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A property of a particular generalized Petersen unit-distance graph

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A generalized Petersen graph is a graph with $2n$ vertices, where each vertex has degree 3 and there are $3n$ edges. A unit-distance graph is a graph with every edge of 1 unit length. We study the geometric transformation of a generalized Petersen graph into a generalized Petersen unit-distance graph and the rotation angles of the n-pointed star of the generalized Petersen unit-distance graph. Then, we obtain the properties of the generalized Petersen unit-distance graph and the rotation angles of the n-pointed star of the generalized Petersen unit-distance graph by using geometric transformations, trigonometric functions, and the rule of sine and cosine, along with similar polygons.

Keywords: *generalized Petersen graph; unit-distance; rotation angles.*

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1. Introduction

The generalized Petersen graph is the general structure of the Petersen graph, denoted by $GP(n,k)$ for integers $n \geqslant 3$ and $1 \leqslant k \leqslant \left\lfloor \frac{(n-1)}{2} \right\rfloor$. It consists of $2n$ vertices and $3n$ edges, as shown in Figure 1. It can be drawn in a way that has n mirror reflection edges and n-fold rotational symmetry so that it consists of an outer n-sided regular polygon and an inner n-pointed star. However, each edge may not have a length of one unit.

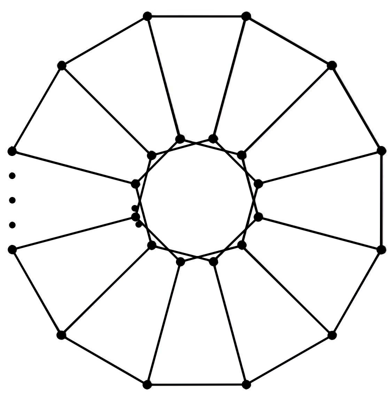

Fig. 1. A generalized Petersen graph.

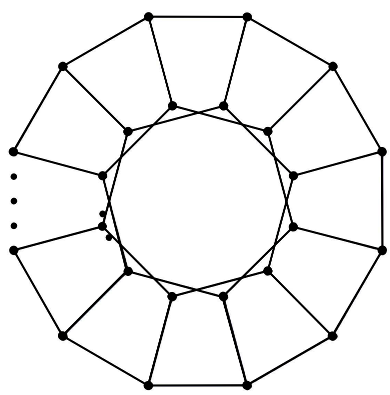

Fig. 2. A generalized Petersen graph with the remaining n edges having equal lengths but shorter than 1 unit.

A unit-distance graph is widely studied in recent years. For example, chromatic number [1–3], product of graphs [4,5], planarity [6,7] and regularity [8]. In 2019, Griffiths [9] geometrically transformed the Petersen graph into a Petersen unit-distance graph and then studied a geometric property of a particular unit-distance Petersen graph, namely an undirected graph consisting of 10 vertices and 15 edges. We extend to a generalized Petersen unit-distance graph by starting with resizing the outer

n-sided polygon so that its edges are one-unit long. Then, we enlarge the inner n-pointed star until each edge of the star is also one-unit long. The resultant graph can be seen in Figure 2.

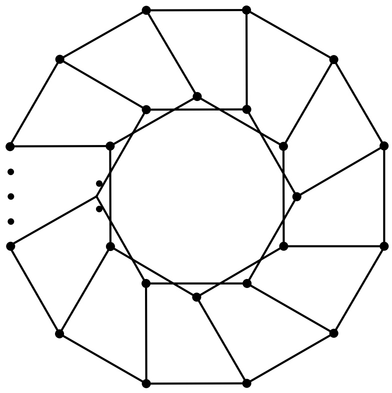

Fig. 3. A generalized Petersen unit-distance graph.

After that, we rotate the enlarged n-pointed star clockwise around the center of the circle that surrounds the outer n-sided polygon until we obtain a generalized Petersen unit-distance graph with $3n$ edges have a length of one unit, as shown in Figure 3.

