MATHEMATICAL MODELING AND COMPUTING, Vol. 12, No. 2, pp. 415–424 (2025)

Parameter estimation for two-tank system using multiparametric programming

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(Received 18 December 2024; Revised 14 April 2025; Accepted 18 April 2025)

Multiparametric programming simplifies optimization problems by allowing solutions to be expressed as functions of varying parameters. This method enhances the accuracy and efficiency of parameter estimation, which is crucial for dynamic systems. This study presents a novel parameter estimation approach for two-tank system using multiparametric programming (MPP). Two-tank system is widely employed in various applications, necessitating efficient management of liquid levels for safe and effective operation. Accurate parameter estimation in tank systems is critical for optimal control; however, conventional estimation methods often encounter complexities and computational demands that can result in delays and inaccuracies in diagnosing system parameters. The proposed method employs MPP to accurately identify parameters of the tank system. In this approach, the two-tank system is modeled using ordinary differential equations that represent the dynamics of liquid levels in each tank. The Karush–Kuhn–Tucker method is utilized to solve the parameter estimation problem, deriving an explicit solution with particular focus on the cross-sectional area of the outlet pipe. The estimated parameter values are calculated using the state variables of the two-tank system, facilitating a comprehensive evaluation of the proposed method. The results demonstrate that the proposed method accurately estimates parameters using an explicit solution, offering faster computations and reduced complexity compared to conventional optimization methods.

Keywords: *parameter estimate; tank system; multiparametric programming.*

2010 MSC: 90C31, 62F10 **DOI:** 10.23939/mmc2025.02.415

1. Introduction

Parameter estimation is a critical component in control systems, directly affecting the accuracy, stability, and performance of various processes. It involves identifying system parameters that best describe the dynamics of the system, allowing for accurate modeling and control. In applications like fluid dynamics, electrical circuits, or chemical processes, the precise estimation of these parameters is essential for optimizing system operations. When parameters are inaccurately estimated, control strategies can be compromised, leading to inefficiencies, unsafe conditions, or operational failures. Consequently, developing robust and efficient parameter estimation techniques is a priority in fields where dynamic system modeling is vital.

The parameter estimation plays a particularly important role in the tank system. Tank systems are prevalent in a wide range of industries, from small household water tanks to large-scale industrial applications such as chemical processing, wastewater treatment, and petrochemical facilities [1]. A two-tank system is one of the most commonly studied configurations, often used to regulate liquid levels and ensure the desired flow rates [2]. Maintaining appropriate liquid levels is essential for system

This work was supported by grant Fundamental Research Grant Scheme (FRGS) support under grant number FRGS/1/2022/TK08/UNIMAP/02/92 from the Ministry of Higher Education (MOHE) Malaysia and University Malaysia Perlis.

safety and operational efficiency, as improper control can lead to issues like overfilling, equipment damage, or even hazardous spills. For example, incidents at chemical plants, such as the benzene leak in Wuyi, China, have demonstrated the severe consequences of tank overflows due to human error and inadequate control measures [3]. These events underscore the importance of accurately estimating parameters that govern the flow and level dynamics in tank systems to prevent similar incidents.

Although human error often contributes to such accidents, prioritizing preventive measures is essential to minimize the risk of overfilling. This includes accurately estimating the water parameters involved to enhance safety protocols. The parameters involved in controlling the liquid flow rate are considered critical issues. A level parameter is crucial in process industries. An excessively high level can alter the reaction equilibrium, damage equipment, or cause an overflow of costly or hazardous materials. Conversely, if the level is too low, it may lead to issues with initiating the sequential process [4]. Therefore, it is vital to manage the flow between two tanks and maintain the liquid level within the tank system by accurately estimating the parameters involved in the process.

Traditional parameter estimation techniques have been widely used in the study of tank systems. Common methods include Bayesian estimation, least squares approaches, and recursive algorithms such as the Recursive Least Squares (RLS) and Kalman Filter. Bayesian estimation integrates prior knowledge from parameter distributions with data-driven insights from the likelihood function [5]. This method generates a posterior distribution that updates information for parameter estimation in Bayesian approaches. Another method is least squares, which numerically estimates parameters by fitting a function to a dataset [6]. It aims to achieve optimal results by minimizing the sum of squared errors (SSE). Both Bayesian and least squares methods can be computationally demanding for different reasons. Bayesian estimation involves iterative computations to update the posterior distribution, which can be time-consuming, especially for complex models or large datasets. Least squares estimation also requires substantial computational resources, especially when optimizing models with many parameters or using advanced techniques to minimize SSE.

