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Vitaliy Korendiy[1], Oleksandr Kachur[2], Mykhailo Pylyp[3], Roman Karpyn[4], Andy Augousti[5], Olena Lanets[6]

[1] Department of Technical Mechanics and Engineering Graphics, Lviv Polytechnic National University, 12, S. Bandery Str., Lviv, Ukraine, E-mail: vitalii.m.korendii@lpnu.ua, ORCID 0000-0002-6025-3013
[2] Department of Technical Mechanics and Engineering Graphics, Lviv Polytechnic National University, 12, S. Bandery Str., Lviv, Ukraine, E-mail: oleksandr.y.kachur@lpnu.ua, ORCID 0000-0003-2263-6360
[3] Department of Technical Mechanics and Engineering Graphics, Lviv Polytechnic National University, 12, S. Bandery Str., Lviv, Ukraine, E-mail: mykhailo.v.pylyp@lpnu.ua, ORCID 0009-0003-7964-0624
[4] Department of Applied Mathematics, Lviv Polytechnic National University, 12, S. Bandery Str., Lviv, Ukraine, E-mail: roman.b.karpyn@lpnu.ua, ORCID 0009-0005-1695-492X
[5] Department of Mechanical Engineering, Kingston University, Kingston Upon Thames, KT1 1LQ, London, United Kingdom, E-mail: augousti@kingston.ac.uk, ORCID 0000-0003-3000-9332
[6] Department of Mechanical Engineering, Kingston University, Kingston Upon Thames, KT1 1LQ, London, United Kingdom, E-mail: o.lanets@kingston.ac.uk, ORCID 0000-0001-7149-0957

DESIGN AND KINEMATIC ANALYSIS OF A ROBOTIC MANIPULATOR FOR CONTROLLING FIRE MONITORS

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Abstract. *Problem statement.* Conventional firefighting methods expose personnel to significant risks, particularly in hazardous environments. Robotic systems, specifically manipulators for controlling fire monitors, offer a safer and more efficient alternative by enabling precise delivery of extinguishing agents. However, their effective deployment necessitates a thorough understanding of their kinematic capabilities and limitations. *Purpose*. This research aims to conduct a comprehensive design and kinematic analysis of a five-degree-of-freedom (5-DOF) articulated robotic manipulator tailored for controlling fire monitors. The study focuses on establishing its foundational kinematic model, evaluating its workspace, and verifying its motion capabilities to lay the groundwork for advanced robotic firefighting systems. *Methodology.* The research involved the conceptual design of an all-revolute joint manipulator. The kinematic analysis was performed using the matrix transformation method to derive the forward kinematic equations. These equations define the position and orientation of the end-effector (fire monitor nozzle) based on joint variables. Numerical simulations of the gripper's motion under various predefined joint input scenarios were conducted using Mathematica software to verify the derived equations. Furthermore, the manipulator's operational workspace and motion were simulated and visualized using SolidWorks CAD/CAE software. *Findings (results).* The kinematic analysis successfully yielded the transformation matrices and explicit equations for the end-effector's coordinates. Numerical simulations in Mathematica validated the correctness of these motion equations, demonstrating predictable trajectory generation for different joint inputs. The SolidWorks simulation visually confirmed the manipulator's kinematic behavior and defined its operational workspace, suitable for targeted fire suppression tasks. The 5-DOF configuration was shown to provide substantial maneuverability for aiming a fire monitor. *Originality (novelty).* The work provides a detailed kinematic characterization and simulation-based validation of a specific 5-DOF manipulator configuration intended for fire monitor control. While building on established robotic principles, its novelty lies in the focused application and detailed kinematic groundwork for this specific firefighting task, bridging the gap between general manipulator theory and the practical requirements of fire monitor operation. It offers a foundational model that can be leveraged for more complex, dynamic, and control system designs in firefighting robotics. *Practical value.* The research provides essential kinematic data and a validated model crucial for the design and development of effective robotic firefighting systems. The findings can inform the engineering of manipulators capable of precise and agile fire monitor control, leading to improved firefighter safety, enhanced operational efficiency in hazardous environments, and more effective fire suppression through accurate delivery of extinguishing agents. *Scopes of further investigations.* Future research will focus on dynamic modeling to account for link masses, inertias, and jet reaction forces; development of robust control systems; integration with perception systems (e.g., thermal cameras) for autonomous operation; coupling with jet trajectory models for enhanced accuracy; structural optimization for harsh environments; and experimental validation with a physical prototype.

Keywords: firefighting systems, articulated arm, degrees of freedom, workspace simulation, matrix method, motion equations, end-effector positioning, trajectory generation, emergency response, nozzle aiming.

Introduction

> *Daniela Rus (Executive Director, CSAIL, MIT):*
> *"We are entering an era where machines not only build but also think, learn, and collaborate. This is a revolution in engineering."*
> *(About the synthesis of machine learning and robotics in manufacturing)*

The domain of firefighting is undergoing a significant transformation, driven by the urgent need to mitigate the inherent dangers and operational limitations faced by human firefighters. Conventional firefighting methods, particularly in complex, large-scale, or hazardous environments such as industrial facilities, tunnels, and high-rise buildings, expose personnel to extreme risks. The increasing frequency of challenging fire incidents, including those in dense indoor settings, underscores the imperative for innovative solutions. Robotic systems have emerged as a pivotal technology in this evolution, offering the potential to dramatically enhance firefighter safety, improve operational efficiency, and increase the overall effectiveness of fire suppression efforts. These systems can perform tasks in environments too perilous for humans, thereby augmenting human capabilities and reducing direct exposure to harm.

Problem Statement

Within the broader scope of firefighting robotics, a critical area of focus is the development and application of robotic manipulators specifically engineered to control fire monitors – essentially, remotely operated water or foam cannons. The efficacy of any firefighting operation hinges on the precise delivery of extinguishing agents to the seat of the fire. Robotic manipulators, when coupled with fire monitors, can provide the necessary accuracy in aiming and controlling the water or foam jet, which is paramount for rapid fire suppression, minimizing collateral water damage, and optimizing the use of extinguishing resources. The development of robotic systems for firefighting has garnered significant research interest, focusing on enhancing efficiency, safety, and precision in fire suppression. A critical aspect of these systems is the accurate control of fire monitors, which necessitates a thorough understanding of water jet dynamics and robotic manipulator kinematics. The research presented in this paper seeks to contribute to this specialized field by addressing the challenges associated with the design and kinematic performance of robotic manipulators for controlling fire monitors.

Review of Modern Information Sources on the Subject of the Paper

Several studies have focused on improving the visual perception and control of fire robots. For instance, Zhu, Pan, and Zhao [1] proposed an improved near-field computer vision system for predicting the falling position of a water jet, a crucial element for intelligent fire robots. Building on this, Pan et al. [2] developed a visual predictive control system for fire monitors that incorporates a time delay model of the fire extinguishing jet, addressing the complexities of real-time control. The foundational understanding of water jet trajectory is also a significant area of research. Zhu et al. [3] conducted studies on water jet trajectory models for fire monitors using both simulation and experimental approaches. Similarly, Lin et al. [4] introduced a two-stage water jet landing point prediction model for intelligent water shooting robots, aiming to improve targeting accuracy. Hou et al. [5] further contributed by developing models to predict both the jet trajectory and the intensity drop point of fire monitors. The physical characteristics of the nozzle also play a role; Vahedi Tafreshi and Pourdeyhimi [6] investigated the effects of nozzle geometry on waterjet breakup at high Reynolds numbers, which can influence jet cohesion and range. More recently, Fan, Deng, and Liu [7] researched modeling and landing point prediction technology for water jet trajectories from fire trucks in large-scale scenarios, highlighting the need for robust prediction across various operational conditions.

