ISSN 2707-1898 (print)

Український журнал інформаційних технологій
Ukrainian Journal of Information Technology
http://science.lpnu.ua/uk/ujit
https://doi.org/10.23939/ujit2025.01.140
Article received 14.03.2025 p.
Article accepted 01.05.2025 p.
UDC 004.422:536.24

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MATHEMATICAL MODELING AND ANALYSIS OF TEMPERATURE REGIME IN DIGITAL DEVICES DUE TO SURFACE HEATING

Linear and nonlinear mathematical models for determining the temperature field and, subsequently, for analyzing temperature regimes in an isotropic plate due to near-surface thermal loading have been developed. For the case of a thermosensitive plate (the thermophysical parameters of the structural material depend on temperature), the Kirchhoff transformation has been applied, using which the nonlinear heat conduction equation and nonlinear boundary conditions have been linearized, and as a result, a linear second-order differential equation with partial derivatives and a discontinuous right-hand side and partially linearized boundary conditions have been obtained. For the final linearization of the boundary conditions, the temperature has been approximated by the spatial coordinate on the boundary surface of the thermosensitive plate by a segment-constant function, which made it possible to obtain a linear boundary problem with respect to the Kirchhoff variable. To solve the obtained boundary value problems, the integral Fourier transform was used and, as a result, analytical and analytical-numerical solutions in the form of improper convergent integrals were obtained. For a thermally sensitive medium, as an example, the linear dependence of the thermal conductivity coefficient of the structural material of the structure on temperature was chosen, which is often used to solve many practical problems. Software tools have been developed, using which a numerical analysis of the behavior of temperature as a function of spatial coordinates for given values of geometric and thermophysical parameters has been performed, and on this basis the behavior of the temperature field has been geometrically depicted. The developed linear and nonlinear mathematical models for determining the temperature field in spatial environments with near-surface heating make it possible to analyze their thermal stability and on this basis it is possible to prevent overheating, which can cause failure not only of structural units and their individual elements, but also of the electronic device as a whole.

Keywords: temperature field; isotropic plate; thermal conductivity of the material; convective heat transfer; surface heating; thermal sensitivity of the material; heat flux; ideal thermal contact.

Introduction

The temperature field in microelectronic devices is an important factor that affects their operation and reliability. An increase in the temperature in the device can lead to a decrease in its efficiency, increased energy consumption, and a reduction in its operating life. Therefore, determining the behavior of the temperature field is an important task in microelectronics. The temperature field in microelectronic devices is formed as a result of the passage of electric current through electronic components. This is due to the fact that when current flows, electrons collide with each other and with atoms of the material, which leads to the generation of heat. As a result, the temperature in the device increases and a temperature field is formed. Experimental determination of the temperature field in microelectronic devices is performed using various methods, in particular, such as thermal impedance microscopy, thermometry, based on the photoelectric effect, thermal microscopy, infrared microscopy, interferometry. Each of these methods has its own advantages and disadvantages and is used to determine the temperature field under certain conditions. Observation of the behavior of the temperature field in microelectronic devices allows not only to determine the temperature in the device, but also to analyze its distribution in it. In the future, this makes it possible to develop designs of microelectronic devices with optimal temperature operating conditions. For example, it is possible to estimate the level of heat release in individual components of microelectronic devices and determine the distribution of heat from active elements to thermal areas, which prevents overheating and damage to the device. To analyze the temperature field in microelectronic devices, computer simulations are widely used, which allow determining the temperature field in devices using mathematical models of thermal processes. As a result, it is possible to predict the temperature modes of operation of devices and determine their design parameters for effective operation. One of the important factors affecting the temperature regime in microelectronic devices is the location of components on the board. In particular, components located in the center of the board can heat up more than components located at the edges, due to the difference in heat dissipation. Also, increasing the density of components on the board can lead to an increase in temperature and deterioration of device performance. An important aspect of studying the behavior of the temperature field in microelectronic devices is ensuring effective heat dissipation, which ensures a decrease in temperature and contributes to increasing their efficiency and reliability. As a result, heat dissipation is a significant factor that leads to a decrease in the reliability of microelectronic devices. The relative influence of temperature is the highest (55 %) compared to humidity (19 %), vibration (20 %) and dust (6 %) [1]. If there is no effective heat dissipation, the device overheats and becomes unusable. Various technologies are used to ensure effective heat dissipation, in particular, heat dissipation materials, fans, thermal pipes, thermoelectric modules, etc. In addition, to ensure effective heat dissipation in microelectronic devices, methods such as coating the surface of components with heat dissipation materials, using radiators, thermal pastes and heat pumps are used. An important task is also to control the temperature regime in microelectronic devices. For this purpose, temperature sensors are used that measure the temperature at individual points of the device, which makes it possible to control the level of heating of the device and take measures to reduce the temperature in a timely manner. Consequently, the study of the temperature field in microelectronic devices is an important task that allows ensuring the effective operation of the device and increasing its reliability. The use of modern methods of analyzing temperature regimes and technologies for effective heat dissipation contributes to the process of developing more efficient and reliable microelectronic devices.

Consequently, the development of mathematical models of the heat conduction process is an urgent problem, since as a result of the operation of modern electronic devices, they are subjected to thermal loads. As a result of intense heating, significant temperature gradients arise that contribute to overheating, which leads to the failure of both individual elements and assemblies and the device as a whole. To prevent this, it is necessary to establish permissible temperature regimes for the effective operation of devices. Without conducting expensive experiments for isotropic media with near-surface heating, the presented research results make it possible to achieve this.