Recent research has investigated various parameter estimation techniques, such as genetic algorithm (GA) optimization, to accurately determine system parameters [7]. GA-based methods have shown improvements in simulated tank level accuracy and enhanced control strategies like proportional integral control. However, this approach's complexity arises when parameter estimation is based on input signals covering only a limited operational range, which can result in inaccuracies across the entire system's behaviour. Another method, presented in [8], employs the second-order sliding mode technique for estimating control valve ratios, which are essential for tank process control. The precision of this approach is crucial for designing robust control systems, such as the proposed state estimation scheme for a quadruple tank process, which ensures finite-time convergence and accurate liquid level control. However, this technique assumes that the system behaves as a minimum phase system, potentially leading to instability or poor performance if non-minimum phase characteristics are present.

In [9], a recursive estimation method is combined with a zonotopic approach to improve fault detection in a quadruple tank system. The Zonotopic Recursive Least Squares (ZRLS) estimator uses a zonotope-based strategy to address parametric uncertainty by minimizing the weighted Frobenius radius and optimizing the estimator's gain through a cost function associated with the error covariance matrix. This method enhances parameter estimation accuracy, which is crucial for effective fault detection. A different approach in [10] uses a two-stage parameter estimation framework for Hammerstein nonlinear ARX systems. The first stage estimates the parameters of a nonlinear static block, while the second stage applies the Recursive Least Squares (RLS) method to estimate the parameters of the linear dynamic ARX block. The ARX model's parameters, including autoregressive and exogenous input coefficients, are updated recursively by minimizing the error between the actual system output and the predicted output, using a gain vector and covariance matrix.

The Kalman filter is a widely used technique for parameter estimation, with its extended form (EKF) and the unscented Kalman filter (UKF) being particularly popular in handling nonlinear systems. In [11], the EKF is applied for estimating the internal states of a water distribution system. The approach integrates the EKF with a state-space model to create an equation framework for conserving mass and momentum, allowing the estimation of system flows and pressures even with limited sensor data. Although the EKF improves estimation accuracy, its application to transient hydraulic models requires extensive datasets and a detailed numerical grid, posing significant computational challenges for real-time implementation, especially in medium to large systems. Similarly, [12] utilize the EKF in a continuous-discrete format for real-time parameter estimation of supercapacitors. This method is crucial for understanding and modeling supercapacitor behaviour under varying conditions. The implicit integration approach employed in this implementation ensures convergence, which is particularly beneficial for handling under-sampled and nonlinear models. These studies demonstrate the capabilities and limitations of EKF in different applications, highlighting the need for careful consideration of computational demands in real-time scenarios.

To address these challenges, the proposed study employs multiparametric programming (MPP) as a novel approach for parameter estimation in two-tank systems. MPP differs from conventional methods by formulating explicit solutions as functions of varying parameters, allowing for optimization without the need for repeated iterations [13]. This explicit approach not only reduces computational complexity but also ensures faster computation times. The study models the two-tank system using ordinary differential equations to represent the dynamics of liquid levels, while the Karush–Kuhn–Tucker conditions are applied to derive an explicit solution for estimating key parameters, such as the cross-sectional area of the outlet pipe. By considering a wide range of possible parameter values, MPP enhances the robustness of the control system, offering a reliable method for maintaining desired liquid levels under varying conditions. This novel application of MPP aims to demonstrate its potential advantages over traditional optimization techniques, such as improved accuracy and reduced computational demand for two-tank systems.

