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NUMERICAL MODELING AND DETERMINATION OF THE STRESS-STRAIN STATE OF RAILWAY WHEELS WITH THE PRESENCE OF SUBSURFACE PHYSICAL TYPE STRESS CONCENTRATORS

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Abstract. *Problem statement.* The problem of establishing the stress-strain state of carriage wheels with the presence of subsurface inclusions is considered.

Purpose. The purpose of this article is to build a methodology to establish dependence between the parameters of subsurface inclusions and the contact strength of railway wheels.

Methodology. Based on modern models of the mechanics of a deforming solid of non-local (gradient) media, approaches to assessing the strength of structures and methods of mathematical modeling, the spatial distribution of the field of displacements, stresses and softening that occurs in the elements of carriage wheels under operational friction loads has been established. For the computer implementation of the task, the Python version of the free Fenics finite element analysis package was used.

Findings. Conducted studies of this applied problem have shown that the presence of an inclusion reduces the size of the zone with increased strength characteristics; in addition, on the surface of the inclusion, the base metal, the level of stress increases, which leads to the formation of defects in this area.

Originality. It was established that the field of displacement of the body with the inclusion in the problem under consideration is distorted slightly, which may indicate the fact that cracks are formed not by the mechanism of separation or shear, but due to the formation of zones with lower strength parameters, in particular, zones with increased damage.

Practical value. To eliminate the negative impact of inclusions, it is recommended to technologically form locally gradient structures that make it possible to neutralize the negative impact of inclusions on the formation of a stressed-strained state of the structure.

Jeff Immelt (former CEO of GE):
"The future of manufacturing is digital. Intelligent machines, real-time data, and advanced analytics will define the next industrial era."
(About the digitization of machine engineering processes)

Scopes of further investigations. Further investigations will be conducted in the field of determining the relationship between contact strength and properties of the inclusion zone area.

Keywords: railway wheels, contact strength, finite element method, subsurface inclusion.

Introduction. Features of the operation of friction nodes in railway transport

Wear in the friction nodes is the main type of failure that is characteristic of the modern rolling stock of the 1520 mm track. According to open scientific and technical sources, friction nodes determine the efficiency and reliability of the railway industry by 80-90%. The ability to withstand contact loads in the wheel-rail system without losing the necessary reliability parameters is the main limiting factor that does not allow increasing the carrying capacity of wagons, their speed, and reducing operating costs for track and rolling stock maintenance.

> *Joe Kaeser (CEO of Siemens AG)*
> *"Industry 4.0 is not just about upgrading mechanical engineering. It's about rethinking manufacturing altogether, where data becomes the key resource.."*
> *(About Industry 4.0 conception)*

To increase the operational reliability of tribological interfaces in railway transport, research into the following tribological problems is considered promising:

1. mechano-thermochemical processes in the contact zone of the wheel and rail;
2. structural features of the crew part of rolling stock;
3. track construction;
4. technologies and means for lubricating the wheel rim and rail;
5. technologies for restoring worn wheel rims;
6. creation and use of polymer materials;
7. tribomonitoring.

To this range of problems, we will also add research and prediction of the behavior of structural elements with surface and subsurface stress concentrators, which have a geometric or physical nature. It is precisely such concentrators in the majority that act as the sources of the initiation of cracks and the destruction of parts and structures of rolling stock. By geometric concentrators in this work, we will understand the violation of the integrity of the material of different shapes. As physical stress concentrators, we will take the elements of details in which there is a sharp (jump-like) change in properties. At the physical level, these can be inclusions of a different nature in the material of details, which most often have a technological or operational origin. And although the content of such inclusions can be tenths or hundredths of a percent, they are responsible for 70..90% of the development of microcracks.

Therefore, the study of contact pairs of elements of rolling stock structures containing physical stress concentrators is an urgent task of modern mechanics of railway transport, as it makes it possible to predict the behavior of such objects in various operating conditions (including over-standard ones).

Problem statement. Typical defects of railway wheels

Currently, a sufficiently clear classification of railway wheel defects is adopted [1].

The most common defects are: sliders (flats), burn, ridges, crushing of the rim, and chipping.

If for the first two types of defects the cause is a violation of the rules of operation of rolling stock, then the cause of other wheel defects is the presence of physical structural non-locality of technological or operational origin. Physical structural nonlocality means the formation of zones with a sharp change in properties ("inclusions") that have different properties compared to the base metal. Most often, inclusions appear in the material of parts already in the process of their manufacture. At the same time, under intense loads accompanying the operation of railway wheels, the gradual formation of "white spots" ("white zones"), which can also be considered as inclusions, is possible.

The cause of this phenomenon is most often thermomechanical damage to the surface and subsurface layers of the wheel due to faulty braking equipment, which leads to intense heating of the surface layer of the metal above the critical temperature of structural transformations, as well as its rapid and uneven cooling.

When repairing rolling stock, areas with defects in wheel pairs are most often eliminated by turning, parts are rejected if the defects are larger than permissible sizes.