The object of research is linear and nonlinear heat conduction processes in isotropic spatial media that are subjected to near-surface heating.

The subject of the study is linear and nonlinear mathematical models of the heat conduction process and methods for determining analytical and analytical-numerical solutions of the corresponding boundary value problems for isotropic spatial media with near-surface heating.

The purpose of the work is to develop linear and nonlinear mathematical models of heat conduction for an isotropic plate with near-surface heating, as a result of which it will be possible to increase the accuracy of determining temperature fields, which will further affect the effectiveness of design methods for modern electronic devices.

To achieve this goal, it is necessary to perform the following main research tasks:

- analyze the main literary sources in the direction of developing linear and nonlinear mathematical models of heat conduction;
- indicate the object of the study and its linear and nonlinear mathematical models;
- indicate the method of linearization of the nonlinear mathematical model;
- obtain an analytical solution of the linear and analytical-numerical solution of the nonlinear boundary value problems of heat conduction;
- develop algorithms and software tools for their numerical implementation for analyzing temperature regimes in an isotropic plate with near-surface thermal heating.

Analysis of major studies and publications. The study of temperature regimes in structures of both homogeneous and heterogeneous materials is a subject of interest to many scientists. The importance of taking these regimes into account is important for establishing the physical and chemical properties of materials, in particular for the case of significant temperature fluctuations that are inherent in heat conduction processes. Temperature changes cause transformations of material properties, which complicates the determination of temperature distribution, thermal loads and thermoelastic state of structures.

In [2], a method for modeling heat transfer in porous materials with temperature-dependent properties is considered, which is relevant for structures of complex architecture. This approach can be applied to electronic technology, since modern electronic devices with components containing foreign semi-through inclusions face similar challenges in the field of thermal conductivity. It is important to develop mathematical models based on analytical and numerical methods to predict the thermal behavior of such devices, which will contribute to their more efficient operation and increased reliability. The use of the modeling method does not allow to take into account local thermal disturbances, which often occur in devices with foreign semi-through inclusions.

Analytical solutions are given [3] for the distribution of temperature, displacements and stresses in layered rectangular plates with a simple support, which are subjected to thermomechanical loads. The properties of the materials of the layers take into account the temperature dependence.

The analytical solutions given do not describe local thermomechanical loads, which limits their application in problems with real operating conditions.

Reconstruction of the temperature field from limited observations is important for the thermal regulation of electronic equipment. To solve such a problem, a deep learning method combining adaptive UNet and fine multilayer perceptron (MLP) is given in [4]. The method allows to transform the problem of reconstruction of the temperature field into the problem of image-to-image regression. Adaptive UNet reconstructs the general temperature field, while MLP specializes in accurate prediction of zones with large temperature gradients. The results of numerical experiments performed using finite element modeling data show that the maximum absolute errors of the reconstructed temperature field are less than 1 °K. The method has also been tested for different locations of heat sources and observation points. The disadvantage of this approach is the need for a significant amount of data for training the model, which is not easy to provide in real conditions.

Thermomechanical loads of fixed columns for longitudinal thermal heating with different boundary conditions were analyzed [5]. The temperature distribution is determined by the differential quadrature method (DQM). A segmental model of a column with a uniform temperature distribution is used to analyze the deflection. The critical load and deflection mode are determined by the transfer matrix method based on the Euler – Bernoulli theory. The obtained results are confirmed by comparison with literature data and FEM. The influence of temperature and material properties on deflection and critical load is studied. The main drawback of the presented approach is a simplified model for determining the temperature distribution, which does not take into account the appearance of significant temperature gradients as a result of critical temperature loading.

In [6], the main equations and a data set of the thermal model for predicting temperature fields and heating rates when applying localized laser treatments to the Fe-C-Ni alloy are presented. The model takes into account the transient properties of the material and the relationship between temperature and microstructure with an emphasis on the phase dependence of thermal parameters and hysteresis in the phase change. The model provides temperature fields that are consistent with experimental microstructures in laser-affected zones. The presented model can be applied to other materials that demonstrate solid-state transformations during laser processing. The thermophysical parameters are averaged, which leads to errors in the obtained results.

In the article [7], a temperature field model was developed to control the shape of a steel plate during roller hardening. The cooling mechanism was analyzed and heat transfer coefficients were obtained for each surface. The model is based on the heat conduction equation, which allows us to investigate the uniformity of plate cooling. A plate shape control structure was developed and tested experimentally. However, the results show certain errors in modeling for a homogeneous medium.

In [8], the influence of control parameters on dimensionless velocity, temperature, skin friction and local heat transfer rate for two thermal boundary conditions: Newtonian heating and convection was investigated. The thermophysical properties of the fluid remain constant throughout the study for a constant plate surface temperature. Geometric mapping allows analyzing the behavior of the heat flux and temperature distribution with respect to the influence of dimensionless parameters. The studies confirm the influence of boundary conditions on the heat transfer rate, with Newtonian heating leading to an increase in the rate, and convective heating leading to a decrease. This is explained by the heating at the boundary during Newtonian heating, which improves the transfer of thermal energy, in contrast to convective heating. As a result, heat is dissipated due to the moving fluid, which limits the transfer rate. The thermophysical properties of the fluid are generally considered constant, which does not reflect the real conditions during heat treatment. The thermophysical parameters of the fluid may depend on temperature and other factors, and failure to take this into account in the model may lead to significant errors in the research results.