The broader context of autonomous firefighting robots encompasses various technological advancements. Rakib and Sarkar [8] detailed the design and fabrication of an autonomous firefighting robot equipped with multisensory fire detection and a PID controller. The integration of artificial intelligence for enhanced fire detection capabilities in firefighting robots was explored by Ramasubramanian, Muthukumaraswamy, and Sasikala [9]. Innovative robotic designs have also emerged, such as the aerial hose-type robot proposed by Ando et al. [10], which utilizes a water jet for propulsion and firefighting. Control strategies for water cannons on unmanned fireboats have also been investigated, with Gao, Xie, and Wang [11] presenting a method based on EGWO-ADFUZZY.

Central to the effective operation of these robotic systems is the design and kinematic analysis of their manipulators. Huczala et al. [12] discussed the initial estimation of a robotic manipulator's kinematic structure as a vital input for its synthesis. Eliot et al. [13] provided a detailed design and kinematic analysis of an articulated robotic manipulator, a common configuration in such applications. The development of multipurpose mobile manipulators for tasks including autonomous firefighting has been demonstrated by Basiri et al. [14]. Advances in sensor integration and machine learning continue to push the boundaries, as shown by Tephila et al. [15], who developed a deep learning and machine vision-based robot for fire detection and control. Tanyıldızı [16] presented the design, control, and stabilization of a transformable wheeled firefighting robot featuring a fire-extinguishing, ball-shooting turret, showcasing specialized end-effector designs.

Kinematic motion control for mobile manipulators acting as firefighters was addressed by Syam et al. [17]. De Santis, Siciliano, and Villani [18] explored fuzzy trajectory planning and redundancy resolution specifically for a firefighting robot operating in tunnel environments. The structural and kinematic analysis of different manipulator types, such as pantograph-type manipulators with three degrees of freedom, has been studied by Korendiy, Zinko, and Cherevko [19]. Raut, Rathod, and Ruiwale [20] conducted a forward kinematic analysis of a robotic manipulator with unique triangular prism structured links. A comprehensive review of automatic fire water monitor systems was provided by Duan and Hou [21], summarizing the state of the art in this specific area.

The reliability and performance of robotic manipulators are also critical considerations. Pandey and Zhang [22] performed a system reliability analysis of robotic manipulators considering random joint clearances, which can affect precision. Korendiy et al. [23] analyzed the kinematic characteristics of a mobile caterpillar robot equipped with a SCARA-type manipulator. Zhang, Li, and Wang [24] focused on the kinematic modeling and performance analysis of a 5-DoF robot for industrial automation, principles of which can be applied to firefighting manipulators. Liu, Chen, and Wang [25] detailed the design and research of an articulated tracked firefighting robot, emphasizing mobility in challenging terrains. Gowda, Katti, and H. [26] utilized software tools like Roboanalyzer for the kinematic analysis of a 4R robot manipulator. Further work by Korendiy et al. [27] focused on optimizing the structural parameters of robotic systems to ensure efficiency and reliability in production environments, which has parallels in demanding firefighting applications. Recent advancements include the use of deep neural networks for inverse kinematics, control, and planning for robotic manipulators, as reviewed by Calzada-Garcia et al. [28]. Çetinkaya, Yildirim, and

Yildirim [29] applied artificial neural networks for the trajectory analysis of 6-DOF industrial robot manipulators. Finally, the integration of SLAM (Simultaneous Localization and Mapping) and flame image recognition for indoor autonomous inspection and firefighting robots was demonstrated by Li et al. [30], while Aliff et al. [31] documented the development process of a firefighting robot named QRob. Guo et al. [32] contributed to the field with the design of a small wheel-foot hybrid firefighting robot focusing on infrared visual fire recognition. This body of work underscores the multifaceted research efforts towards developing sophisticated robotic manipulators for effective fire monitor control.

Objectives and Problems of Research

The primary aim of this research is to conduct a comprehensive design and kinematic analysis of a robotic manipulator specifically tailored for the effective control and aiming of fire monitors. This study seeks to lay the groundwork for developing advanced robotic firefighting systems capable of precise and agile operation. A specific objective of this research is the development of a conceptual design and detailed structural layout for a robotic manipulator optimized for firefighting tasks. This involves defining a suitable kinematic architecture, such as an articulated manipulator with an appropriate number of degrees of freedom, to provide the necessary dexterity for aiming a fire monitor. The design must also consider the fire monitor as the end-effector, accommodating its weight, dimensions, and the significant reaction forces generated during high-pressure water or foam discharge, while ensuring robust operation in potentially hazardous firefighting environments.

Another key objective is to establish a precise mathematical model of the manipulator's kinematics using established methodologies, such as the matrix transformation method. This encompasses assigning local coordinate frames to each link of the manipulator according to systematic conventions, deriving the forward kinematic equations to define the position and orientation of the fire monitor's nozzle, and formulating the inverse kinematic problem to determine the joint variables required for accurate fire targeting. Furthermore, this research aims to analyze and characterize the operational workspace of the designed manipulator. This involves determining the reachable volume and orientation capabilities of the fire monitor when mounted on the manipulator and identifying any limitations or dead zones within the workspace to ensure comprehensive coverage for firefighting scenarios. Finally, an important objective is to perform numerical simulations and virtual prototyping to validate the kinematic model and assess the manipulator's performance. This includes utilizing software tools to verify the derived forward and inverse kinematic solutions and simulating the manipulator's motion as it guides the fire monitor along representative trajectories, allowing for the visualization and evaluation of its kinematic behavior.

One of the key research problems this study aims to address is the identification of an optimal kinematic configuration for firefighting. This involves determining a structure that balances extensive reach, high maneuverability for targeting fires in complex three-dimensional spaces, structural rigidity to counteract water jet reaction forces, and a practical design for deployment. A significant challenge is the development or adaptation of computationally efficient and reliable algorithms for solving the inverse kinematics problem for the chosen manipulator configuration. This is crucial for enabling responsive and accurate control of the fire monitor, particularly for tracking moving fire fronts or adjusting aim in dynamic situations. The investigation of potential kinematic singularities within the manipulator's workspace also presents a research problem. Identifying these points, where the manipulator may lose degrees of freedom or experience excessively high joint velocities, and developing strategies for their avoidance or mitigation are critical for robust and safe operation.

Ensuring the high fidelity of the developed kinematic models and simulation results in representing the intended physical system is another problem to be addressed. This requires careful verification of mathematical derivations and simulation setups to build confidence in the predicted performance before any physical implementation. By addressing these objectives and problems, this research aims to contribute to the advancement of robotic systems for firefighting, providing a solid foundation for the design and control of manipulators capable of enhancing the safety and effectiveness of fire suppression efforts.

Design and Operational Peculiarities of the Robotic Manipulator

The robotic manipulator (Fig. 1), intended for the precise control and aiming of a fire monitor, is conceptualized with an articulated, link-based structure employing an angular (or all-revolute) kinematic scheme. This design choice, where all mobile joints are rotational, inherently provides high maneuverability and flexibility within the operational workspace, which is critical for firefighting applications. The manipulator can be conceptualized for either stationary deployment, with its base directly anchored to a fixed foundation or support structure, or for mobile integration, where the base is mounted onto a wheeled or tracked platform to extend its operational reach in diverse scenarios.