In [10], Zitnik, Horvat, and Pisanski studied there are only 13 generalized Petersen graphs can be transformed into a generalized Petersen unit-distance graph such as $GP(5,2)$, $GP(6,2)$, $GP(7,2)$, $GP(7,3)$, $GP(8,2)$, $GP(8,3)$, $GP(9,2)$, $GP(9,3)$, $GP(9,4)$, $GP(10,2)$, $GP(10,3)$, $GP(11,2)$ and $GP(12,2)$.

Therefore, we separate the consideration of the rotation angle of the n-pointed star in the unit-distance generalized Petersen graph into three parts as follows:

1) The rotation angle for the generalized Petersen graph $GP(n,2)$, where $n \in \{5,6,7,\ldots,12\}$.

2) The rotation angle for the generalized Petersen graph $GP(n,3)$, where $n \in \{7,8,9,10\}$.

3) The rotation angle for the generalized Petersen graph $GP(9,4)$.

The objective of the study is to determine the rotation angle of the n-pointed star that transforms a generalized Petersen graph into a generalized Petersen unit-distance graph.

2. Computations

2.1. The rotation angle for the generalized Petersen unit-distance graph $GP(n,2)$, where $n \in \{5,6,7,\ldots,12\}$

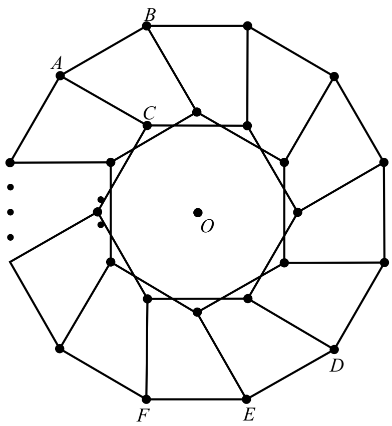

Fig. 4. A partially-labelled Generalized Petersen unit-distance graph $GP(n,2)$.

First, we consider triangle DEF to find the length of the diagonal of the n-sided polygon, and we obtain that triangle DEF is an isosceles triangle because $DE = EF = 1$ and $\widehat{DEF} = \frac{(n-2)\pi}{n}$. It follows that $\widehat{DFE} = \widehat{FDE} = \frac{\pi - \frac{(n-2)\pi}{n}}{2} = \frac{\pi}{n}$.

By the law of sines, we obtain that $\frac{DF}{\sin \widehat{DEF}} = \frac{DE}{\sin \widehat{DFE}}$ and then,

$$DF = \frac{\sin \frac{(n-2)\pi}{n}}{\sin \frac{\pi}{n}} = \frac{\sin\left(\pi - \frac{2\pi}{n}\right)}{\sin \frac{\pi}{n}} = \frac{\sin \frac{2\pi}{n}}{\sin \frac{\pi}{n}} = \frac{2 \sin \frac{\pi}{n} \cos \frac{\pi}{n}}{\sin \frac{\pi}{n}} = 2 \cos \frac{\pi}{n}.$$

Therefore, the diagonal of the n-sided polygon has a length $2 \cos \frac{\pi}{n}$.

Next, consider the n-pointed star. Then, draw edges connecting each vertex of the n-pointed star, resulting in an inner n-sided polygon. It follows, that each diagonal of this inner n-sided polygon has a length of 1 unit.

The ratio of the radius of the circle circumscribed in the outer n-sided polygon to the diagonal lengths of such n-sided polygon is equal to the distance from O to the vertices of the inner n-sided polygon. That is, $OA : DF = OC : 1$.

Next, we consider triangle AOB to determine the length of side OA. We see that since OA and OB are radii of the circle that circumscribes the n-sided polygon that means $OA = OB$. Therefore, triangle AOB is an isosceles triangle. Since $\widehat{AOB} = \frac{2\pi}{n}$, we have that $\widehat{OAB} = \widehat{OBA} = \frac{\pi - \frac{2\pi}{n}}{2} = \frac{(n-2)\pi}{2n}$.