Thermal modeling of electronic devices is one of the most important tools for assessing their reliability in different operating modes. In [9], a thermal model of electronic devices is presented, which is based on experimental temperature measurement data obtained by an infrared camera. These data are input for the developed mathematical model, which is based on the finite difference method and some known physical dependencies. The developed model was verified by comparing the simulation data with the experimentally obtained data. It can be used to study the thermal behavior of the device under different operating conditions. The temperature distribution was determined experimentally, which introduces an error into the developed mathematical model based on the finite difference method. As a result, the obtained results contain significant errors.

In the article [10], dynamic compact thermal models for predicting the case temperature of portable devices, in particular smartphones and laptops, are proposed based on the convolution method. The models allow for quick determination of the case temperature, taking into account the stepwise response of each heat source, but are limited to two devices and are experimental. The model contributes to improving thermal design and determining the temperature control strategy at the early stages of development. The developed model is experimental and does not allow determining the temperature regimes for more than two portable electronic devices.

A solution for the steady-state reaction of thick cylinders subjected to pressure and external heat flux on the inner surface is presented [11]. The effect of the temperature gradient on the deformation of the medium is not taken into account, which significantly worsens the accuracy of the model.

A functional defect causes an increase in temperature and thermal stresses in thermoelectric materials, which reduces the reliability of the devices. In [12], the thermoelectric-elastic fields around an elliptical defect in a two-dimensional thermoelectric plate were investigated using the complex variable method. The results show that the temperature at the defect tip increases with increasing size and can exceed the melting point of the material, and the stresses can exceed the yield strength. This is important for the analysis of material failure.

A thermal analysis of cylinders of different thicknesses made of functionally graded materials, which are under the influence of inhomogeneous heat fluxes concentrated on the inner and outer layers, was performed [13], [14]. The presented studies do not allow to analyze the thermal state of the cylinders for local thermal perturbation.

Functionally graded materials with continuous change of properties are useful for thermal protection and biomedical applications. In the case of a thin coating on a substrate, the usual mesh discretization is ineffective. In the developed method [15] of approximate transfer, the finite difference concept is used to transfer the boundary conditions from the coating to the substrate. This allows numerically considering only the substrate with convection conditions using the hybrid finite element method. The method has been tested for various types of coatings and can be used to develop thermal conductivity models in electronic devices containing individual nodes and their elements with semi-through inclusions. The use of the finite difference concept to transfer boundary conditions may limit the accuracy of numerical calculations, especially for complex systems with continuous change of properties.

In [16], the authors simplify the nonlinear three-dimensional heat conduction problem by reducing it to the Laplace equation using an intermediate function. A generalized triple function is proposed and a general solution of the Laplace equation is obtained. Three specific problems are analyzed: it is shown that the heat flux of the nonlinear problem coincides with the results for the linear one, while the temperature field is different. On the flat defect boundary, the heat flux has a singularity, and its intensity is proportional to the root of the fourth power of the defect width. The disadvantage of this approach is that simplifying the nonlinear problem to the Laplace equation can lead to a loss of accuracy in determining the temperature field, since the nonlinearity inherent in the initial problem is not always adequately reproduced by a linear model.

Existing methods have been improved and new approaches have been developed for creating mathematical models that allow analyzing heat transfer in piecewise homogeneous media [17]. Planar and spatial heat transfer models are presented, in which the differential equations contain coefficients depending on the thermophysical properties of the phases and the geometric structure. Approaches are presented for determining analytical and analytical-numerical solutions of boundary value problems of heat conduction [18]. Heat transfer processes occurring in homogeneous and layered structures with foreign inclusions of canonical form are analyzed [19]. Linear and nonlinear mathematical models are developed for determining the temperature field and analyzing temperature regimes in isotropic media with local thermal heating [20]. Analytical solutions are obtained and algorithms for numerical implementation of the temperature distribution in spatial coordinates are developed. The obtained results make it possible to analyze heat transfer processes and increase the thermal resistance of structures.

Research results and their discussion

The object of the study and its mathematical models. An isotropic plate with a thickness of 2δ with thermally insulated front surfaces $|z| = \delta$, referred to a Cartesian rectangular coordinate system (*Oxyz*), in the near-surface region $\Omega_0 = \{(x, y, z): |x| \le H, 0, |z| \le \delta\}$ of which uniformly distributed internal heat sources with a specific power $q_0 = const$ are concentrated, is considered. Convective heat exchange with the environment with a constant temperature t_c=const occurs on the boundary surface of the layer $L_+ = \{(x, l, z): |x| < \infty, |z| \le \delta\}$ according to Newton's law, and boundary conditions $L_- = \{(x, -l, z): |x| < \infty, |z| \le \delta\}$ of the second kind are set on its other surface (fig. 1).