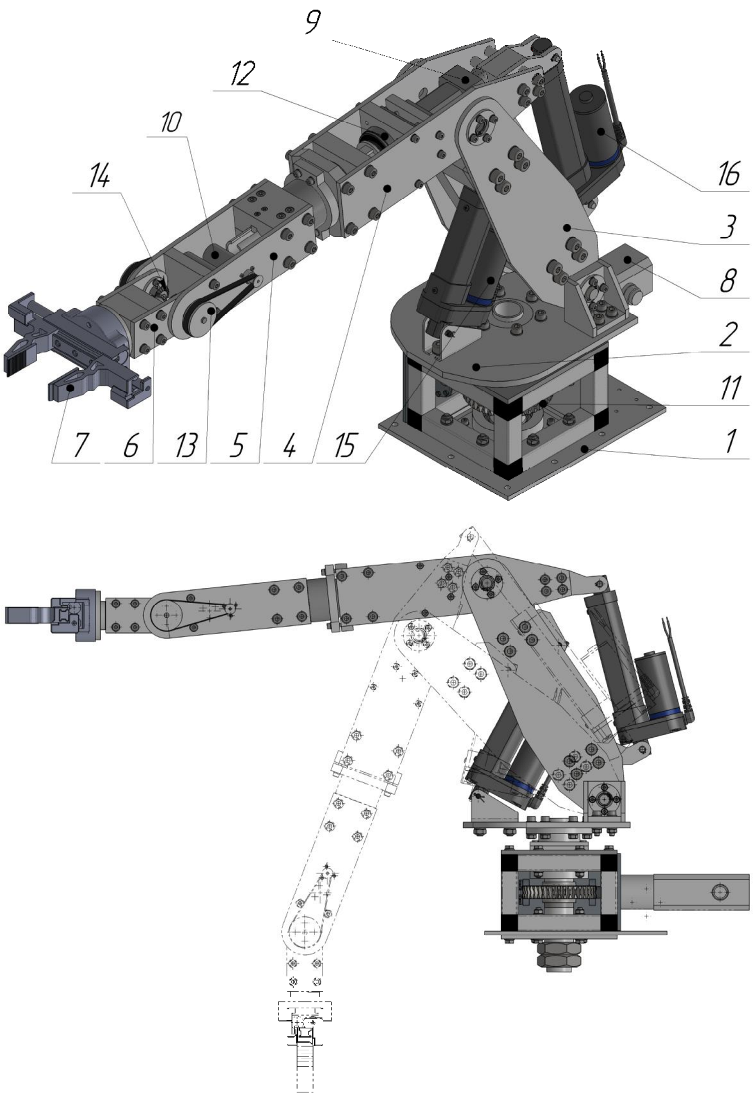

Fig. 1. General design of the developed robotic manipulator:
1 – manipulator base; 2 – horizontal rotation drive; 3 – shoulder link; 4 – elbow link;
5 – forearm link; 6 – wrist link; 7 – gripper; 8, 9, 10 – electric motors; 11 – worm gear; 12 – turning mechanism; 13 – toothed belt drive; 14 – revolute kinematic pair (bearing assembly); 15, 16 – linear electric actuators

The fundamental structural elements of the proposed manipulator are designed to ensure robust and precise operation. A base plate serves as the primary foundation, providing stability and a mounting point for the entire manipulator assembly. The first major motion is achieved through a horizontal rotation drive, typically incorporating an electric motor and a gear reduction mechanism (such as a worm gear), which allows the entire manipulator to sweep and cover a wide azimuthal range around its vertical axis.

The manipulator arm itself consists of several key links. The shoulder, or first link, connects to the horizontal rotation drive through a revolute joint. Its angular position relative to the horizontal plane (pitch) is controlled by a dedicated drive mechanism, often a linear actuator, enabling elevation and depression of the arm. The elbow, or second link, is connected to the shoulder via another revolute joint. The angle of the elbow relative to the shoulder is managed by a separate linear actuator, allowing the manipulator to extend and retract, thereby adjusting the reach and configuration of the arm. The forearm, or third link, connects to the elbow. A distinct feature of this segment is its ability to rotate about its longitudinal axis, driven by an electric motor and a suitable turning mechanism. This axial rotation (roll) provides an additional degree of freedom crucial for orienting the end-effector. The wrist assembly is connected to the forearm through a revolute joint, with its angular position relative to the forearm being controlled by an electric motor, potentially through a belt drive or similar transmission. For applications such as controlling a fire monitor, the wrist assembly is envisioned to provide multiple degrees of freedom. While the specific configuration can vary, the design intent is to achieve sufficient dexterity (potentially up to three independent rotations at the wrist: pitch, yaw, and roll) to allow for arbitrary orientation of the fire monitor nozzle in three-dimensional space. The fire monitor itself, serving as the end-effector, would be securely mounted to this wrist assembly. The design must account for the weight of the fire monitor and the substantial reaction forces generated during the discharge of water or firefighting agents at high pressure.

The primary operational movements of the manipulator, enabling effective fire monitor control, include: rotation of the entire manipulator structure about its vertical base axis; pitching motion of the shoulder joint to elevate or lower the arm; pitching motion of the elbow joint to extend or retract the arm; axial rotation of the forearm; and multi-axis rotation of the wrist to precisely orient the fire monitor nozzle. These movements are coordinated by a control system that receives input commands (either from an operator or an autonomous system) and translates them into precise actuations of the respective joint motors and actuators. Key design considerations for such a manipulator include compactness, to allow for deployment in potentially constrained spaces, and modularity, which can facilitate maintenance and adaptation to different types of fire monitors or operational requirements. Crucially, all drive mechanisms, transmission elements, and sensitive electronic components must be robustly protected by sealed housings or covers. This is essential to prevent damage from environmental factors encountered during firefighting, such as water, heat, smoke, and debris, and to ensure the safety of any nearby personnel. The overall operational principle involves the control system directing the manipulator's links through a programmed sequence of movements to accurately aim the fire monitor at the designated target, maintain the aim as needed, and adjust the trajectory of the firefighting agent.

Kinematic Analysis Methodology

Kinematic analysis is a cornerstone in the design, research, and operational programming of robotic manipulators, particularly for applications demanding precise end-effector control, such as the aiming of a fire monitor. This analysis provides the fundamental mathematical framework to describe and understand the motion of the manipulator, specifically how the movements of its individual links collectively determine the position and orientation of the fire monitor nozzle in three-dimensional space. A thorough kinematic analysis is indispensable for effective robot control, trajectory planning, and the programming required to execute specific firefighting tasks.

The kinematic analysis of a manipulator typically involves addressing two fundamental and interconnected problems: forward kinematics and inverse kinematics. The forward kinematics problem focuses on determining the position and orientation of the end-effector (in this case, the fire monitor nozzle) relative to a fixed base coordinate system, given the known geometric parameters of the manipulator (such as link lengths and joint offsets) and the current values of its generalized coordinates (i.e., the angles of its revolute joints or displacements of prismatic joints). Furthermore, if the velocities and accelerations of these generalized coordinates are known, forward kinematics can also determine the linear and angular velocity and acceleration of the end-effector. Solving the forward kinematics problem is essential for tasks such as defining the manipulator's reachable workspace and simulating its motion based on given joint inputs. Conversely, the inverse kinematics problem addresses the challenge of determining the appropriate set of generalized coordinates (joint angles/displacements) that the manipulator must adopt to achieve the desired position and orientation of its end-effector in space. If the desired trajectory, including velocities and accelerations of the end-effector, is specified, inverse kinematics aims to find the corresponding time histories of the generalized coordinates and their derivatives. The solution to the inverse kinematics problem is of paramount importance for the control system, as it dictates how the manipulator's actuators must be driven to guide the fire monitor nozzle accurately to a target or along a specified path.

Several methods have been developed to solve these kinematic problems. The geometric method, which relies on trigonometric and geometric relationships, can be intuitive for simpler manipulators but often becomes exceedingly complex for manipulators with multiple degrees of freedom or intricate spatial configurations. Numerical methods are frequently employed for complex kinematic chains where analytical solutions are difficult or impossible to obtain, often involving iterative algorithms. For the analysis of the robotic manipulator designed for fire monitor control, which typically features an articulated (angular) kinematic scheme with multiple degrees of freedom to ensure high flexibility and maneuverability, the matrix transformation method is selected. This method is highly systematic, universal, and particularly effective for spatial manipulators. It involves assigning a local coordinate system to each link of the manipulator and then using homogeneous transformation matrices to describe the geometric relationship (translation and rotation) between successive link coordinate frames. By multiplying these individual transformation matrices, a composite matrix is obtained that defines the position and orientation of the end-effector's coordinate frame relative to the base frame. This approach provides a structured and computationally efficient way to solve both the forward and, often, the inverse kinematics problems for complex manipulators.