By the law of sines, we obtain that $\frac{OA}{\sin \widehat{ABO}} = \frac{AB}{\sin \widehat{AOB}}$ and then,

$$OA = \frac{\sin \frac{(n-2)\pi}{2n}}{\sin \frac{2\pi}{n}} = \frac{\cos\left(\frac{\pi}{2} - \frac{(n-2)\pi}{2n}\right)}{2 \sin \frac{\pi}{n} \cos \frac{\pi}{n}} = \frac{\cos \frac{\pi}{n}}{2 \sin \frac{\pi}{n} \cos \frac{\pi}{n}} = \frac{1}{2 \sin \frac{\pi}{n}}.$$

Since $OA : DF = OC : 1$, it follows that

$$OC = \frac{OA}{DF} = \frac{1}{2\sin\frac{\pi}{n}\cdot 2\cos\frac{\pi}{n}} = \frac{1}{2\sin\frac{2\pi}{n}}.$$

Next, we consider triangle AOC to determine angle $\widehat{AOC}$, which is the rotation angle of the n-pointed star. We see that $AC = 1$, $OA = \frac{1}{2\sin\frac{\pi}{n}}$ and $OC = \frac{1}{2\sin\frac{2\pi}{n}}$.

Let $\theta = \widehat{AOC}$. By the law of cosines, we obtain that

$$\begin{aligned} AC^2 &= OA^2 + OC^2 - 2(OA)(OC)\cos\theta,\\ 1 &= \frac{1}{4\sin^2\frac{\pi}{n}} + \frac{1}{4\sin^2\frac{2\pi}{n}} - \frac{\cos\theta}{4\sin\frac{\pi}{n}\sin\frac{2\pi}{n}},\\ 1 &= \frac{\sin^2\frac{2\pi}{n} + \sin^2\frac{\pi}{n} - 2\sin\frac{\pi}{n}\sin\frac{2\pi}{n}\cos\theta}{4\sin^2\frac{\pi}{n}\sin^2\frac{2\pi}{n}},\\ 4\sin^2\frac{\pi}{n}\sin^2\frac{2\pi}{n} &= \sin^2\frac{2\pi}{n} + \sin^2\frac{\pi}{n} - 2\sin\frac{\pi}{n}\sin\frac{2\pi}{n}\cos\theta. \end{aligned}$$

Let $A = \sin\frac{\pi}{n}$ and $B = \sin\frac{2\pi}{n}$. It follows that

$$\begin{aligned} 4A^2B^2 &= A^2 + B^2 - 2AB\cos\theta,\\ \cos\theta &= \frac{A^2+B^2-4A^2B^2}{2AB},\\ \theta &= \arccos\left(\frac{A^2+B^2-4A^2B^2}{2AB}\right). \end{aligned}$$

Hence, the rotation angle of the n-pointed star that transforms the generalized Petersen graph $GP(n,2)$ into a generalized Petersen unit-distance graph is $\arccos\left(\frac{A^2+B^2-4A^2B^2}{2AB}\right)$, where $A = \sin\frac{\pi}{n}$ and $B = \sin\frac{2\pi}{n}$.

2.2. The rotation angle for the generalized Petersen unit-distance graph $GP(n,3)$, where $n \in \{7,8,9,10\}$

First, we consider triangle EFG, and then we see that triangle EFG is an isosceles triangle because $EF = FG = 1$ and $\widehat{EFG} = \frac{(n-2)\pi}{n}$. It follows that $\widehat{FEG} = \widehat{FGE} = \frac{\pi - \frac{(n-2)\pi}{n}}{2} = \frac{\pi}{n}$.

Next, we consider triangle DGE to determine the length of the diagonal of the n-sided polygon. We see that $4DE = 1$, $\widehat{DGE} = \frac{\pi}{n}$ and $\widehat{DEG} = \frac{(n-2)\pi}{n} - \frac{\pi}{n} = \frac{(n-3)\pi}{n}$.

By the law of sines, we obtain that $\frac{DG}{\sin\widehat{DEG}} = \frac{DE}{\sin\widehat{DGE}}$ and then,

$$\begin{aligned} DG &= \frac{\sin\frac{(n-3)\pi}{n}}{\sin\frac{\pi}{n}} = \frac{\sin\left(\pi - \frac{3\pi}{n}\right)}{\sin\frac{\pi}{n}} = \frac{\sin\frac{3\pi}{n}}{\sin\frac{\pi}{n}}\\ &= \frac{3\sin\frac{\pi}{n} - 4\sin^3\frac{\pi}{n}}{\sin\frac{\pi}{n}} = 3 - 4\sin^2\frac{\pi}{n}. \end{aligned}$$

Therefore, the diagonal of the n-sided polygon has a length $3-4\sin^2\frac{\pi}{n}$.