However, the issues of establishing the reliability, strength, and durability of wheels with inclusion-type defects, and the influence of the geometric and physical parameters of the inclusions on the permissible loads in their presence, remain unexplored.

The influence of inclusions on the performance of parts

Inclusions are chemical compounds of metals with non-metals (oxygen, sulfur, nitrogen, carbon, phosphorus).

Inclusions should be classified according to several criteria: origin, moment of formation, chemical composition, atomic structure, microstructure, melting point, and mechanical behavior [2].

Based on their origin, they can be divided into two groups: endogenous, exogenous and endo-exogenous.

Endogenous inclusions are formed as a result of reactions between steel components, a decrease in the solubility of components in liquid and solid steel with decreasing temperature, during monotectic, eutectic, peritectic, eutectoid, and syntectoid transformations, and the addition of deoxidizers and desulfurates to steel. Endogenous inclusions can nucleate homogeneously or heterogeneously - on the surface of pre-existing inclusions or growing steel crystals. The method of formation of inclusions is determined by the composition of the steel, as well as thermodynamic and kinetic factors.

Exogenous inclusions are products of the destruction of the lining of steel-smelting units and steel-pouring devices, particles of charge, exothermic mixtures, sand, and refined slags.

Endo-exogenous inclusions are formed as a result of the interaction of inclusions of the first and second types.

Based on their composition, nonmetallic inclusions can be divided into several main groups: 1) simple oxides; 2) complex; 3) silicates; 4) sulfides; 5) phosphides; 6) nitrides; 7) hydrides; 8) carbides; 9) selenides; 10) tellurides; 11) borides.

The structure of steel can contain various inclusions that are formed as a result of reactions of steel components with oxygen, sulfur, and nitrogen. In addition, inclusions of calcium, magnesium, titanium, and chromium oxides are often found. The formation of sulfides in steel is determined by the affinity of its components for sulfur, which increases from iron to aluminum, chromium, vanadium, manganese, titanium, calcium, and cerium. The appearance of nitrides depends on the affinity of the elements for nitrogen, which increases from chromium to vanadium, aluminum, titanium and zirconium.

Based on their atomic structure, inclusions are divided into crystalline and amorphous. There are inclusions with different types of crystal lattices: cubic, hexagonal, triclinic, trigonal, tetragonal, orthorhombic, monoclinic. There are isomorphic and polymorphic inclusions.

Let us consider in more detail the mechanical properties of inclusions, which depend on the field of deformations and stresses near the inclusions. The difference in the physical, mechanical and chemical properties of the inclusion and the steel matrix leads to the appearance of stresses at their interface, the magnitude of which depends on a number of factors: the coherence of the inclusion and the matrix, which is determined by the degree of correspondence of their crystal structures and the energy of the interface; shape and size of the inclusion; the distance between inclusions, which determines their mutual influence; volume fraction of inclusions; distribution of inclusions. The combined action of these factors determines the role of inclusions as stress concentrators. In the absence of external influence, there will be residual stresses in the steel due to the presence of inclusions. These stresses are caused by interfacial stresses at the inclusion-matrix interfaces, phase hardening caused by transformations in steel and particles with temperature changes, and the concentration of defects in the crystalline structure. Thermal stresses, which arise due to the difference in the coefficients of thermal expansion of the matrix and the inclusion, also play an important role. As a result, the relaxation of thermal stresses that arise at the inclusion-matrix boundaries during cooling after crystallization, as well as after hot deformation or annealing, is not localized at the interfaces, but spreads to adjacent regions of the matrix and inclusion.

Non-metallic inclusions under different conditions of deformation contribute to its localization. Inclusion particles are traditionally considered barriers that prevent the development of intragranular deformation. Deformation inhibition leads to the absorption of lattice dislocations by inclusion-matrix boundaries, which promotes dynamic stress relaxation and changes the properties of interphase boundaries. Localization of deformation near inclusions is associated with their action as stress concentrators in steel. It is of interest to study the processes of localization of deformation near inclusions at different temperatures and strain rates, as well as the interaction of various mechanisms of deformation and relaxation, which will make it possible to evaluate the role of inclusions under different loading conditions.

During plastic deformation, deformation and contact stresses arise at the inclusion-matrix interfaces due to the different deformability of the inclusions and the steel matrix. The inclusion and matrix are a system of a stressed (inclusion) and a plastic layer (matrix) with dislocations at the interface. Plastic deformation is carried out by releasing stress on concentrators with the emission of various types of defects. Deformation and contact (intersurface) stresses relax as a result of the emergence of regions of nonlinear, highly excited states of the steel matrix near the inclusions.

Note that all the inclusions described above have a technological origin, as their appearance depends on the features of the product manufacturing technology.

Less studied, but no less common, are inclusions of exploitative origin. The physical processes of their appearance and the effect on the performance of the wheels are described in [1].

It should be noted that the mechanisms by which the influence of non-metallic inclusions in steels on the processes of destruction of parts takes place are currently open and have not been fully studied. At the same time, it is believed that the influence of inclusions consists in distorting the homogeneity of the field of stresses and deformations in the body due to the formation of stress concentrators. A special distortion of these fields occurs in the transition zone of the inclusion - the base metal. As a result of empirical studies, it was established that such negative processes as the initiation and propagation of cracks, corrosion begin right there. However, the question of analytical or numerical studies of these processes remains open due to their extreme complexity. The question of the effect of the geometric parameters of the inclusions on the performance of the products is also not investigated.