2. Mathematical model of a two-tank system

A schematic diagram of the proposed two-tank system (TTS) is shown in Figure 1. The target is to control the level of liquid in tank 1 and tank 2 with pump. The input is $u(t)$ which is voltage to the pump. This process can be described using nonlinear ordinary differential equations (ODEs) derived from Bernoulli's law of conservation of mass [14], where

$$A\frac{dh}{dt} = Q_{\text{in}} - Q_{\text{out}}. \tag{1}$$

Eq. (1) represents the basic principle of mass conservation, where the rate of change of liquid height, $\frac{dh}{dt}$, in a tank is determined by the difference between the inflow rate, Q_{in}, and the outflow rate, Q_{out}. Hence, the nonlinear ODEs for Tank 1 and Tank 2 are given as follows:

$$\frac{dh_1}{dt} = -\frac{a_1}{A}\sqrt{2gh_1} + \frac{\eta}{A}u(t), \tag{2}$$

$$\frac{dh_2}{dt} = \frac{a_1}{A}\sqrt{2gh_1} - \frac{a_2}{A}\sqrt{2gh_2}, \tag{3}$$

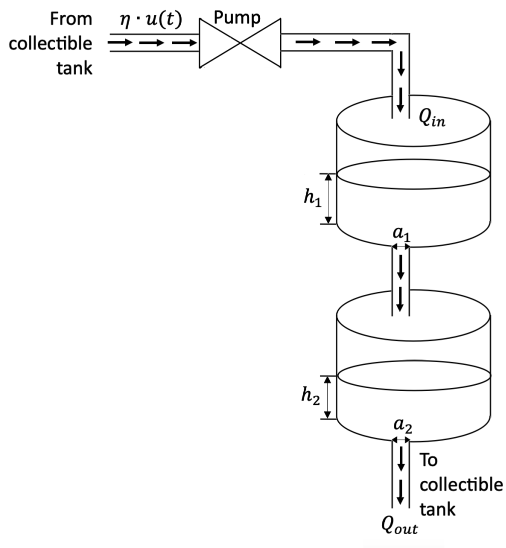

Fig. 1. Two-tank system.

where A is the cross-sectional area of tanks 1 and 2 (assumption: cross-sectional area of tank 1 equal to the cross-sectional area of tank 2), a_i is the cross-sectional area of the outlet pipe in tank i, g is the gravitational force, η is the pump gain. This model allows for the analysis and control of the liquid levels in both tanks by adjusting the voltage $u(t)$ to the pump, thus regulating the flow rates and maintaining the desired system behaviour.

In this study, the state variables representing the liquid heights, h_1 and h_2, are obtained using the fourth-order Runge–Kutta (RK4) method using MATLAB programming, which is defined in Eqs. (4)–

(12),

$$h_i(t+1) = h_i(t) + \frac{\Delta t}{6}\left(k_{1,h_i} + 2k_{2,h_i} + 2k_{3,h_i} + k_{4,h_i}\right), \tag{4}$$

where for h_1 and h_2,

$$k_{1,h_1} = \Delta t\left[-\frac{a_1}{A}\sqrt{2gh_1} + \frac{\eta}{A}u(t)\right], \tag{5}$$

$$k_{2,h_1} = \Delta t\left[-\frac{a_1}{A}\sqrt{2g\left(h_1 + \frac{k_{1,h_1}}{2}\right)} + \frac{\eta}{A}u(t)\right], \tag{6}$$

$$k_{3,h_1} = \Delta t\left[-\frac{a_1}{A}\sqrt{2g\left(h_1 + \frac{k_{2,h_1}}{2}\right)} + \frac{\eta}{A}u(t)\right], \tag{7}$$

$$k_{4,h_1} = \Delta t\left[-\frac{a_1}{A}\sqrt{2g\left(h_1 + k_{3,h_1}\right)} + \frac{\eta}{A}u(t)\right], \tag{8}$$

$$k_{1,h_2} = \Delta t\left[\frac{a_1}{A}\sqrt{2gh_1} - \frac{a_2}{A}\sqrt{2gh_2(t)}\right], \tag{9}$$

$$k_{2,h_2} = \Delta t\left[\frac{a_1}{A}\sqrt{2gh_1} - \frac{a_2}{A}\sqrt{2g\left(h_2 + \frac{k_{1,h_2}}{2}\right)}\right], \tag{10}$$

$$k_{3,h_2} = \Delta t\left[\frac{a_1}{A}\sqrt{2gh_1} - \frac{a_2}{A}\sqrt{2g\left(h_2 + \frac{k_{2,h_2}}{2}\right)}\right], \tag{11}$$

$$k_{4,h_2} = \Delta t\left[\frac{a_1}{A}\sqrt{2gh_1} - \frac{a_2}{A}\sqrt{2g\left(h_2 + k_{3,h_2}\right)}\right]. \tag{12}$$

3. Parameter estimation using multiparametric programming

A general formulation for parameter estimation often involves minimizing the sum of squares of the differences between observed and predicted values, a method known as the least squares technique. This approach is widely utilized due to its simplicity and efficiency in handling linear and nonlinear models. The sum of squares method provides an optimal estimate when the errors in the observations are normally distributed with constant variance, making it a robust tool in statistical analysis and machine learning applications.