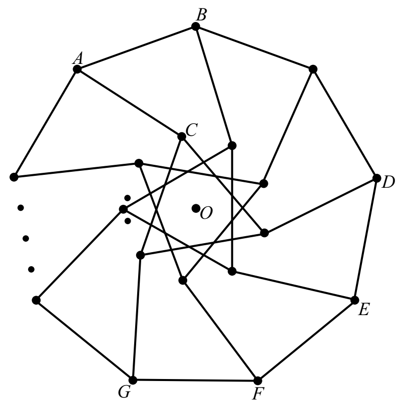

Fig. 5. A partially-labelled Generalized Petersen unit-distance graph $GP(n,3)$.

Next, consider the n-pointed star. Then, draw edges connecting each vertex of the n-pointed star, resulting in an inner n-sided polygon. It follows that each diagonal of this inner n-sided polygon has a length of 1 unit.

The ratio of the radius of the circle circumscribed in the outer n-sided polygon to the diagonal lengths of such n-sided polygon is equal to the distance from O to the vertices of the inner n-sided polygon. That is, $OA : DG = OC : 1$.

Next, we consider triangle AOB to determine the length of side OA. Since OA and OB are radii of the circle that circumscribes the n-sided polygon that means $OA = OB$. We obtain that, triangle AOB is an isosceles triangle. Since $\widehat{AOB} = \frac{2\pi}{n}$, we have that $\widehat{OAB} = \widehat{OBA} = \frac{\pi - \frac{2\pi}{n}}{2} = \frac{(n-2)\pi}{2n}$.

By the law of sines, we obtain that $\frac{OA}{\sin \widehat{ABO}} = \frac{AB}{\sin \widehat{AOB}}$ and then,

$$OA = \frac{\sin \frac{(n-2)\pi}{2n}}{\sin \frac{2\pi}{n}} = \frac{\cos\left(\frac{\pi}{2} - \frac{(n-2)\pi}{2n}\right)}{2\sin\frac{\pi}{n}\cos\frac{\pi}{n}} = \frac{\cos\frac{\pi}{n}}{2\sin\frac{\pi}{n}\cos\frac{\pi}{n}} = \frac{1}{2\sin\frac{\pi}{n}}.$$

Since $OA : DG = OC : 1$, it follows that

$$OC = \frac{OA}{DG} = \frac{1}{\left(2\sin\frac{\pi}{n}\right)\left(3-4\sin^2\frac{\pi}{n}\right)} = \frac{1}{\left(3\sin\frac{\pi}{n} - 4\sin^3\frac{\pi}{n}\right)} = \frac{1}{2\sin\frac{3\pi}{n}}.$$

Next, we consider triangle AOC to determine angle $\widehat{AOC}$, which is the rotation angle of the n-pointed star. We see that $AC = 1$, $OA = \frac{1}{2\sin\frac{\pi}{n}}$ and $OC = \frac{1}{2\sin\frac{3\pi}{n}}$.

Let $\theta = \widehat{AOC}$. By the law of cosines, we obtain that

$$\begin{aligned} AC^2 &= OA^2 + OC^2 - 2(OA)(OC)\cos\theta, \\ 1 &= \frac{1}{4\sin^2\frac{\pi}{n}} + \frac{1}{4\sin^2\frac{3\pi}{n}} - \frac{\cos\theta}{4\sin\frac{\pi}{n}\sin\frac{3\pi}{n}}, \\ 1 &= \frac{\sin^2\frac{3\pi}{n} + \sin^2\frac{\pi}{n} - 2\sin\frac{\pi}{n}\sin\frac{3\pi}{n}\cos\theta}{4\sin^2\frac{\pi}{n}\sin^2\frac{3\pi}{n}}, \\ 4\sin^2\frac{\pi}{n}\sin^2\frac{3\pi}{n} &= \sin^2\frac{3\pi}{n} + \sin^2\frac{\pi}{n} - 2\sin\frac{\pi}{n}\sin\frac{3\pi}{n}\cos\theta. \end{aligned}$$