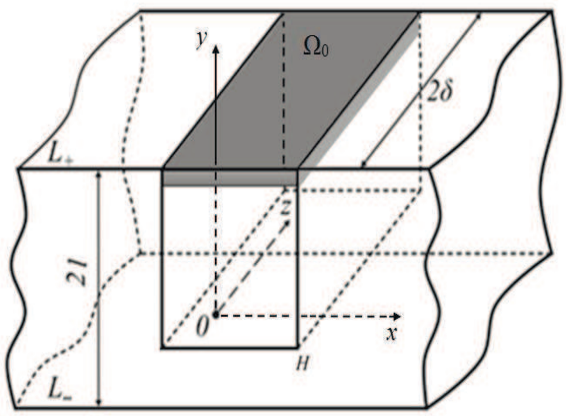


Fig. 1. Isotropic plate under the influence of near-surface heating

In the above structure, it is necessary to determine the temperature distribution $t(x, y)$ in the spatial coordinates x and y, which is obtained by solving the heat conduction equation

$$\Delta\theta(x, y) = -\frac{q_0}{\lambda} S_-(H - |x|)\delta(y - l) \qquad (1)$$

with boundary conditions

$$\theta(x,y)\|_{|x|\to\infty}=0\,,\quad \left.\frac{\partial\theta(x,y)}{\partial x}\right|_{|x|\to\infty}=0\,,$$

$$\lambda\left.\frac{\partial\theta(x,y)}{\partial y}\right|_{y=l}=\alpha\,\theta(x,y)|_{y=l}\,,\quad \left.\frac{\partial\theta(x,y)}{\partial y}\right|_{y=-l}=0\,, \qquad (2)$$

where λ is the thermal conductivity coefficient of the plate; α is the heat transfer coefficient from the surface L_+; $\theta(x,y)=t(x,y)-t_c$; Δ is the Laplace operator in the Cartesian rectangular coordinate system; $S_-(\zeta)$ – asymmetric unit function; $\delta(\zeta)=\frac{dS(\zeta)}{d\zeta}$ – Dirac delta function; $S(\zeta)$ – symmetric unit function;

$$\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\,;\; S_-(\zeta)=\begin{cases}1,\zeta\geq 0,\\ 0,\zeta<0;\end{cases}\qquad S(\zeta)=\begin{cases}1,\zeta>0,\\ 0{,}5,\zeta=0,\\ 0,\zeta<0.\end{cases}$$

The integral Fourier transform along the x coordinate is applied to equation (1) and boundary conditions (2), and as a result, a non-homogeneous ordinary differential equation of the second order with constant coefficients and a discontinuous and singular right-hand side is obtained

$$\frac{d^2\bar{\theta}}{dy^2}-\xi^2\bar{\theta}=-\sqrt{\frac{2}{\pi}}\frac{q_0}{\lambda\xi}\sin H\xi\delta(y-l) \qquad (3)$$

with boundary conditions

$$\left.\frac{d\bar{\theta}(y)}{dy}\right|_{y=l}=\frac{\alpha}{\lambda}\bar{\theta}(y)|_{y=l}\,,\left.\frac{d\bar{\theta}(y)}{dy}\right|_{y=-l}=0\,, \qquad (4)$$

where $\bar{\theta}(y)$ – the transform of the function $\theta(x, y)$; $\bar{\theta}(y)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{i\xi x}\theta(x,y)dx$; ξ – integral Fourier transform parameter, $i^2=-1$.

The general solution of equation (3) is defined as

$$\bar{\theta}(y)=c_1e^{\xi y}+c_2e^{-\xi y}-\sqrt{\frac{2}{\pi}}\frac{q_0}{\lambda\xi^2}\sin H\xi sh\xi(y-l)S(y-l)\,,$$

here c_1 i c_2 – constants of integration.

Boundary conditions (4) were used and on this basis the constants of integration were found and a partial solution of the problem (3)–(4) was obtained

$$\bar{\theta}(y)=\frac{q_0}{\xi\sqrt{2\pi}}\sin H\xi\left[\frac{ch\xi(y+l)}{P(\xi)}-\frac{2}{\lambda\xi}sh\xi(y-l)S(y-l)\right], (5)$$

where

$$P(\xi)=\lambda\xi sh2\xi l-\alpha ch2\xi l\,.$$

The inverse integral Fourier transform was applied to relation (5) and as a result the solution of problem (1)–(2) was obtained in the following form:

$$\theta(x,y)=\frac{q_0}{\pi}\int_0^{\infty}\frac{\cos\xi x}{\xi}\sin H\xi\left[\frac{ch\xi(y+l)}{P(\xi)}-\frac{2sh\xi(y-l)}{\lambda\xi}S(y-l)\right]d\xi\,. \quad (6)$$

As a result, the desired temperature field in the plate, caused by near-surface heating, is expressed by formula (6), from which the temperature value at any point is obtained.

An isotropic thermosensitive plate (thermophysical parameters depend on temperature) is considered (Fig. 1).

In the given structure, it is necessary to determine the temperature distribution $t(x, y)$ in the spatial coordinates x and y, which is obtained by solving the nonlinear heat conduction equation

$$div\left[\lambda(t)gradt(x,y)\right]=-q_0S_-(H-|x|)\delta(y-l)\,, \qquad (7)$$

with boundary conditions

$$t(x,y)\|_{|x|\to\infty}=0\,,\quad \left.\frac{\partial t(x,y)}{\partial x}\right|_{|x|\to\infty}=0\,,\quad \left.\frac{\partial t(x,y)}{\partial y}\right|_{y=-l}=0\,,$$

$$\lambda(t)\left.\frac{\partial t(x,y)}{\partial y}\right|_{y=l}=\alpha(t|_{y=l}-t_c)\,, \qquad (8)$$

where $\lambda(t)$ is the thermal conductivity coefficient of the thermally sensitive plate.

Kirchhoff transformations are considered

$$\vartheta(x,y)=\frac{1}{\lambda^0}\int_0^{t(x,y)}\lambda(\zeta)d\zeta\,. \qquad (9)$$

Here λ^0 is the reference coefficient of thermal conductivity of the plate material.