Kinematic Diagram and Motion Equations of the Manipulator

Figure 2 shows the kinematic diagram of a manipulator with five degrees of freedom, which corresponds to the proposed design of the industrial robot depicted in Fig. 1. This number of degrees of freedom means that the manipulator can perform movements along five independent coordinates. Analyzing the diagram, the following key components and their characteristics can be identified:

1. Links. Link 1 (*OA*): a rotating link that serves as the manipulator's base. Link 2 (*AB*): the manipulator's shoulder, connected to the base by a revolute kinematic pair at point *A*. Link 3 (*BC*): the forearm, connected to the shoulder by a revolute kinematic pair at point *B*. Link 4 (*CD*): the wrist, connected to the forearm by a revolute kinematic pair at point *C*. Link 5 (*DE*): the gripper, connected to the wrist by a revolute kinematic pair at point *D*.

2. Kinematic pairs. All kinematic pairs in this kinematic diagram are revolute ones, which provide the manipulator with high maneuverability and flexibility.

3. Generalized coordinates: φ_0 – angle of rotation of link 1 (base platform) relative to the base coordinate system; φ_1 – angle of rotation of link 2 (shoulder) relative to link 1 (base platform); φ_2 – angle of rotation of link 3 (forearm) relative to link 2 (shoulder); φ_3 – angle of rotation of link 4 (wrist) relative to link 3 (forearm); φ_4 – angle of rotation of link 5 (gripper) relative to link 4 (wrist).

4. Coordinate systems. Within the matrix transformation method, a local coordinate system is introduced for each link. Figure 2 depicts the following coordinate systems: $x_0y_0z_0$ – the base coordinate system, fixedly attached to the base of link 1 (pedestal *O*); $x_1y_1z_1$ – a local coordinate system, rigidly attached to link 2 (shoulder) at joint *A*; $x_2y_2z_2$ – a local coordinate system, rigidly attached to link 3 (forearm) at joint *B*; $x_3y_3z_3$ – a local coordinate system, rigidly attached to link 4 (wrist) at joint *C*; $x_4y_4z_4$ – a local coordinate system, rigidly attached to link 5 (gripper) at joint *D*; $x_5y_5z_5$ – an additional coordinate system, rigidly attached to the end of the gripper (point *E*).

5. Geometric parameters. The diagram indicates the following parameters: l_{OA} – horizontal distance from the base *O* to joint *A*; h_{OA} – vertical distance from the base *O* to joint *A*; l_{AB} – length of link 2 (shoulder); l_{BC} – length of link 3 (forearm); l_{CD} – length of link 4 (wrist); l_{DE} – length of link 5 (gripper).

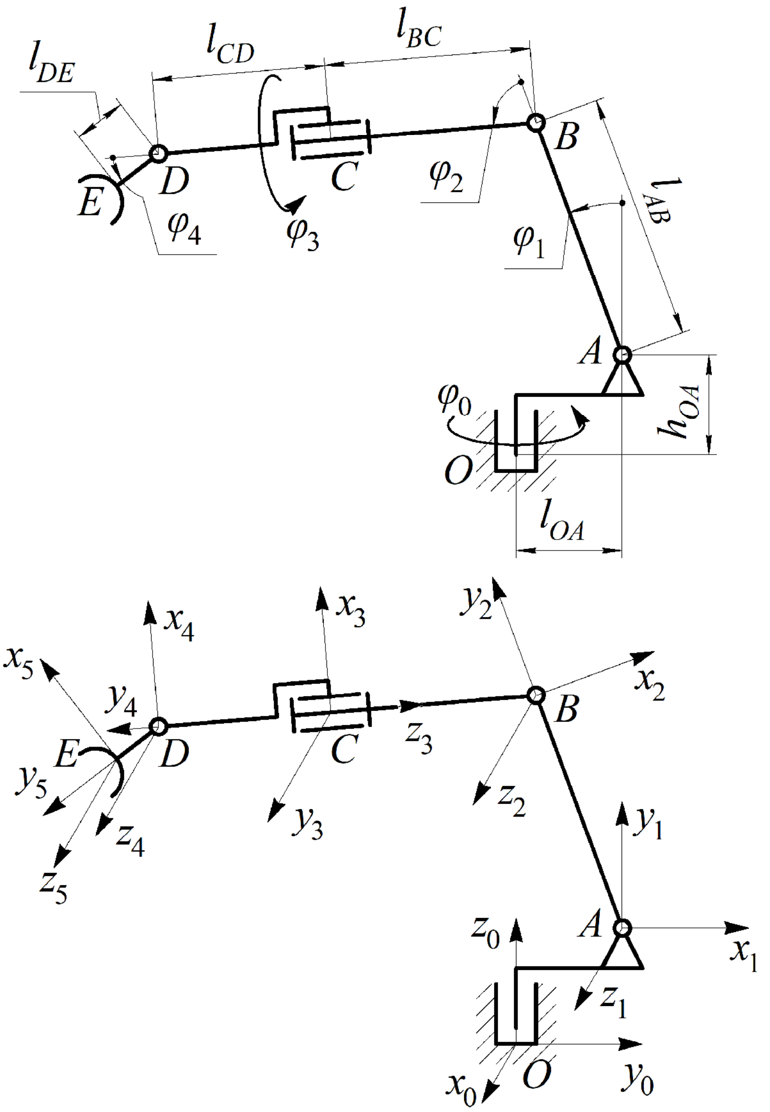

Fig. 2. Kinematic and calculation diagrams of the designed manipulator

To perform the kinematic analysis of this manipulator using the matrix transformation method, it is necessary to determine the coordinate transformation matrices between adjacent coordinate systems. These matrices will take into account the geometric parameters of the links and the generalized coordinates. Using these matrices, it is possible to determine the position and orientation of the gripper in the base coordinate system (direct kinematics problem), as well as to find the necessary values of the generalized coordinates to achieve a given position (or motion trajectory) of the gripper in space (inverse kinematics problem). In accordance with the methodology for performing the kinematic analysis of the manipulator mechanism using the matrix transformation method, as described in the previous section of this work, the appropriate coordinate systems were selected, for which the following conditions are met. The z_0 axis passes along the axis of the revolute kinematic pair (pedestal) *O*. The z_1 axis is directed along the axis of the revolute kinematic pair (joint) *A*. The z_2 axis passes in the direction of the axis of the revolute kinematic pair (joint) B. The z_3 axis is directed along the axis of the revolute kinematic pair (joint) *C*. The z_1 axis coincides with the axis of the revolute kinematic pair (joint) *D*. The directions of the x_i and y_i axes, as well as the origins of the respective coordinate systems, are presented in Fig. 2. The types of all kinematic pairs of the manipulator mechanism and the corresponding parameters of the transformation matrices for the chosen coordinate systems are given in Table 1.

Table 1

Types of all kinematic pairs of the manipulator mechanism and the corresponding parameters of the transformation matrices for the selected coordinate systems

Kinematic pair	Pair type	Link number	Parameters of the transformation matrices			
O (joining the links 0, 1)	Revolute	0	-	-	-	-
A (joining the links 1, 2)	Revolute	1	φ_0	90°	h_{OA}	l_{OA}
B (joining the links 2, 3)	Revolute	2	φ_1	0	0	l_{AB}
C (joining the links 3, 4)	Revolute	3	φ_2	90°	0	l_{BC}
D (joining the links 4, 5)	Revolute	4	φ_3	–90°	0	$-l_{CD}$
E (virtual joint)	Revolute	5	φ_4	0	0	l_{DE}