Let $A = \sin\frac{\pi}{n}$ and $B = \sin\frac{3\pi}{n}$. It follows that

$$\begin{aligned} 4A^2B^2 &= A^2 + B^2 - 2AB\cos\theta, \\ \cos\theta &= \frac{A^2 + B^2 - 4A^2B^2}{2AB}, \\ \theta &= \arccos\left(\frac{A^2 + B^2 - 4A^2B^2}{2AB}\right). \end{aligned}$$

Hence, the rotation angle of the n-pointed star that transforms the generalized Petersen graph $GP(n, 3)$ into a generalized Petersen unit-distance graph is $\arccos\left(\frac{A^2+B^2-4A^2B^2}{2AB}\right)$, where $A = \sin\frac{\pi}{n}$ and $B = \sin\frac{3\pi}{n}$.

2.3. The rotation angle for the generalized Petersen unit-distance graph $GP(9, 4)$

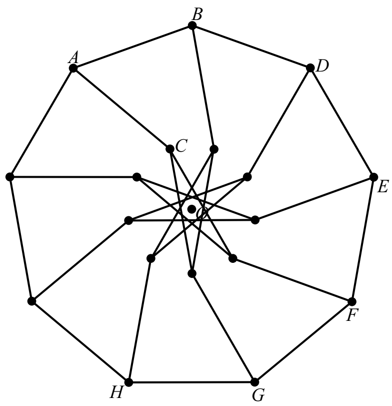

Fig. 6. A partially-labelled Generalized Petersen unit-distance graph $GP(9, 4)$.

First, we consider triangle FGH, and we obtain that triangle FGH is an isosceles triangle because $FG = GH = 1$ and $\widehat{FGH} = \frac{7\pi}{9}$. It follows that $\widehat{GFH} = \widehat{GHF} = \frac{\pi - \frac{7\pi}{9}}{2} = \frac{\pi}{9}$.

To consider triangle EFH, we found that $EF = 1$, $\widehat{EHF} = \frac{\pi}{9}$, $\widehat{EFH} = \frac{7\pi}{9} - \frac{\pi}{9} = \frac{2\pi}{3}$ and $\widehat{FEH} = \pi - \frac{\pi}{9} - \frac{2\pi}{3} = \frac{2\pi}{9}$.

Next, we consider triangle DEH to determine the length of the diagonal of the 9-sided polygon. Then, $DE = 1$, $\widehat{DHE} = \frac{\pi}{9}$ and $\widehat{DEH} = \frac{7\pi}{9} - \frac{2\pi}{9} = \frac{5\pi}{9}$.

By the law of sines, we obtain that $\frac{DH}{\sin\widehat{DEH}} = \frac{DE}{\sin\widehat{DHE}}$ and then,

$$\begin{aligned} DH &= \frac{\sin\frac{5\pi}{9}}{\sin\frac{\pi}{9}} = \frac{\sin\frac{4\pi}{9}}{\sin\frac{\pi}{9}} = \frac{4\sin\frac{\pi}{9}\cos\frac{\pi}{9}\cos\frac{2\pi}{9}}{\sin\frac{\pi}{9}} \\ &= 4\cos\frac{\pi}{9}\cos\frac{2\pi}{9} = 2\left(\cos\frac{\pi}{3} + \cos\frac{\pi}{9}\right) = 1 + 2\cos\frac{\pi}{9}. \end{aligned}$$

Therefore, the diagonal of the 9-sided polygon has a length $1 + 2\cos\frac{\pi}{9}$.

Next, consider the 9-pointed star. Then, draw edges connecting each vertex of the 9-pointed star, resulting in an inner 9-sided polygon. It follows that each diagonal of this inner 9-sided polygon has a length of 1 unit.

The ratio of the radius of the circle circumscribed in the outer 9-sided polygon to the diagonal lengths of such 9-sided polygon is equal to the distance from O to the vertices of the inner 9-sided polygon. That is, $OA : DH = OC : 1$.

