	2.7	0.7	1.3	instability	stability
Cycles *I–V with six benchmarks (starting benchmark Rp 3)*										
4	4	8	0.0	4.0	1.8	2.5	infinite	0.6	stability	instability
5	8	10	0.0	–3.0	2.5	2.8	infinite	0.9	stability	instability
6	7	7	–3.0	–1.0	2.4	2.4	0.8	2.4	instability	stability
1	10	8	0.0	2.0	2.8	2.5	infinite	1.3	stability	stability
2	5	9	2.0	3.0	2.0	2.7	1.0	0.9	stability	instability

The results of the calculations indicate that, throughout all measurement cycles of elevation measurements in the leveling network, there is only one stable benchmark: Benchmark No. 4, along with the initial benchmark, Benchmark No. 3.

Now, let us compare these findings regarding the stability of the leveling network benchmarks with the results obtained earlier using a more accurate method. This method is based on determining weights and performing matrix calculations through the solution of linear equations by the LU decomposition [Gantmakher, 1967].

Below is an algorithm for solving the specified problem of searching for stable benchmarks in the elevation network. It includes the corresponding data points required for the program code to solve the problem.

1. Output data in code *L* (route lengths)

Contains the lengths of leveling routes in kilometers. This data is used to calculate weights, where shorter lengths yield greater weight. *N* (number of measurements) represents the number of measurements taken for each route.

It is considered in the weight calculation for a more accurate estimate *h* (elevation between benchmarks) is found in Table 1, where each row denotes the elevation (in mm) between benchmarks from a separate measurement cycle. Each column corresponds to the route between two benchmarks. H_{Rp} represents the elevation of the starting point.

2. Calculate average elevations

The calculate average *H* function computes the average elevations for each route.

3. Calculate route weights

This step determines the weight for each route, taking into account the number of measurements (*n*) and the route length (*L*).

4. Build the coefficient matrix A

The buildCoefficientMatrix function creates a coefficient matrix (A) where the route weights are placed along the diagonal:

$$A = \begin{bmatrix} w_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & w_n \end{bmatrix},$$

where $w_i = {}^{n_i}\!/\!_{L_i}$.

5. Form a right-hand side vector

Forms a right-hand side vector *B* for the system of equations $A{\cdot}\Delta H = B$ (in our case $A \cdot \Delta H = -\overline{h}$).

6. Solve the system of equations

Solve the system of linear equations $A{\cdot}\Delta H = B$. Result: an array (in millimeters) containing corrections to the heights of each benchmark: $\Delta H_i = -{}^{\overline{h_i}}\!/\!_{w_i}$.

7. Determination of the benchmark stability

This section calculates the stability of each benchmark as the ratio of the correction ΔH_i in the benchmark height to the weight value w_i. The result is an array showing the relative stability of the benchmarks in millimeters.

8. Calculation of the corrected absolute heights

This step adds the corrections ΔH to the initial height $Rp1$ and converts them to meters.

9. Results that the program outputs (Table 5)

Average elevations (*H*) denote the average value of the elevations for each route. Corrections to heights (Δ*H*) present the value of the corrections made to the benchmark heights. Corrected absolute heights (m). Benchmark stability: This shows the relative stability of each benchmark.

Table 5

Calculation results

Benchmark	Average elevations, mm		Height corrections (Δ*H*), mm		Corrected absolute heights, m		Benchmark stability	
Variant	1	2	1	2	1	2	1	2
Rp 1	118.0	521.6	–19.3	–41.7	10.4570	10.0120	3.1	3.3
Rp 2	747.8	93.2	–78.2	–14.1	10.3981	10.0396	8.1	2.1
Rp 3	–0.2	–454.4	0.0	44.3	10.4764	10.0989	0.0	4.3
Rp 4	–0.4	126.8	0.0	–10.6	10.4764	10.0430	0.0	0.9
Rp 5	–134.8	–125.0	12.3	15.5	10.4887	10.0692	1.1	1.9
Rp 6	–730.4	–162.2	34.0	13.9	10.5104	10.0677	1.6	1.2