This paper aims to estimate the cross-sectional area of the outlet pipe using multiparametric programming to manage the liquid levels in the two-tank system. To accomplish this, an explicit algorithm is formulated using MPP based on the developed parameter estimation optimization. The objective of parameter estimation is to reduce the error between h_1 and h_2 by deriving the cross-sectional area of the outlet pipe as a function. The formula of MPP is outlined below.

The objective of parameter estimation is to address the following:

$$\varepsilon_{\mathrm{MPP}} = \min_{\theta, h_1(t), h_2(t)} \sum_{t\in T}\left(\hat{h}_1(t+1) - h_1(t+1)\right)^2 + \left(\hat{h}_2(t+1) - h_2(t+1)\right)^2.$$

Subject to:

$$\text{Eqs. (2) and (3)},$$

$$h_1(0) = 0, \quad h_2(0) = 0.$$

The nonlinear ODEs of TTS in Eqs. (2) and (3) are converted to algebraic equations using the Euler method. The algebraic equation is given as

$$\frac{h_1(t+1) - h_1(t)}{\Delta t} = -\frac{a_1}{A}\sqrt{2gh_1(t)} + \frac{\eta}{A}u(t),$$
$$\frac{h_2(t+1) - h_2(t)}{\Delta t} = \frac{a_1}{A}\sqrt{2gh_1(t)} - \frac{a_2}{A}\sqrt{2gh_2(t)}.$$

The objective function of parameter estimation in Eq. (4) is solved using the Karush–Kuhn–Tucker (KKT) conditions, where the Lagrangian function is given by

$$L = G + \sum_{i \in I} \lambda_i m_i,$$

where

$$m_1 = \frac{h_1(t+1) - h_1(t)}{\Delta t} + \frac{a_1}{A}\sqrt{2gh_1(t)} - \frac{\eta}{A}u(t) = 0,$$

$$m_2 = \frac{h_2(t+1) - h_2(t)}{\Delta t} - \frac{a_1}{A}\sqrt{2gh_1(t)} + \frac{a_2}{A}\sqrt{2gh_2(t)} = 0,$$

$$G = \big(\hat{h}_1(t+1) - h_1(t+1)\big)^2 + \big(\hat{h}_2(t+1) - h_2(t+1)\big)^2,$$

$$\frac{\partial G}{\partial a_1} = 0,$$

$$\frac{\partial G}{\partial a_2} = 0.$$

The equations above are solved analytically in Mathematica and the solution is given by

$$\alpha_1 = -\frac{\sqrt{gh_1(t)}\big(Ah_1(t) - A\hat{h}_1(t+1) - \Delta t\,\eta\,u\big)}{\sqrt{2}\,\Delta t\,g\,h_1(t)},$$

$$\alpha_2 = -\frac{\sqrt{gh_2(t)}\big(Ah_1(t) - A\hat{h}_1(t+1) + Ah_2(t) - A\hat{h}_2(t+1) - \Delta t\,\eta\,u\big)}{\sqrt{2}\,\Delta t\,g\,h_2(t)}.$$

The estimated model parameters, a_1 and a_2, are calculated using the measurements, h_1 and h_2. Figure 2 outlines the methodology by illustrating how multiparametric programming (MPP) functions as an offline parameter estimation technique. MPP achieves this by deriving explicit functions, enabling precise estimation of system parameters. This ensures that the predicted behaviour of the system closely matches its actual performance. The accuracy of MPP allows for effective real-time evaluation of parameters, which supports precise online monitoring and timely adjustments, thereby enhancing overall system performance and alignment with real-world outcomes.