Next, we consider triangle AOB to determine the length of side OA. Since OA and OB are radii of the circle that circumscribes the n-sided polygon that means $OA = OB$. We have triangle AOB is an isosceles triangle. Since $\widehat{AOB} = \frac{2\pi}{9}$, we have that $\widehat{OAB} = \widehat{OBA} = \frac{\pi - \frac{2\pi}{9}}{2} = \frac{7\pi}{18}$.

By the law of sines, we obtain that $\frac{OA}{\sin\widehat{ABO}} = \frac{AB}{\sin\widehat{AOB}}$ and then,

$$OA = \frac{\sin\frac{7\pi}{18}}{\sin\frac{2\pi}{9}} = \frac{\cos\left(\frac{\pi}{2} - \frac{7\pi}{18}\right)}{2\sin\frac{\pi}{9}\cos\frac{\pi}{9}} = \frac{\cos\frac{\pi}{9}}{2\sin\frac{\pi}{9}\cos\frac{\pi}{9}} = \frac{1}{2\sin\frac{\pi}{9}}.$$

Since $OA : DH = OC : 1$, it follows that

$$OC = \frac{OA}{DH} = \frac{1}{\left(2\sin\frac{\pi}{9}\right)\left(1 + 2\cos\frac{\pi}{9}\right)} = \frac{1}{2\sin\frac{\pi}{9} + +2\sin\frac{2\pi}{9}} = \frac{1}{2\left(2\sin\frac{\pi}{6}\cos\frac{\pi}{18}\right)} = \frac{1}{2\cos\frac{\pi}{18}} = \frac{1}{2\sin\frac{4\pi}{9}}.$$

Then, we consider triangle AOC to determine angle $\widehat{AOC}$, which is the rotation angle of the 9-pointed star. It follows that $AC = 1$, $OA = \frac{1}{2\sin\frac{\pi}{9}}$ and $OC = \frac{1}{2\sin\frac{4\pi}{9}}$.

Let $\theta = \widehat{AOC}$. By the law of cosines, we obtain that

$$AC^2 = OA^2 + OC^2 - 2(OA)(OC)\cos\theta$$

$$1 = \frac{1}{4\sin^2\frac{\pi}{9}} + \frac{1}{4\sin^2\frac{4\pi}{9}} - \frac{\cos\theta}{4\sin\frac{\pi}{9}\sin\frac{4\pi}{9}}$$

$$1 = \frac{\sin^2\frac{4\pi}{9} + \sin^2\frac{\pi}{9} - 2\sin\frac{\pi}{9}\sin\frac{4\pi}{9}\cos\theta}{4\sin^2\frac{\pi}{9}\sin^2\frac{4\pi}{9}}$$

$$4\sin^2\frac{\pi}{9}\sin^2\frac{4\pi}{9} = \sin^2\frac{4\pi}{9} + \sin^2\frac{\pi}{9} - 2\sin\frac{\pi}{9}\sin\frac{4\pi}{9}\cos\theta$$

$$\cos\theta = \frac{\sin^2\frac{4\pi}{9} + \sin^2\frac{\pi}{9} - 4\sin^2\frac{\pi}{9}\sin^2\frac{4\pi}{9}}{2\sin\frac{\pi}{9}\sin\frac{4\pi}{9}}$$

$$\theta = \arccos\left(\frac{\sin^2\frac{4\pi}{9} + \sin^2\frac{\pi}{9} - 4\sin^2\frac{\pi}{9}\sin^2\frac{4\pi}{9}}{2\sin\frac{\pi}{9}\sin\frac{4\pi}{9}}\right) = \frac{\pi}{9}.$$

Hence, the rotation angle of the 9-pointed star that transforms the generalized Petersen graph $GP(9,4)$ into a generalized Petersen unit-distance graph is $\frac{\pi}{9}$.

3. Application

Inspired by generalized Petersen unit-distance graph, an electrical circuit design requires the connections between any two components to maintain equal distances. This design enhances the good performance and reduces potential issues. For example, (i) Resistors, capacitors, and electronic chips are arranged symmetrically and orderly, with equal spacing between each component. This arrangement allows for a smooth flow of electrical current. (ii) The golden traces on the circuit board are designed in balanced and consistent geometric pattern. All traces have equal lengths, which helps to avoid issues such as noise interference and signal errors, and (iii) The overall design is balanced in terms of component placement and lengths of connection, making the circuit durable and reliable in various situations.