Conclusions

The article discusses two primary approaches for evaluating the stability benchmarks of the Altitude Base. These approaches include methods that focus on selecting the most stable benchmark or calculating the average value of all benchmarks, as well as both "relative" and "absolute" assessment methods. Analyzing these methods enables the selection of the most suitable approach based on the specific observation conditions and the accuracy of measurements. However, there is a need to improve methods that allow for more accurate assessment of the benchmark stability, especially on industrial sites, where deformation factors can be more complex. A comparison of approximate and strict methods for assessing the benchmark stability based on measurement results showed that using the A. Kostekhel's method is one of the most efficient for determining the benchmark stability. This strict method enables accurate and reliable determination of the benchmark displacement, though it may require more computational resources. In contrast, approximate methods are usually easier to implement, but their accuracy may be insufficient for complex cases. Analyzing the differences in elevations between leveling cycles allows for a more precise assessment of the benchmark stability. By determining the maximum error in elevations and comparing it with the obtained data, we can more accurately classify benchmarks as stable or unstable. Employing this technique for each measurement cycle helps in selecting the most stable benchmark, which is critically important for ensuring the accuracy and reliability of the results. The studies conducted demonstrate that the two methods considered in the article produced the same results for assessing benchmark stability.

To ensure maximum accuracy and reliability in monitoring structure deformations, it is recommended to use a combination of different methods for assessing the benchmark stability. The proposed universal and strict algorithm enhances the reliability of results when assessing benchmark stability. This is especially important for repeated geodetic measurements in leveling networks on large construction sites, as it helps determine the settlements of buildings and structures.

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ПРО СТІЙКІСТЬ РЕПЕРІВ ВИСОТНОЇ ОСНОВИ

Мета дослідження – оцінювання стійкості реперів висотної основи через порівняння наближеного та строгого методів опрацювання результатів повторних циклів нівелювання. Дослідження передбачає аналіз методів, що ґрунтуються на виборі найстабільнішого репера, згідно із принципами "відносної" і "абсолютної" оцінки. Метою є також виявлення найефективнішого підходу до математичного опрацювання результатів нівелювання в умовах вільних геодезичних мереж, які характеризуються високим ризиком помилкової ідентифікації стабільних точок. Насамперед, це стосується проблеми початкового вибору поверхні відліку, щодо якої необхідно розраховувати позначки реперів та їх вертикальні зміщення, а також самої оцінки стабільності реперів. Методика та результати. У статті розглянуто різні методи оцінювання стійкості реперів, які можна класифікувати на "відносні" та "абсолютні" залежно від критеріїв точності вимірювання. Подано математичні розрахунки для результатів вимірювань перевищень між реперами у кількох циклах нівелювання нівелірної мережі промислового майданчика, а також визначено граничні похибки для кожного репера. В результаті виконаного аналізу виявлено репери, що відповідають вимогам стійкості, та сформульовані критерії для визначення відносної стійкості та нестійкості реперів. Порівняння наближеного та строгого методів оцінювання стійкості реперів на основі результатів вимірювань показало, що застосування наближеного методу А. Костехеля є одним із найефективніших для визначення стійкості реперів. Строгий метод дає змогу точно і надійно визначити зміщення реперів, але може потребувати більше обчислювальних ресурсів. Виконані дослідження показали, що розглянуті в статті два методи узгоджуються за результатами визначення стійкості реперів. Наукова новизна. Стаття містить корисну інформацію для забезпечення точності моніторингу деформацій і стабільності будівельних конструкцій. Із використанням запропонованого універсального алгоритму і відкритого програмного коду опрацювання результатів вимірювань у повторних циклах нівелювань отримано надійну оцінку стійкості реперів висотної основи, що важливо під час виконання повторних геодезичних вимірювань на великих будівельних майданчиках для точного визначення осідань будівель і споруд.

Ключові слова: висотна стійкість репера, методи аналізу стійкості реперів, осідання будівель, вільна висотна мережа.

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