The transformation matrices A_i for the corresponding coordinate systems are as follows:

$$A_1=\begin{bmatrix}\cos\theta_1 & -\sin\theta_1\cos\alpha_1 & \sin\theta_1\sin\alpha_1 & a_1\cos\theta_1\\ \sin\theta_1 & \cos\theta_1\cos\alpha_1 & -\cos\theta_1\sin\alpha_1 & a_1\sin\theta_1\\ 0 & \sin\alpha_1 & \cos\alpha_1 & S_1\\ 0 & 0 & 0 & 1\end{bmatrix}=$$

$$=\begin{bmatrix}\cos\varphi_0 & -\sin\varphi_0\cos 90^\circ & \sin\varphi_0\sin 90^\circ & l_{OA}\cdot\cos\varphi_0\\ \sin\varphi_0 & \cos\varphi_0\cos 90^\circ & -\cos\varphi_0\sin 90^\circ & l_{OA}\cdot\sin\varphi_0\\ 0 & \sin 90^\circ & \cos 90^\circ & h_{OA}\\ 0 & 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}\cos\varphi_0 & 0 & \sin\varphi_0 & l_{OA}\cdot\cos\varphi_0\\ \sin\varphi_0 & 0 & -\cos\varphi_0 & l_{OA}\cdot\sin\varphi_0\\ 0 & 1 & 0 & h_{OA}\\ 0 & 0 & 0 & 1\end{bmatrix};$$

$$A_2=\begin{bmatrix}\cos\theta_2 & -\sin\theta_2\cos\alpha_2 & \sin\theta_2\sin\alpha_2 & a_2\cos\theta_2\\ \sin\theta_2 & \cos\theta_2\cos\alpha_2 & -\cos\theta_2\sin\alpha_2 & a_2\sin\theta_2\\ 0 & \sin\alpha_2 & \cos\alpha_2 & S_2\\ 0 & 0 & 0 & 1\end{bmatrix}=$$

$$=\begin{bmatrix}\cos\varphi_1 & -\sin\varphi_1\cos 0^\circ & \sin\varphi_1\sin 0^\circ & l_{AB}\cdot\cos\varphi_1\\ \sin\varphi_1 & \cos\varphi_1\cos 0^\circ & -\cos\varphi_1\sin 0^\circ & l_{AB}\cdot\sin\varphi_1\\ 0 & \sin 0^\circ & \cos 0^\circ & 0\\ 0 & 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}\cos\varphi_1 & -\sin\varphi_1 & 0 & l_{AB}\cdot\cos\varphi_1\\ \sin\varphi_1 & \cos\varphi_1 & 0 & l_{AB}\cdot\sin\varphi_1\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix};$$

$$A_3=\begin{bmatrix}\cos\theta_3 & -\sin\theta_3\cos\alpha_3 & \sin\theta_3\sin\alpha_3 & a_3\cos\theta_3\\ \sin\theta_3 & \cos\theta_3\cos\alpha_3 & -\cos\theta_3\sin\alpha_3 & a_3\sin\theta_3\\ 0 & \sin\alpha_3 & \cos\alpha_3 & S_3\\ 0 & 0 & 0 & 1\end{bmatrix}=$$

$$=\begin{bmatrix}\cos\varphi_2 & -\sin\varphi_2\cos 90^\circ & \sin\varphi_2\sin 90^\circ & l_{BC}\cdot\cos\varphi_2\\ \sin\varphi_2 & \cos\varphi_2\cos 90^\circ & -\cos\varphi_2\sin 90^\circ & l_{BC}\cdot\sin\varphi_2\\ 0 & \sin 90^\circ & \cos 90^\circ & 0\\ 0 & 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}\cos\varphi_2 & 0 & \sin\varphi_2 & l_{BC}\cdot\cos\varphi_2\\ \sin\varphi_2 & 0 & -\cos\varphi_2 & l_{BC}\cdot\sin\varphi_2\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix};$$

$$A_4=\begin{bmatrix}\cos\theta_4 & -\sin\theta_4\cos\alpha_4 & \sin\theta_4\sin\alpha_4 & a_4\cos\theta_4\\ \sin\theta_4 & \cos\theta_4\cos\alpha_4 & -\cos\theta_4\sin\alpha_4 & a_4\sin\theta_4\\ 0 & \sin\alpha_4 & \cos\alpha_4 & S_4\\ 0 & 0 & 0 & 1\end{bmatrix}=$$

$$=\begin{bmatrix}\cos\varphi_3 & -\sin\varphi_3\cos(-90^\circ) & \sin\varphi_3\sin(-90^\circ) & l_{CD}\cdot\cos\varphi_3\\ \sin\varphi_3 & \cos\varphi_3\cos(-90^\circ) & -\cos\varphi_3\sin(-90^\circ) & l_{CD}\cdot\sin\varphi_3\\ 0 & \sin(-90^\circ) & \cos(-90^\circ) & 0\\ 0 & 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}\cos\varphi_3 & 0 & -\sin\varphi_3 & l_{CD}\cdot\cos\varphi_3\\ \sin\varphi_3 & 0 & \cos\varphi_3 & l_{CD}\cdot\sin\varphi_3\\ 0 & -1 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix};$$

$$A_5=\begin{bmatrix}\cos\theta_5 & -\sin\theta_5\cos\alpha_5 & \sin\theta_5\sin\alpha_5 & a_5\cos\theta_5\\ \sin\theta_5 & \cos\theta_5\cos\alpha_5 & -\cos\theta_5\sin\alpha_5 & a_5\sin\theta_5\\ 0 & \sin\alpha_5 & \cos\alpha_5 & S_5\\ 0 & 0 & 0 & 1\end{bmatrix}=$$

$$=\begin{bmatrix}\cos\varphi_4 & -\sin\varphi_4\cos 0^\circ & \sin\varphi_4\sin 0^\circ & l_{DE}\cdot\cos\varphi_1\\ \sin\varphi_4 & \cos\varphi_4\cos 0^\circ & -\cos\varphi_4\sin 0^\circ & l_{DE}\cdot\sin\varphi_1\\ 0 & \sin 0^\circ & \cos 0^\circ & 0\\ 0 & 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}\cos\varphi_4 & -\sin\varphi_4 & 0 & l_{DE}\cdot\cos\varphi_1\\ \sin\varphi_4 & \cos\varphi_4 & 0 & l_{DE}\cdot\sin\varphi_1\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}.$$

The overall transformation matrix T_5 between coordinate systems 0 and 5 is equal to the product:

$$T_5=A_1\cdot A_2\cdot A_3\cdot A_4\cdot A_5.$$

This product was calculated using the Mathematica software. Considering the rather cumbersome expression for the overall transformation matrix T_5 between coordinate systems 0 and 5, let us limit the presentation to only the last column of the corresponding matrix, which defines the coordinates of the end-effector (point E) in the base coordinate system "0" ($x_0y_0z_0$):

$$x_E=\left(l_{OA}-l_{AB}\cdot\sin\varphi_1-\left(l_{BC}+l_{CD}+l_{DE}\cdot\cos\varphi_4\right)\cdot\sin\left(\varphi_1+\varphi_2\right)-l_{DE}\cdot\sin\varphi_4\cdot\sin\varphi_3\cdot\sin\left(\varphi_1+\varphi_2\right)\right)\cdot\cos\varphi_0-$$
$$-l_{DE}\cdot\sin\varphi_4\cdot\cos\left(\varphi_1+\varphi_2\right)\cdot\cos\left(\varphi_0+\varphi_3\right);$$

$$y_E=\left(l_{OA}-l_{AB}\cdot\sin\varphi_1-\left(l_{BC}+l_{CD}+l_{DE}\cdot\cos\varphi_4\right)\cdot\sin\left(\varphi_1+\varphi_2\right)-l_{DE}\cdot\sin\varphi_4\cdot\sin\varphi_3\cdot\sin\left(\varphi_1+\varphi_2\right)\right)\cdot\sin\varphi_0-$$
$$-l_{DE}\cdot\sin\varphi_4\cdot\cos\left(\varphi_1+\varphi_2\right)\cdot\sin\left(\varphi_0+\varphi_3\right);$$

$$z_E=h_{OA}+l_{AB}\cdot\cos\varphi_1+\left(l_{BC}+l_{CD}+l_{DE}\cdot\cos\varphi_4\right)\cdot\cos\left(\varphi_1+\varphi_2\right)-l_{DE}\cdot\sin\varphi_4\cdot\cos\varphi_3\cdot\sin\left(\varphi_1+\varphi_2\right).$$

By substituting the geometric parameters (l_{OA}, h_{OA}, l_{AB}, l_{BC}, l_{CD}, l_{DE}) of the designed robotic manipulator into the equations derived above and specifying the parameters of change for the corresponding generalized coordinates (φ_0, φ_1, φ_2, φ_3, φ_4), we can determine the trajectory of the gripper's motion in space and model its kinematic characteristics.