Expression (9) is differentiated by the variables x and y and as a result therelationis obtained

$$\lambda^0\frac{\partial\vartheta(x,y)}{\partial x}=\lambda(t)\frac{\partial t(x,y)}{\partial x},\lambda^0\frac{\partial\vartheta(x,y)}{\partial y}=\lambda(t)\frac{\partial t(x,y)}{\partial y},$$

taking into account which the original equation (7) and boundary conditions (8) are transformed to the following form:

$$\Delta\vartheta=-\frac{q_0}{\lambda^0}S_-(H-|x|)\delta(y-l)\,, \qquad (10)$$

$$\left.\frac{\partial\vartheta(x,y)}{\partial x}\right|_{|x|\to\infty}=0\,;\;\left.\frac{\partial\vartheta(x,y)}{\partial y}\right|_{y=-l}=0\,; \qquad (11)$$

$$\frac{\partial\vartheta(x,y)}{\partial y}|_{y=l}=\frac{\alpha}{\lambda^0}(t(x,y)|_{y=l}-t_c)\,. \qquad (12)$$

As a result of such transformations linear differential equations with partial derivatives of the second order with respect to the function $\vartheta(x,y)$ with discontinuous and singular right-hand side, boundary conditions (11) and quasi-linear boundary condition (12) are obtained.

The temperature $t(x,h)$ is approximated as a function of the spatial coordinate x by a segment-constant function in the form

$$t(x,h)=t_1+\sum_{k=1}^{m-1}(t_{j+1}-t_j)S_-(x-x_k)\,, \qquad (13)$$

where $x_k\in(0;x^*); x_1\leq x_2\leq ...\leq x_{m-1};\;\; t_k(j=\overline{1,m})$ are the unknown approximate values of temperature $t(x,h)$; m is the number of partitions of the interval (0; x^*); x^* is the abscissa value for which the temperature reaches the value t_c (it is found from the corresponding linear problem).

The integral Fourier transform in the coordinate x is applied to equation (10) and boundary conditions (11), (12) taking into account the relation (13) and an ordinary second-order differential equation with constant coefficients and a singular right-hand side is obtained

$$\frac{d^2\bar{\vartheta}}{dy^2} - \xi^2\bar{\vartheta} = -\sqrt{\frac{2}{\pi}}\frac{q_0}{\lambda^0\xi}\sin H\xi\delta(y-l) \quad (14)$$

and linear boundary conditions

$$\left.\frac{d\bar{\vartheta}(y)}{dy}\right|_{y=-l} = 0, \left.\frac{d\bar{\vartheta}(y)}{dy}\right|_{y=l} = \frac{\alpha D(\xi)}{\sqrt{2\pi}\lambda^0\xi}, \quad (15)$$

where $\bar{\vartheta}(y) = \frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{\infty} e^{i\xi x}\vartheta(x,y)dx$ – the transform of the function $\vartheta(x,y)$;

$$D(\xi) = i\sum_{j=1}^{m-1}(t_{j+1}-t_j)(e^{i\xi x_j} - e^{i\xi x_{j-1}}).$$

The general solution of equation (14) is obtained in the form

$$\bar{\vartheta}(y) = c_1 e^{\xi y} + c_2 e^{-\xi y} - \sqrt{\frac{2}{\pi}}\frac{q_0}{\lambda^0\xi^2}\sin H\xi sh\xi(y-l)S(y-l)$$

and using the boundary conditions (15) the integration constants c_1, c_2 are determined and as a result – the solution of the problem (14)–(15)

$$\bar{\vartheta}(y) = \frac{1}{\sqrt{2\pi}\lambda^0\xi^2}\left[q_0\sin H\xi(D(\xi,y) - \right.$$
$$\left. - 2sh\xi(y-l)S(y-l)) + \alpha D(\xi,y)D(\xi)\right]. \quad (16)$$

Here $D(\xi,y) = \frac{ch\xi(l+y)}{sh2\xi l}$.

The inverse integral Fourier transform is applied to relation (16) and the expression for the linearizing function $\vartheta(x,y)$ is defined in the following form:

$$\vartheta(x,y) = \frac{1}{\pi\lambda^0}\int\limits_0^{\infty}\frac{1}{\xi^2}\left\{q_0\sin H\xi\cos\xi x\left[D(\xi,y) - \right.\right.$$
$$\left.\left. - 2sh\xi(y-l)S(y-l)\right] + \alpha D(\xi,y)D(\xi,x)\right\}d\xi,. \quad (17)$$

Where

$$D(\xi,x) = \sin\xi x\sum_{j=1}^{m-1}(t_{j+1}-t_j)(\cos\xi x_j - \cos\xi x_{j-1}) -$$
$$-\cos\xi x\sum_{j=1}^{m-1}(t_{j+1}-t_j)(\sin\xi x_j - \sin\xi x_{j-1}).$$

The desired temperature field $t(x, y)$ for the given structure is determined using the obtained nonlinear algebraic equation, taking into account the temperature dependence of the thermal conductivity coefficient of the structural materials of the plate in relations (9), (17) and by performing certain mathematical transformations.

Partial example. The dependence of the thermal conductivity coefficient on the temperature of the structural material of the plate is given in the form of the relation

$$\lambda = \lambda^0(1-kt), \quad (18)$$

where k is the temperature coefficient of thermal conductivity of the plate material.