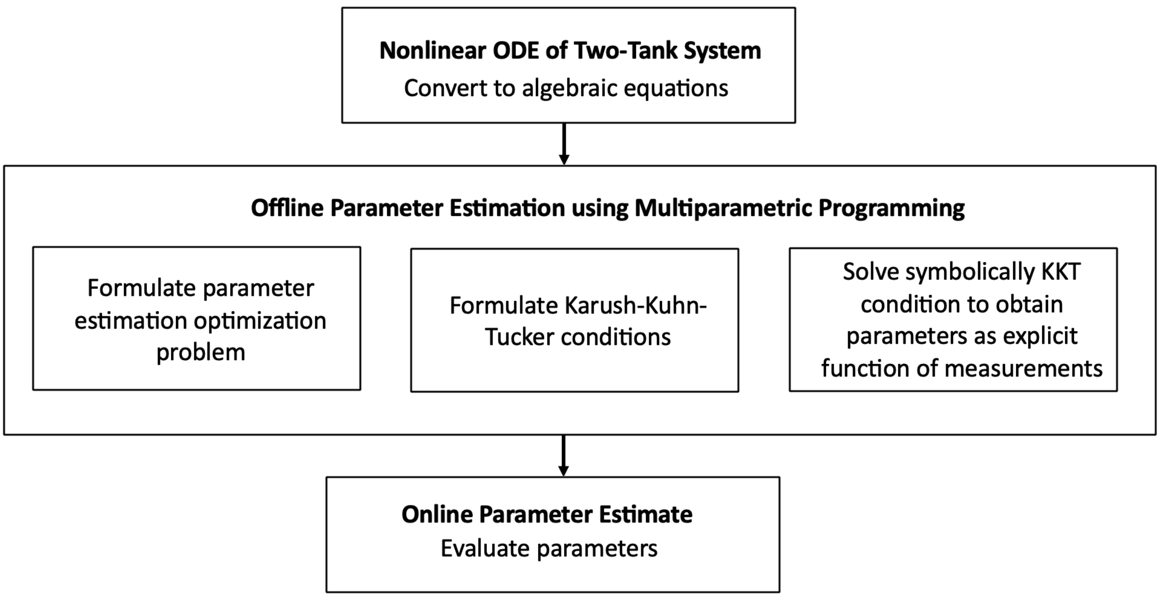

Fig. 2. Summary of the proposed research.

4. Results and discussion

The parameters involved in the two-tank system can be referred to in Table 1. Random noise is introduced into the system to test the performance and reliability of the proposed method. The state variable profile for measured and model prediction values of liquid levels at the tanks are shown in Figures 3 and 4. These figures demonstrate that the liquid levels h_1 and h_2 approach a steady state. This behavior aligns with the expected dynamics of a two-tank system, where the liquid levels increase quickly until reaching equilibrium. The model predictions closely match the measured values, as indicated by the minimal deviation. The consistency between the model and measured data suggests that the parameter estimation and modeling approach used for the system is effective.

Table 1. The parameter values of the two-tank system.

Parameters	**Description**	**Values**
A	The cross-sectional area of the tank	$0.01389\ \mathrm{m}^2$
α_1	The cross-sectional area of the outlet pipe in tank 1	$0.1245\ \mathrm{m}^2$
α_2	The cross-sectional area of the outlet pipe in tank 2	$0.1245\ \mathrm{m}^2$
g	Gravity	$9.81\ \mathrm{m/s}^2$
η	Pump gain	0.1194

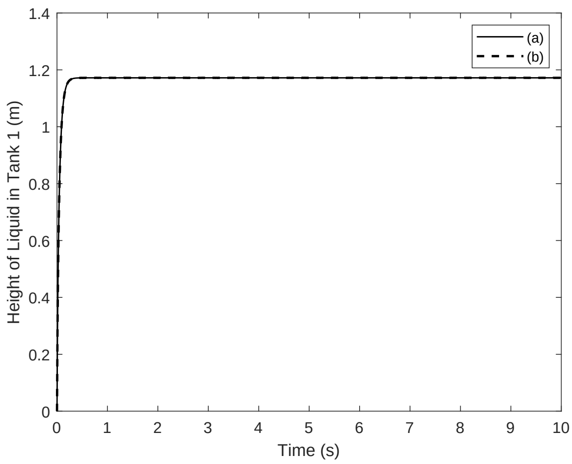

Fig. 3. State variable profile for the liquid level at tank 1, h_1 ($\boldsymbol{a}$) the model predicted value and ($\boldsymbol{b}$) measured value.