This design improves efficiency and durability of electrical circuits, particularly in systems that require high precision and stability. For more details, see in [11–13].

4. Conclusion

From the previous computations, we found the rotation angle of the n-pointed star to transform the generalized Petersen graph into a generalized Petersen unit-distance graph as Table 1.

Table 1. This table shows the rotation angle of the n-pointed star that makes it a generalized Petersen unit-distance graph.

Types of the generalized Petersen graph	The rotation angle of the n-pointed star after consideration
$GP(n,2)$, where $n \in \{5,6,7,\ldots,12\}$	$\arccos\left(\frac{A^2+B^2-4A^2B^2}{2AB}\right)$, where $A=\sin\frac{\pi}{n}$ and $B=\sin\frac{2\pi}{n}$
$GP(n,3)$, where $n \in \{7,8,9,10\}$	$\arccos\left(\frac{A^2+B^2-4A^2B^2}{2AB}\right)$, where $A=\sin\frac{\pi}{n}$ and $B=\sin\frac{3\pi}{n}$
$GP(9,4)$	$\frac{\pi}{9}$

Besides previous thirteen generalized Petersen graphs, other generalized Petersen graphs also be transformable into a unit-distance graph. For example, a graph $GP(10,4)$, which is isomorphic to an I-graph $I(10,2,3)$. We see that the I-graph can be represented as a unit-distance graph with rotational symmetry, as shown in Figure 7. Therefore, other generalized Petersen graphs might be transformable into a generalized Petersen unit-distance graph by providing a unit-distance I-graph and finding its rotation angles that could be more than one angle.

For further research, we investigate rotation angles of the generalized Petersen graphs rather than thirteen generalized Petersen unit-distance graphs by considering the I-graph instead.

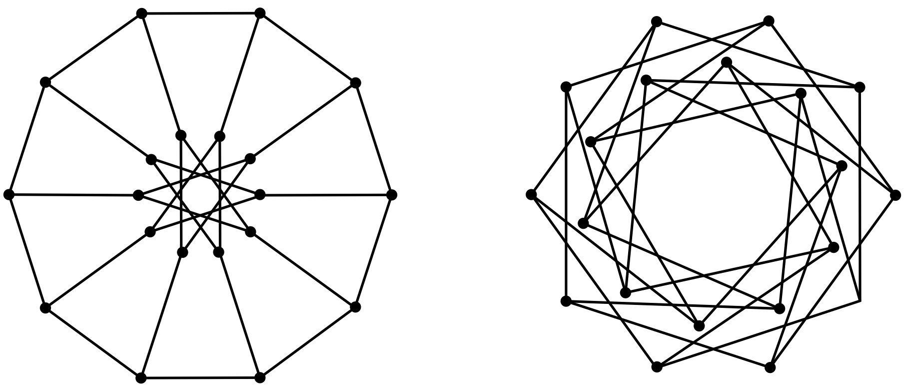

Fig. 7. A generalized Petersen graph $GP(10,4)$ and I-graph $I(10,2,3)$.

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Властивість окремого узагальненого графа одиничних відстаней Петерсена

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Узагальнений граф Петерсена — це граф із $2n$ вершинами, де кожна вершина має степінь 3 і є $3n$ ребер. Граф одиничної відстані — це граф, в якому кожне ребро має довжину 1 одиницю. Досліджено геометричне перетворення узагальненого графа Петерсена в узагальнений граф одиничних відстаней Петерсена та кути повороту n-кінцевої зірки узагальненого графа одиничних відстаней Петерсена. Потім отримано властивості узагальненого графа одиничних відстаней Петерсена та кути обертання n-кінцевої зірки узагальненого графа одиничних відстаней Петерсена за допомогою геометричних перетворень, тригонометричних функцій і правила синуса та косинуса, а також подібних многокутників.

Ключові слова: *узагальнений граф Петерсена; одиниця відстані; кути повороту.*