Using expressions (9), (18), an expression for determining the temperature $t(x, y)$

$$t(x,y) = \frac{1}{k}\left(1 - \sqrt{1-2k\vartheta(x,y)}\right). \quad (19)$$

Numerical experiment and analysis of the obtained results. Silicon was chosen as the material of the plate. In the temperature range [0 °C; 1127 °C], the dependence of the thermal conductivity coefficient of silicon on temperature was obtained by interpolation in the form

$$\lambda(t)=67.9\frac{Wt}{m\cdot degree}(1–0.0005\frac{1}{degree}t) \quad (20)$$

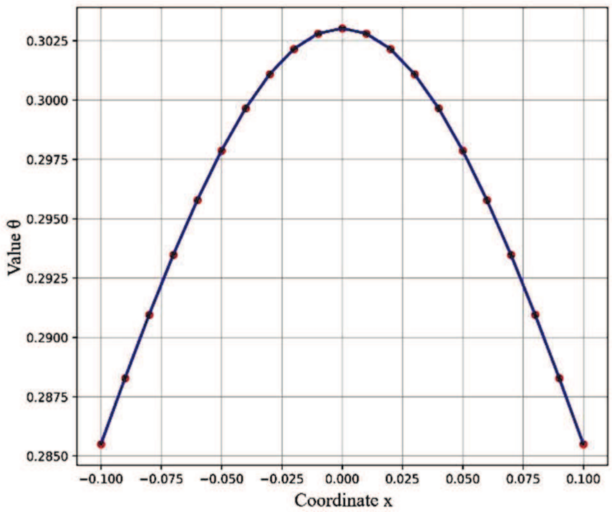
which is a partial case of the relation (18).

According to formulas (6), (19), numerical calculations of the temperature distribution in spatial coordinates in the plate were performed for a constant value of the thermal conductivity coefficient for silicon (λ=67.9 W/(m·degree) at a temperature t = 27 °C) and a linearly variable one (relation (20)). The following input data values were chosen: q_0=200 W/m^3; l=0.1 m; H=0.05 m; α=17.64 W/(m^2·degree). The temperature change depending on the spatial coordinates x for y=0 (Fig. 2, *a*) and y for x=0.05 (Fig. 2, *b*) is illustrated. The behavior of the curves shows that the temperature as a function of the spatial coordinates is smooth and monotonic and reaches maximum values in the region where near-surface heat sources are concentrated. Numerical calculations were performed with an accuracy of 10^{-6}.

The results obtained for the selected medium material (silicon) with a linear temperature dependence of the thermal conductivity coefficient differ from the results obtained for a constant thermal conductivity coefficient by 2 %. Their insignificant difference is explained by the fact that the value of the temperature coefficient of thermal conductivity for silicon, as shown by the relation (20), is small.

Discussion of the research results. The boundary value problems of thermal conductivity are formulated in accordance with the real physical process, which is studied in environments with near-surface heating. Due to this, the differential equations of thermal conductivity and boundary conditions clearly describe the mathematical models of the stationary process of thermal conductivity, which correspond to the given physical model.

The theory of generalized functions was applied in the studies, which made it possible to effectively describe near-surface thermal heating. As a result, the differential equations and boundary conditions contain a discontinuous right-hand side. The given mathematical models of thermal conductivity are simplified, but they can be improved for more complex physical processes.



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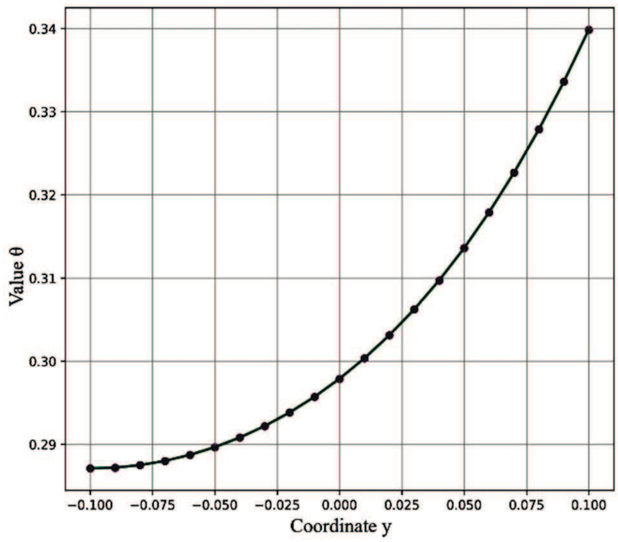

b

Fig. 2. Dependence of temperature t(x,y) on spatial x (a) and y (b) coordinates

The architecture of digital devices of modern electronic technology contains individual nodes and their elements in the form of structures with near-surface heating. Therefore, the development of mathematical models of the thermal conductivity process is an urgent task for the analysis of thermal stability, which makes it possible to prevent their overheating.

Scientific novelty of the obtained research results – linear and nonlinear mathematical models of thermal conductivity for isotropic spatial media in the form of a plate with near-surface heating have been developed. A method for linearizing the nonlinear mathematical model of thermal conductivity has been presented and analytical and analytical-numerical solutions of the corresponding linear and nonlinear boundary value problems have been obtained in a closed form.

Practical significance of the research results – based on the obtained analytical and analytical-numerical solutions of linear and nonlinear boundary value problems of thermal conductivity for isotropic thermally active spatial media with near-surface heating, computational algorithms and software tools for their numerical implementation have been developed for analyzing temperature regimes in individual structural elements and assemblies of electronic devices in order to predict their operating modes, identify unknown parameters and increase thermal stability, which increases their service life.