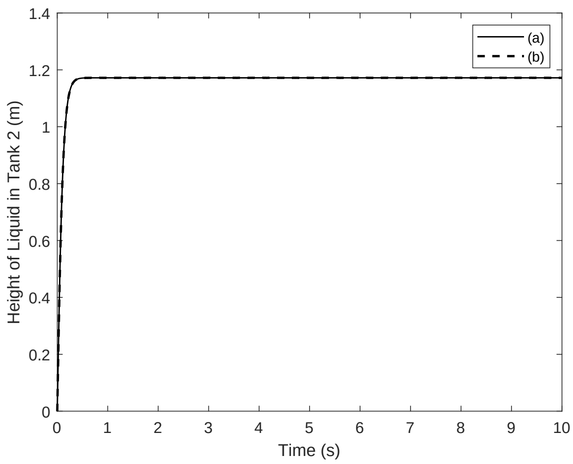

Fig. 4. State variable profile for the liquid level at tank 2, h_2 ($\boldsymbol{a}$) the model predicted value and ($\boldsymbol{b}$) measured value.

The proposed technique of multiparametric programming (MPP) effectively evaluates the cross-sectional areas of the outlet pipes, α_1 and α_2, in tank 1 and tank 2, respectively. This process is performed using a step size of 0.01 for 10 seconds. Figures 5 and 6 display the estimated parameters of the cross-sectional areas α_1 and α_2 using MPP. The cross-sectional areas can be calculated using MPP, and the figures show that α_1 and α_2 are close to the observed value, $0.1245\ \mathrm{m}^2$. These estimated parameters indicate that the MPP technique accurately obtained the cross-sectional areas.

Table 2. The parameter of cross-sectional area in Tank 1 and Tank 2.

Time (t)	**Cross-sectional area of the outlet pipe in tank 1**	**Cross-sectional area of the outlet pipe in tank 2**
$t \leqslant 50$ s	Normal	Normal
$50\ \mathrm{s} < t \leqslant 100$ s	Decrease 10%	Normal
$100\ \mathrm{s} < t \leqslant 150$ s	Normal	Decrease 10%
$150\ \mathrm{s} < t \leqslant 200$ s	Decrease 10%	Decrease 10%
$200\ \mathrm{s} < t \leqslant 250$ s	Increase 10%	Normal
$250\ \mathrm{s} < t \leqslant 300$ s	Normal	Increase 10%
$300\ \mathrm{s} < t \leqslant 350$ s	Increase 10%	Increase 10%

To demonstrate the application of parameter estimation for the TTS, the cross-sectional areas α_1 and α_2 are changed as shown in Table 2, and the model is simulated to obtain model and measured data as shown in Figures 7 and 8. Noise has been added to the system as random data to evaluate the effectiveness of the proposed method using multiparametric programming. Figure 9 and 10 show the evaluation of estimated cross-sectional areas α_1 and α_2, respectively, using MPP. When the value of α_1 decreases, the pipe's cross-sectional area becomes smaller, causing more water to remain in tank 1, thereby increasing the water level h_1. This reduction in flow to tank 2 results in a decrease in h_2. Other than that, the pipe's cross-sectional area changes, α_2 impacts the water flow in tank 2 like the effect observed in tank 1 when α_1 changes. The overall imbalance starting when the water in tank 1 keeps increasing can cause the water to exceed its capacity, potentially leading to overflow. Such an overflow can disrupt the equilibrium of the entire two-tank system, potentially leading to operational inefficiencies or system failures.

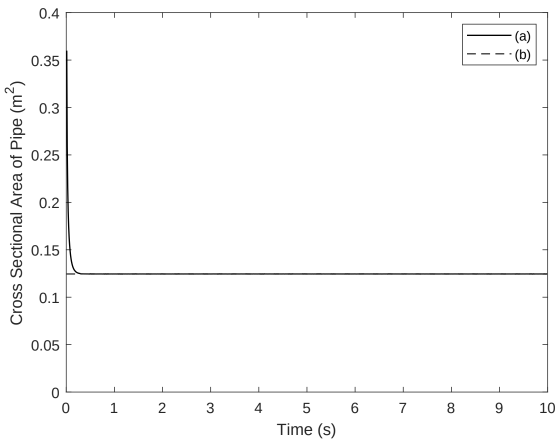

Fig. 5. The cross-sectional area of the outlet pipe, α_1: (***a***) estimated parameter using MPP and (***b***) actual value.