Numerical Simulation of the Gripper Motion Using Mathematica Software

To investigate the kinematic characteristics of the robotic manipulator and analyze the motion of its gripper, a numerical simulation was performed using the Mathematica software, which allows for symbolic and numerical computations, plotting graphs, and creating interactive documents, making it a convenient tool for engineering calculations. The mathematical model is based on the kinematic equations obtained from the analysis of the manipulator's kinematic diagram (see previous section). Using the matrix transformation method, analytical expressions were derived to find the coordinates of the gripper in the base coordinate system. To implement the model in Mathematica, the following steps will be performed: 1) inputting initial data, specifically, the values of the manipulator's geometric parameters (link lengths), as well as the laws of change of the generalized coordinates over time; 2) defining the coordinate transformation matrices between adjacent coordinate systems, taking into account the geometric parameters and generalized coordinates; 3) calculating the gripper coordinates: using matrix transformations, the gripper coordinates in the base coordinate system are calculated for different time instances; 4) plotting graphs of the gripper coordinates versus time and its motion trajectory.

The geometric parameters of the manipulator links are as follows: $l_{OA} = 0.182$ m, $h_{OA} = 0.364$ m, $l_{AB} = 0.581$ m, $l_{BC} = 0.325$ m, $l_{CD} = 0.273$ m, $l_{DE} = 0.231$ m.

To conduct the numerical simulation and verify the correctness of the derived gripper motion equations, let us consider the following few simple cases of changing the generalized coordinates in the time interval 0…3 s:

1. Case of manipulator rotation around the vertical axis (Fig. 3, *a*): $\varphi_0 = 1.04 \cdot t$, $\varphi_1 = 0$, $\varphi_2 = 90°$, $\varphi_3 = 0$, $\varphi_4 = 0$;

2. Case of manipulator rotation around the vertical axis and shoulder tilt (Fig. 3, *b*): $\varphi_0 = 1.04 \cdot t$, $\varphi_1 = 0.36 \cdot t$, $\varphi_2 = 90°$, $\varphi_3 = 0$, $\varphi_4 = 0$;

3. Case of shoulder tilt and forearm extension (Fig. 3, *c*): $\varphi_0 = 0$, $\varphi_1 = 0.36 \cdot t$, $\varphi_2 = \pi/2 – 0.36 \cdot t$, $\varphi_3 = 0$, $\varphi_4 = 0$;

4. Case of manipulator rotation around the vertical axis, shoulder tilt, and forearm extension (Fig. 3, *d*): $\varphi_0 = 1.04 \cdot t$, $\varphi_1 = 0.36 \cdot t$, $\varphi_2 = \pi/2 – 0.36 \cdot t$, $\varphi_3 = 0$, $\varphi_4 = 0$.

In the first case (Fig. 3, *a*), the gripper moves along a trajectory resembling an arc or a segment of a circle in three-dimensional space. The trajectory lies approximately in a plane parallel to the *YZ* plane and has a concavity in the direction of the *Y* axis. Considering the shape of the trajectory, it can be stated that for its realization, the manipulator performs rotation around the *Z* axis, i.e., the manipulator's shoulder rotates around the vertical *Z* axis, ensuring the gripper's movement along an arc.

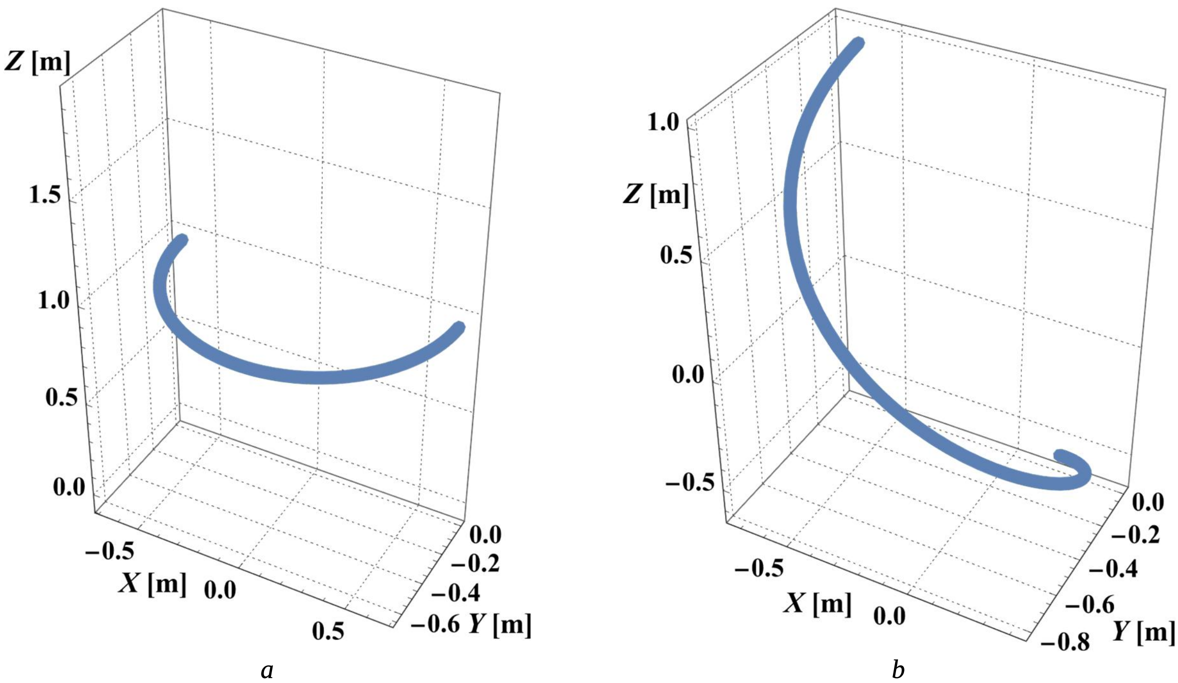

Fig. 3. Results of numerical simulation of gripper motion using Mathematica software (continued on the next page)

In the second case (Fig. 3, *b*), the gripper moves along a curve resembling a spiral or part of a helical line. The trajectory has an ascending character, meaning that the increase in the *Z* coordinate occurs simultaneously with rotation around the *Z* axis and movement in the direction of the *Y* axis. To implement such a trajectory, the manipulator evidently performs the following motions: 1) rotation around the *Z* axis (the manipulator's shoulder rotates around the vertical axis, which ensures the rotational movement of the gripper); 2) change in reach (simultaneously with rotation, the manipulator changes its reach by bending/extending the shoulder at the shoulder joint).