Conclusions

Linear and nonlinear mathematical models for determining temperature fields have been developed, and subsequently for analyzing temperature regimes caused by near-surface thermal heating for structures geometrically described by an isotropic spatial structure in the form of a plate. As a result, the accuracy of determining temperature fields has been increased, which significantly affects the effectiveness of design methods for devices, individual elements and assemblies of which are subjected to near-surface thermal loads. Based on the results of the research, the following main conclusions can be drawn:

1. Using linear and nonlinear mathematical models for determining temperature fields, an analysis of temperature regimes caused by near-surface thermal heating for structures geometrically described by an isotropic spatial structure in the form of a plate has been performed.

2. A method for linearizing the nonlinear boundary value problem of thermal conductivity has been presented and its analytical-numerical solution has been obtained in a closed form.

3. Based on the obtained analytical and analytical-numerical solutions for both linear and nonlinear boundary value problems of heat transfer, computational algorithms and software tools for their numerical implementation have been developed for the analysis of temperature regimes in spatial environments with near-surface heating.

4. It has been established that the thermal sensitivity of structural materials of digital devices should be taken into account. This approach significantly complicates the process of solving the corresponding nonlinear boundary value problems of thermal conductivity, but the sought solutions of these problems more accurately describe the behavior of temperature as a function of spatial coordinates. As a result, it is possible to identify structural materials of devices for which consideration of thermal sensitivity is essential, and therefore the obtained results will be more accurate. The insignificant influence of thermal sensitivity leads to the use of a linear model of thermal conductivity, which simplifies the determination of temperature fields.

References

[1] Zehed Sheikh (1994). Where do you the cooling vents. Electronics cooling. April 18.

[2] Zhang, Zhong; Sun, Ying; Cao, Xiaojian; Xu, Jiajing; Yao, Lu (2024). A slice model for thermoelastic analysis of porous functionally graded material sandwich beams with

temperature-dependent material properties. *Thin-Walled Structures*, vol. 198, 111700. 10.1016/j.tws.2024.111700
[3] Zhang, Z., Zhou, D., Fang, H., Zhang, J., & Li, X. (2021). Analysis of layered rectangular plates under thermo-mechanical loads considering temperature-dependent material properties. *Applied Mathematical Modelling*, 92, 244–260. https://doi.org/10.1016/j.apm.2020.10.036
[4] Xingwen Peng, Xingchen Li, Zhiqiang Gong, Xiaoyu Zhao, Wen Yao (2022). A deep learning method based on partition modeling for reconstructing temperature field. *International Journal of Thermal Sciences*, vol. 182, 107802. https://doi.org/10.1016/j.ijthermalsci.2022.107802
[5] Ren, Yongan; Huo, Ruili; Zhou, Ding; Zhang, Zhong (2023). Thermo-Mechanical Buckling Analysis of Restrained Columns Under Longitudinal Steady-State Heat Conduction. *Iranian Journal of Science and Technology – Transactions of Civil Engineering*, vol. 47, issue 3, 1411–1423. 10.1007/s40996-022-01020-7
[6] H.J. Breukelman, M.J. Santofimia, J. Hidalgo (2023). Dataset of a thermal model for the prediction of temperature fields during the creation of austenite / martensite mesostructured materials by localized laser treatments in a Fe-Ni-C alloy. *Data in Brief*, Vol. 48, 109–110. https://doi.org/10.1016/ j.dib. 2023.109110
[7] Wen Zhang, Min Wu, Sheng Du, Luefeng Chen, Jie Hu, Xuzhi Lai. (2023). Modeling of Steel Plate Temperature Field for Plate Shape Control in Roller Quenching Process. *IFAC-PapersOnLine*, vol. 56, issue 2, 6894–6899. https://doi.org/ 10.1016/j.ifacol.2023.10.493
[8] Zafar Hayat Khan, W. A. Khan, S. M. Ibrahim, F. Mabood, Zaitang Huang (2024). Effects of thermal boundary conditions on Stokes' second problem. *Results in Physics*, Vol. 60, 107662. https://doi.org/10.1016/j.rinp.2024.107662
[9] Evstatieva, N., & Evstatiev, B. (2023). Modelling the Temperature Field of Electronic Devices with the Use of Infrared Thermography. 13th International Symposium on Advanced Topics in Electrical Engineering (ATEE), Bucharest, Romania, pp. 1–5. http://dx.doi.org/10.1109/ ATEE58038. 2023.10108375
[10] Haoran, L., Jiaqi, Y., & Ruzhu, W. (2023). Dynamic compact thermal models for skin temperature prediction of porta-ble electronic devices based on convolution and fitting methods, *International Journal of Heat and Mass Transfer*, vol. 210, 124170, ISSN 0017-9310. https://doi.org/10.1016/ j.ijheatmasstransfer.2023.124170
[11] Ghannad, M., & Yaghoobi, M. P. (2015). A thermoelasticity solution for thick cylinders subjected to thermo-mechanical loads under various boundary conditions. *International Journal of Advanced Design & Manufacturing Technology*, Vol. 8, No. 4, 1–12. https://sanad.iau.ir/journal/ admt/Article/ 534941?jid=534941
[12] Song, H., Song, K., Gao, C. (2019). Temperature and thermal stress around an elliptic functional defect in a thermoelectric material. *Mech. Mater*., 130, 58–64. https://doi.org/10.1016/ j.mechmat.2019.01.008
[13] Yaghoobi, M. P., & Ghannad, M. (2020). An analytical solution for heat conduction of FGM cylinders with varying thickness subjected to non-uniform heat flux using a first-order temperature theory and perturbation technique. *International Communications in Heat and Mass Transfer*, 116, 104684. https://doi.org/10.1016/j.icheatmasstransfer.2020.104684
[14] Eker, M., Yarımpabuç, D., & Celebi, K. (2020). Thermal stress analysis of functionally graded solid and hollow thick-walled structures with heat generation. *Engineering Computations*, 38(1), 371–391. http://dx.doi.org/10.1108/EC-02-2020-0120
[15] Wang, H.; Qin, Q. (2019). Thermal Analysis of a Functionally Graded Coating/Substrate System Using the Approximated Transfer Approach. *Coatings*, 9, 51. https://doi.org/10.3390/ coatings9010051
[16] Qianqian Zhang, Haopeng Song, Cun-Fa Gao. (2023). The 3-D problem of temperature and thermal flux distribution around defects with temperature-dependent material properties. *Thermal Science*, vol. 27, issue 5, part B, 3903–3920. https://doi.org/10.2298/TSCI221003028Z
[17] Havrysh V. I., Kolyasa L. I., Ukhanska O. M., Loik V. B. (2019). Determination of temperature fielde in thermally sensitive layered medium with inclusions. *Naukovyi Visnyk Natsionalnoho Hirnychoho Universetety*, 1, 94–100. http://nvngu.in.ua/index.php/en/archive/on-the-issues/1709-2019/contens-1-2019/geotechnical-and-mining-mechanical-engineering-machine-building/4711-determination-of-temperature-field-in-thermally-sensitive-layered-medium-with-inclusions
[18] Havrysh V. I. (2017). Investigation of temperature fields in a heat-sensitive layer with through inclusion. *Materials Science*, 52(4), 514–521.
[19] Havrysh V. I., Kosach, A. I. (2012). Boundary-value problem of heat conduction for a piecewise homogeneous layer with foreign inclusion. *Materials Science*, 47(6), 773–782. https://link.springer.com/article/10.1007/s11003-012-9455-4
[20] Gavrysh, V., Tushnytskyy, R., Pelekh, Y., Pukach, P., Baranetskiy, Y. (2017). Mathematical model of thermal conductivity for piecewise homogeneous elements of electronic systems. 14 th International Conference The Experiense of Designing and Application of CAD Systems in Microelektronics, CADSM 2017-Proceedings, 333–336. https:// ieeexplore.ieee.org/document/7916146