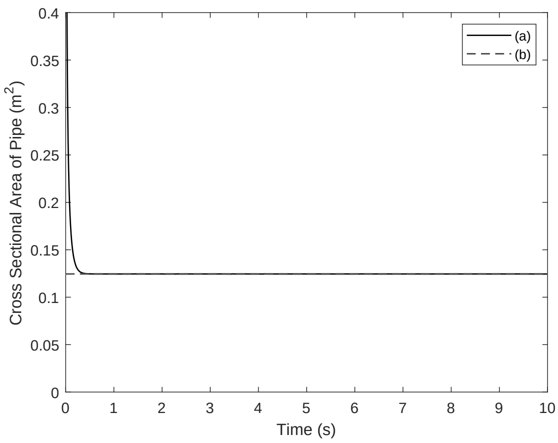

Fig. 6. The cross-sectional area of the outlet pipe, α_2: (***a***) estimated parameter using MPP and (b) actual value.

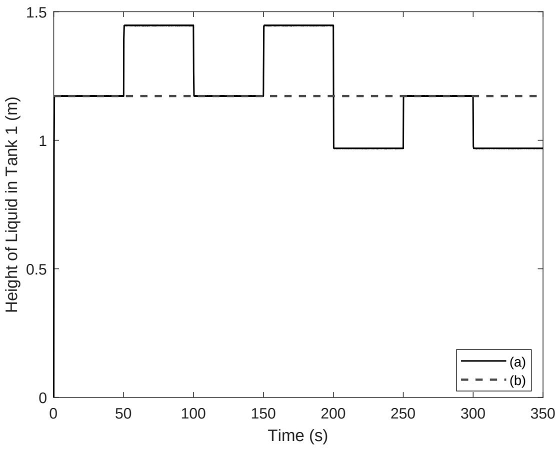

Fig. 7. State variable profile for the liquid level at tank 1, h_1 (***a***) the model predicted value and (***b***) measured value.

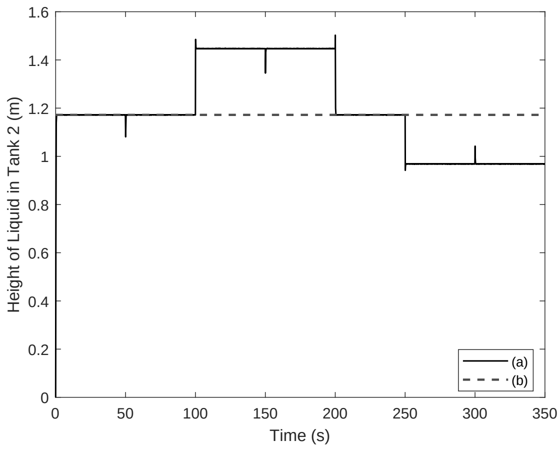

Fig. 8. State variable profile for the liquid level at tank 2, h_2 (***a***) the model predicted value and (***b***) measured value.

Accurately estimating these parameters is essential for preserving the system's stability and ensuring its optimal performance. The precise balance between them is key to maintaining functionality and operational reliability. The use of multiparametric programming not only provides precise parameter estimates but also facilitates computational efficiency, making it suitable for real-time applications. The success of the MPP technique in stabilizing the cross-sectional areas highlights its potential for optimizing system parameters in various engineering applications.

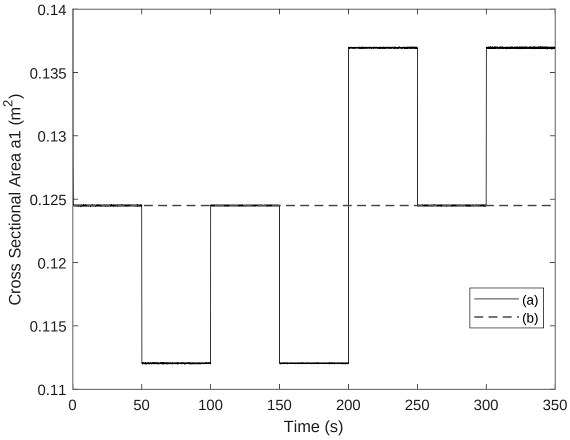

Fig. 9. The cross-sectional area of the outlet pipe, α_1: (***a***) estimated parameter using MPP and (***b***) actual value.