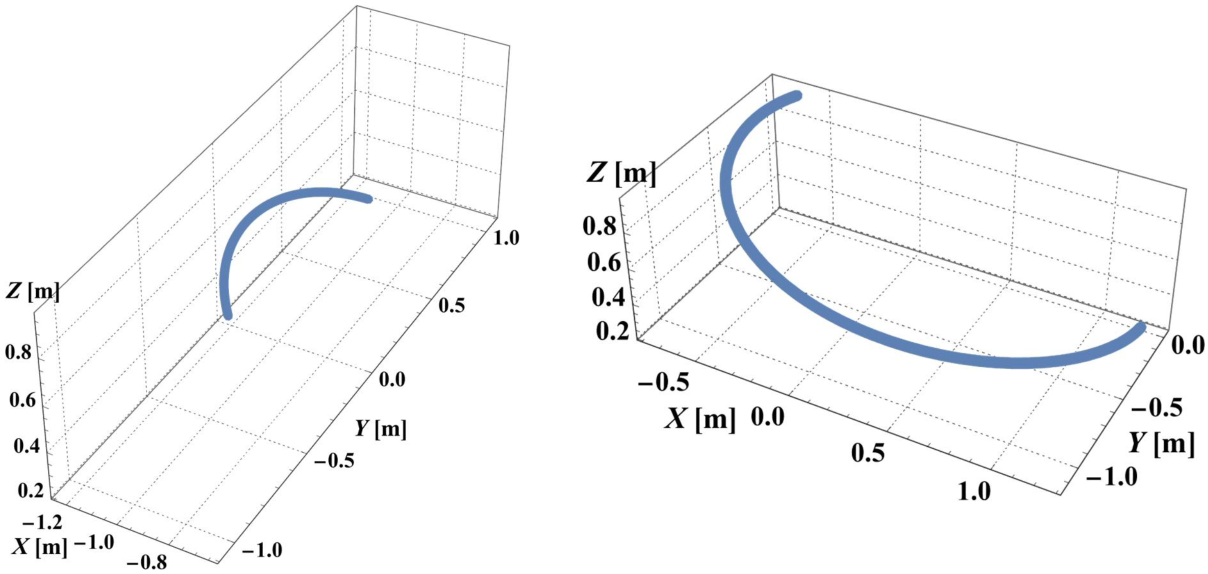

Fig. 3. Results of numerical simulation of gripper motion using Mathematica software (Continuation of Fig. 3)

The next graph (Fig. 3, *c*) displays the trajectory of the robotic manipulator's gripper, which has the shape of an arc or a segment of a circle in three-dimensional space. The trajectory resembles a circular arc located in a plane almost parallel to the *XZ* plane. The arc is situated in the region of negative *X* values and positive *Z* values. The center of the circle, of which the arc is a part, lies approximately on the *Y* axis. The gripper moves clockwise when viewed from the positive *Y*-axis side. We can ascertain that to achieve such a trajectory, the manipulator performs the following actions: 1) rotation around the *Y* axis (the manipulator's shoulder rotates around the *Y* axis, which ensures the gripper's movement along the arc); 2) bending/extending at the elbow (simultaneously with the shoulder's rotation, the forearm can bend or extend, which affects the arc's radius and its position in space).

In the fourth case (Fig. 3, *d*), the gripper's trajectory has the form of an arc or a segment of a circle, located in a plane close to the *XZ* plane. The arc is in the region of positive *Z* values and negative *Y* values. The gripper moves clockwise when viewed from the positive *X*-axis side. To achieve such a trajectory, the manipulator evidently performs the following motions: 1) rotation around the *Z* axis (the manipulator's shoulder rotates around the vertical axis, which ensures the rotational movement of the gripper); 2) rotation around the *X* axis (the manipulator's shoulder rotates around the *X* axis, ensuring the gripper's movement along the arc); 3) bending/extending at the elbow (simultaneously with the shoulder's rotation, the forearm can bend or extend, which affects the arc's radius and its position in space).

Simulation of the Manipulator Operation Zone Using SolidWorks Software

For a detailed investigation of the robotic manipulator's motion and visualization of its operation, a simulation was conducted using the SolidWorks software. SolidWorks is a computer-aided design (CAD) and engineering (CAE) software system that provides extensive capabilities for creating three-dimensional models, performing kinematic and dynamic analysis, as well as visualizing the motion of mechanisms.

Within the scope of the simulation, the following steps were performed: 1) 3D model creation: a detailed three-dimensional model of the robotic manipulator was developed in SolidWorks, taking into account its geometric parameters and design features; 2) Joint definition: appropriate types of joints (revolute, prismatic) corresponding to its kinematic diagram were defined between the manipulator links; 3) Motion definition: for each joint, a law of motion was specified, defining the change in generalized coordinates over time; 4) Simulation launch: after completing the settings, the simulation of the manipulator's motion was launched. SolidWorks automatically calculates the position and orientation of all links at each moment in time, considering the specified laws of motion; 5) Results analysis: as a result of the simulation, an animation of the manipulator's motion was obtained, which clearly demonstrates its operation. SolidWorks also provides tools for analyzing various motion characteristics, such as velocity, acceleration, displacement, forces in the manipulator link joints, energy and dynamic indicators, etc.

The simulation results, presented as a solid model with the manipulator's defined workspace, are shown in Fig. 4. The industrial robotic manipulator is characterized by five degrees of freedom. The simulation results are presented in two projections: frontal and horizontal. Additionally, the figure contains information about the geometric dimensions of the main links of the manipulator and its workspace.

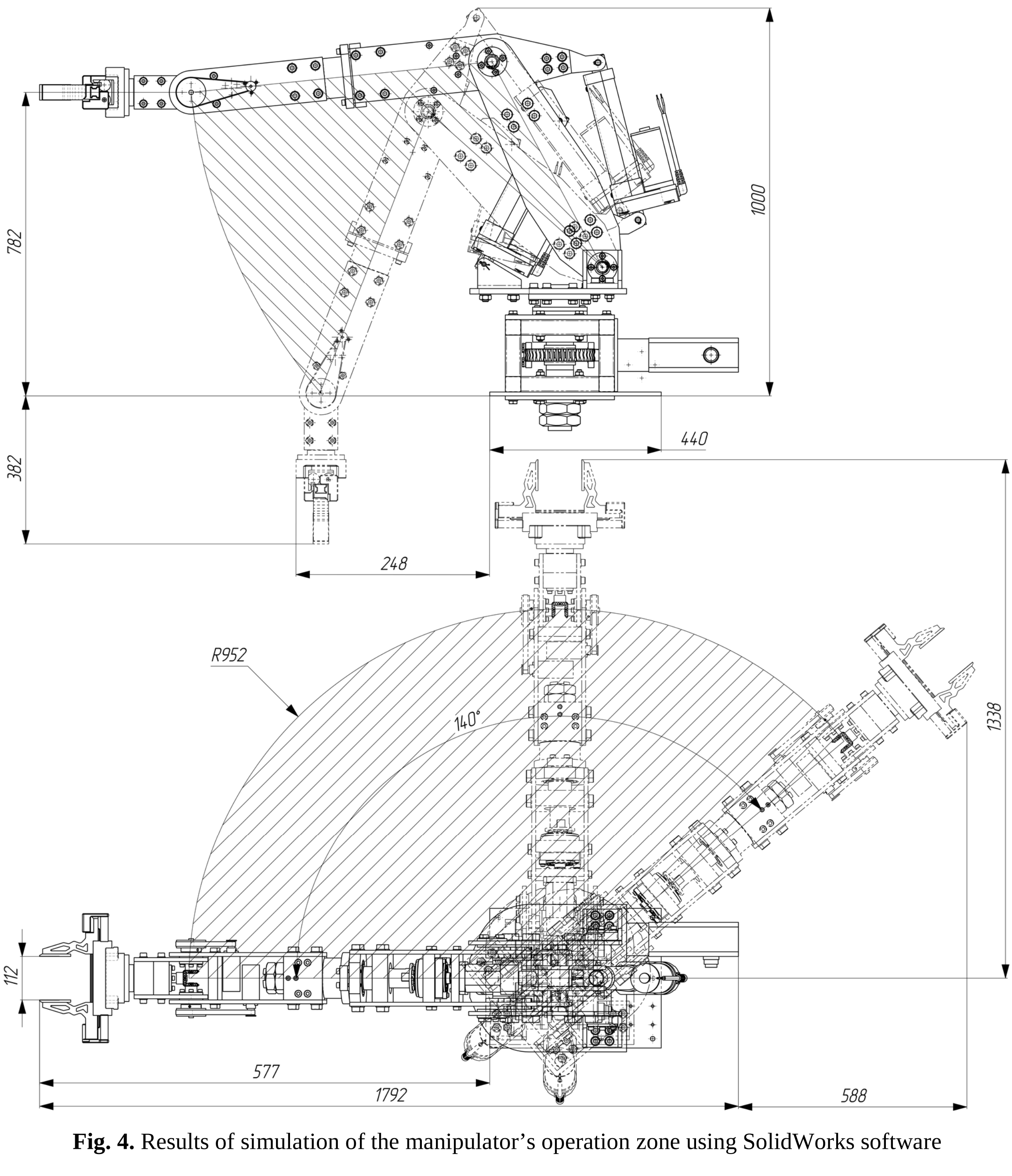

Fig. 4. Results of simulation of the manipulator's operation zone using SolidWorks software

The general design of the proposed robot is as follows. The manipulator has an articulated lever structure with a base, shoulder, forearm, wrist, and gripper. All connections between the links are revolute, which ensures high maneuverability. Figure 4 additionally indicates the main overall dimensions of the manipulator, which can be used to approximately determine the length of the shoulder, the length of the forearm, the length of the wrist, the length of the gripper, the height, width, and depth of the base, etc. Corresponding data were presented in the previous section of this paper.