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МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ТА АНАЛІЗ ТЕМПЕРАТУРНИХ РЕЖИМІВ У ЦИФРОВИХ ПРИСТРОЯХ ВНАСЛІДОК ПРИПОВЕРХНЕВОГО НАГРІВАННЯ

Розроблено лінійну та нелінійну математичні моделі визначення температурного поля, а надалі й аналізу температурних режимів у ізотропній пластині внаслідок приповерхневого теплового навантаження. Для випадку термочутливої пластини (теплофізичні параметри конструкційного матеріалу залежать від температури) застосовано перетворення Кірхгофа, із використанням якого лінеаризовано нелінійне рівняння теплопровідності та нелінійні крайові умови. У результаті отримано лінійне диференціальне рівняння другого порядку із частковими похідними та розривною правою частиною та частково лінеаризовані крайові умови. Для остаточної лінеаризації крайових умов виконано апроксимацію температури за просторовою координатою на межовій поверхні термочутливої пластини сегментно-сталою функцією, що дало змогу одержати лінійну крайову задачу відносно змінної Кірхгофа. Для розв'язування отриманих крайових задач використано інтегральне перетворення Фур'є і, як наслідок, аналітичний та аналітично-числовий розв'язки у вигляді невласних збіжних інтегралів. Для термочутливого середовища, як приклад, вибрано лінійну залежність коефіцієнта теплопровідності конструкційного матеріалу структури від температури, яку часто використовують для розв'язування багатьох практичних задач. Розроблено програмні засоби, з використанням яких виконано числовий аналіз поведінки температури як функції просторових координат для заданих значень геометричних і теплофізичних параметрів. На цій основі геометрично зображено поведінку температурного поля. Розроблені лінійна та нелінійна математичні моделі визначення температурного поля у просторових середовищах із приповерхневим нагріванням дають змогу аналізувати їх термостійкість. На цій основі можна запобігти перегріванню, яке здатне спричинити вихід із ладу не тільки конструкційних вузлів та їх окремих елементів, а й електронного пристрою загалом.

Ключові слова: температурне поле, ізотропна пластина, теплопровідність матеріалу, конвективний теплообмін, приповерхневе нагрівання, термочутливість матеріалу, тепловий потік, ідеальний тепловий контакт.

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Цитування за ДСТУ: Гавриш В. І. Математичне моделювання та аналіз температурних режимів у цифрових пристроях внаслідок приповерхневого нагрівання. Український журнал інформаційних технологій. 2025, т. 7, № 1. С. 140–148.

Citation APA: Havrysh, V. I. (2025). Mathematical modeling and analysis of temperature regime in digital devices due to surface heating. *Ukrainian Journal of Information Technology, 7*(1), 140–148. https://doi.org/10.23939/ujit2025.01.140