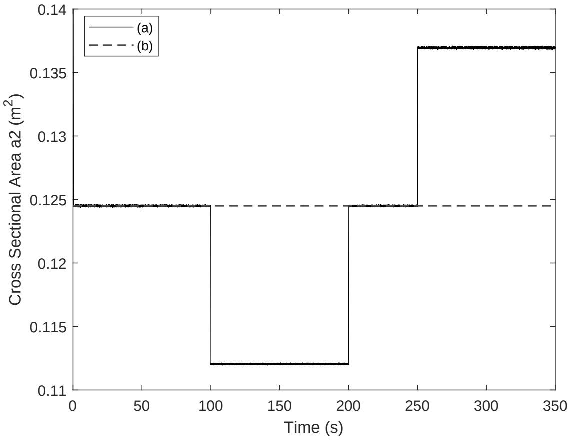

Fig. 10. The cross-sectional area of the outlet pipe, α_2: (***a***) estimated parameter using MPP and (***b***) actual value.

5. Conclusion

The multiparametric programming (MPP) method is proposed to evaluate system parameters in a two-tank system precisely. The explicit solution of parameters was formulated using the MPP technique. The results show that the estimated parameters for cross-sectional areas are close to the actual values, confirming that the proposed method can accurately evaluate these parameters. Therefore, it can detect deviations if the parameters vary from their expected values. These findings highlight the method's potential for timely and accurate detection of parameter variations, which is crucial for maintaining the reliability and efficiency of fluid control systems. The approach significantly enhances system performance by identifying deviations from nominal parameter values, which is critical for predictive maintenance and minimizing operational disruptions. The successful implementation of this method suggests that multiparametric programming can substantially improve the performance and reliability of two-tank systems across various applications, marking a significant advancement in the field of control systems engineering. Future work should focus on incorporating advanced control strategies into the two-tank system, further optimizing liquid-level management and enhancing system stability; implementing controllers, such as PID or model predictive control, could provide real-time adjustments based on the parameters estimated through the MPP method, leading to more responsive and robust liquid level control, ensuring optimal performance under varying operating conditions and maximizing efficiency and safety in fluid management processes.

Acknowledgements

The authors would like to acknowledge the Fundamental Research Grant Scheme (FRGS) support under grant number FRGS/1/2022/TK08/UNIMAP/02/92 from the Ministry of Higher Education (MOHE) Malaysia and Universiti Malaysia Perlis.

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Оцінка параметрів системи з двома резервуарами за допомогою багатопараметричного програмування

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Багатопараметричне програмування спрощує проблеми оптимізації, дозволяючи виражати розв'язки як функції змінних параметрів. Цей метод підвищує точність і ефективність оцінки параметрів, що є критично важливим для динамічних систем. У цьому дослідженні подано новий підхід до оцінки параметрів для системи з двома резервуарами з використанням багатопараметричного програмування (MPP). Система з двома резервуарами широко використовується в різних сферах застосування, що вимагає ефективного керування рівнями рідини для безпечної та ефективної роботи. Точна оцінка параметрів у системі резервуарів має вирішальне значення для оптимального керування; однак звичайні методи оцінювання часто стикаються зі складнощами та обчислювальними вимогами, що може призвести до затримок і неточностей у діагностиці параметрів системи. Запропонований метод використовує MPP для точної ідентифікації параметрів системи резервуарів. У цьому підході система з двох резервуарів моделюється за допомогою звичайних диференціальних рівнянь, які описують динаміку рівнів рідини в кожному резервуарі. Метод Каруша–Куна–Таккера використовується для вирішення задачі оцінки параметрів, виводячи явний розв'язок з особливим акцентом на площі поперечного перерізу вихідної труби. Оціночні значення параметрів розраховуються з використанням змінних стану системи з двома резервуарами, що полегшує комплексну оцінку запропонованого методу. Результати демонструють, що запропонований метод точно оцінює параметри за допомогою явного розв'язку, пропонуючи швидші обчислення та меншу складність порівняно зі звичайними методами оптимізації.

Ключові слова: *оцінка параметра; система з резервуарами; багатопараметричне програмування.*