The frontal and horizontal projections show the manipulator's workspace, which has the shape of a circular sector with a radius of approximately 952 mm and an angle of 140°. This means that the gripper can reach any point within this sector.

Figure 4 also shows a pincer-type gripper with two fingers, designed for manipulating a fire monitor nozzle. The figure also shows some additional structural elements, such as drives, motion transmission elements, etc. The presented model provides a general understanding of the design and geometric parameters of the robotic manipulator and can be further used for more in-depth research into the robot's inertial, strength, and dynamic characteristics.

Discussion of the Carried-out Investigations and Obtained Results

The proposed manipulator, characterized by its five degrees of freedom (DOF) in the analyzed kinematic scheme (Fig. 2) and an articulated structure with all-revolute joints, offers significant potential for fire monitor control. The 5-DOF configuration provides substantial maneuverability, allowing the end-effector – in this case, a fire monitor nozzle – to be aimed accurately within its defined workspace (approximately 952 mm reach with a 140° sweep, as indicated by simulations in Fig. 4). This level of dexterity is a marked improvement over traditional, often manually operated or simple pan-tilt fire monitors, enabling more precise water or suppressant delivery to the seat of a fire. The kinematic analysis provides the mathematical tools necessary for implementing sophisticated control algorithms, which is essential for tasks like tracking a moving fire front or targeting specific hotspots.

While the general design description (referring to Fig. 2.1) mentions a wrist with potentially higher degrees of freedom for arbitrary gripper orientation, the detailed kinematic analysis focused on a 5-DOF system. This simplification is common for foundational kinematic studies. For fire monitor aiming, 5-DOF can be sufficient for directing the jet. However, additional wrist mobility (e.g., a 6th DOF providing full nozzle orientation control) could be advantageous in complex scenarios, such as compensating for hose torsion or navigating around obstacles, and represents an area for expanded analysis.

It is important to note that the current study focuses on kinematics. A fire monitor, especially when discharging water at high pressure, imposes significant static and dynamic loads (weight and reaction forces). Therefore, while the kinematic principles are sound, a practical firefighting application would necessitate a robust structural design and appropriately sized actuators, guided by a thorough dynamic analysis, which is beyond the current scope. The presented kinematic model, however, forms the essential first step for such dynamic considerations.

The methodology employed in this paper, utilizing matrix transformations for kinematic modeling and software tools like Mathematica and SolidWorks for simulation, aligns with standard practices in robotics research, as seen in general manipulator design and analysis studies [13, 20, 24]. The 5-DOF articulated structure is also a common configuration in industrial robotics [24] and has been considered or implemented in various mobile and firefighting robot designs [17, 25].

This work contributes a detailed kinematic characterization of a specific manipulator design intended for fire monitor applications. While numerous studies focus on the broader aspects of firefighting robots, such as autonomous navigation, fire detection, and overall system integration [8, 9, 14, 15, 30, 32], the fundamental ability to accurately direct the firefighting agent via a manipulator is crucial. This paper provides that foundational kinematic understanding. For instance, the multipurpose mobile manipulator by Basiri et al. [14] or the articulated tracked robot by Liu, Chen, and Wang [25] would rely on precise kinematic models similar to the one developed here for their manipulator subsystems.

The research on water jet trajectory and prediction [1, 2, 3, 4, 5, 7] is complementary to this study. Accurate manipulator control, stemming from a well-defined kinematic model, is a prerequisite for effectively utilizing such jet trajectory prediction models to ensure the suppressant reaches its intended target. This work provides the means to control the initial vector of the jet, which is a critical input for these trajectory models. Similarly, understanding nozzle geometry effects [6] can inform the design of the end-effector integrated with the proposed manipulator.

Compared to advanced control strategies using AI or fuzzy logic [18, 28, 29], this work provides the underlying kinematic plant model. The derived forward and inverse kinematic equations are essential for the implementation and testing of such intelligent control algorithms. For example, the fuzzy trajectory planning for a firefighting robot in tunnels by De Santis et al. [18] or the neural network-based approaches for inverse kinematics reviewed by Calzada-Garcia et al. [28] could leverage the type of kinematic model developed herein. The work by Syam et al. [17] on kinematic motion control for mobile firefighter manipulators directly underscores the importance of the detailed kinematic analysis undertaken in this paper.

The current study opens several avenues for future research, building upon the established kinematic foundation:

1. Dynamic modeling and analysis: A critical next step is to develop a dynamic model of the manipulator. This would involve considering link masses, inertias, actuator characteristics, and, importantly, the reaction forces from the fire monitor jet. Such a model is essential for actuator sizing, control system design, and ensuring structural integrity.

2. Control system development and implementation: Based on the inverse kinematics and dynamic model, robust joint-level and trajectory-tracking controllers can be designed and simulated. This would involve exploring classical control techniques (e.g., PID) and modern control approaches.

3. Integration with perception systems: Future work should focus on integrating the manipulator control with sensor systems, such as thermal imaging cameras and vision systems [1, 2, 4, 15], to enable autonomous fire detection, target acquisition, and tracking.

4. Coupling with jet trajectory models: The manipulator's kinematic model should be integrated with water jet trajectory models [1, 2, 3, 4, 5, 7] to create a comprehensive simulation environment that predicts the actual impact point of the firefighting agent, allowing for closed-loop aiming correction.

5. Structural optimization and robust design: The physical design of the manipulator needs to be optimized for the harsh conditions of firefighting, considering factors like heat resistance, water ingress protection, and overall robustness [22, 27].

6. Experimental validation: The construction of a physical prototype and experimental validation of the kinematic models, workspace, and control strategies are crucial to demonstrate real-world applicability and refine the design.

7. Mobile platform integration: For enhanced versatility, the manipulator could be integrated onto a mobile platform. This would introduce challenges related to coordinated motion control of the platform and manipulator [14, 17, 23, 25].

8. Advanced control and AI: Exploring advanced control techniques, including machine learning and AI for intelligent decision-making, adaptive control in response to changing fire conditions, and obstacle avoidance [9, 18, 28, 29, 30], would significantly enhance the system's autonomy and effectiveness.

Conclusions

The paper has presented the design and a detailed kinematic analysis of an articulated robotic manipulator, culminating in the derivation of its motion equations and simulation of its workspace. The primary aim was to establish a foundational understanding of such a manipulator's capabilities, particularly envisioning its application in the precise control of fire monitors. The successful development of the forward kinematic model using matrix transformations and its verification through numerical modeling in Mathematica software and 3D CAD/CAE simulations in SolidWorks software confirm the viability of the proposed kinematic structure and analytical approach.

In general, the design and kinematic analysis presented in this paper provides a solid and essential foundation for the development of a robotic manipulator system for fire monitor control. While the current work focuses on fundamental kinematics, it paves the way for extensive further research encompassing dynamics, control, sensor integration, and experimental validation, ultimately contributing to the advancement of safer and more efficient firefighting technologies.

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