Research of the stress-strain state of wheels with a circular inclusion.

To study the distribution of stresses, deformations, and strength parameters of wheels with the presence of a subsurface stress concentrator of physical origin, consider the following 2-dimensional formulation of the contact problem.

Let the structure fragment be an elastic body modeled by a rectangular area with dimensions (X_1, X_2), is in the conditions of contact friction loads, which have a Hertzian distribution.

Note that the issue of choosing the form of power load in the contact zone of the railway wheel-rail is an open and not fully resolved problem. Nevertheless, most studies on the mechanics and tribology of railway structures focus specifically on the form of the Hertz distribution of the contact force load in the interaction zone.

The size of the contact zone is *D*, the load is equal to *P*:

$$\int_D p(x)\,dx = P \,, \tag{1}$$

where $p(x)$ - distributed load.

The relationship between the tangential and normal components of the load is subject to Coulomb's law:

$$q(x) = \mu p(x), \tag{2}$$

where μ- coefficient of friction.

The body has a circular inclusion V_1 (V_1 is only a designation of the inclusion) with a radius of *R*, which is located at a depth H from the surface (Fig. 1).

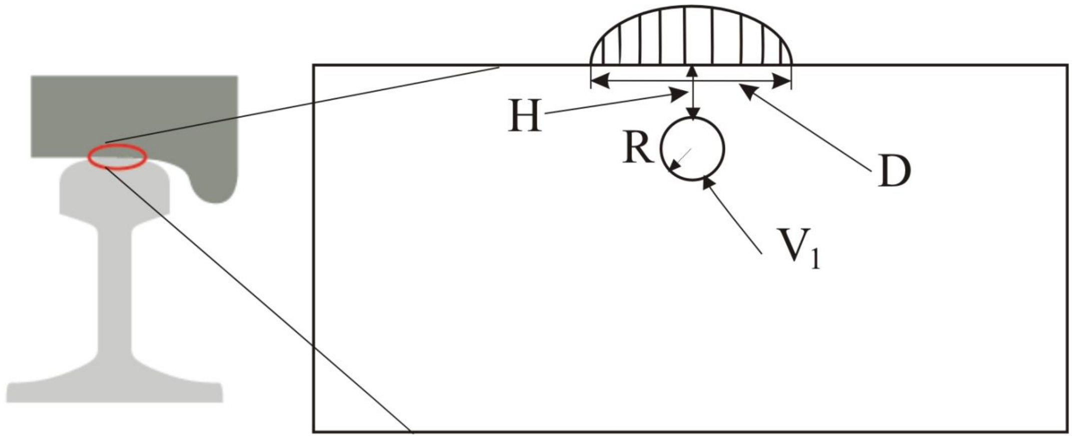

Fig. 1. Scheme of setting the contact problem for a body with an inclusion:
H – depth of inclusion, R – radius of inclusion, V_1 – inclusion, D – zone diameter

The entire studied area will be denoted as V_0.

Accordingly, we have the following distribution of properties in the structure under study:

$$\begin{cases} E_0,\mu_0,\sigma_0^* \in V_0/V_1 \\ E_1,\mu_1,\sigma_1^* \in V_1 \end{cases}, \qquad (3)$$

where E_i, μ_i, σ_i^* - respectively, the modulus of elasticity, Poisson's ratio, the strength limit of the corresponding area, $i=0,1$. The study was conducted in the case of the presence of oxides in the wheel material.

In accordance with the conditions of wheel-rail interaction for numerical calculations, we will take the following values: P=12.5 t, D = 13 mm, R = 2 mm, H = 3 mm, $E_0 = 2.2\cdot 10^{11}$ Pa, $E_1 = 1.2\cdot 10^{11}$ Pa, $\sigma_0^* = 847$ MPa, $\sigma_1^* = 471$ MPa. We will take the coefficient of friction equal to 0, since it has a smaller effect on the general picture of the weakening of the structure under the conditions of frictional interaction than the main force load. We will take Poisson's ratio equal to 0.25, since it is less structurally sensitive.

As a model of a continuous environment, we will take the continuum model constructed in [3]:

$$\vec{\nabla}\cdot\left(K(x)\left(\vec{\nabla}\cdot\vec{u}\right)\hat{I} + 2G(x)\left(\vec{\nabla}\otimes\vec{u} - \frac{1}{3}\left(\vec{\nabla}\cdot\vec{u}\right)\hat{I}\right)\right) = 0\,, \qquad (4)$$

where $\vec{u}$ - displacement vector, $\hat{I}$ - unit tensor, $\vec{\nabla}$ - Hamilton's differential operator, $K(x)$ - volumetric compression module, $G(x)$ - shear modulus, $\otimes$ - tensor product symbol, $\cdot$ - scalar product symbol.

The advantage of the mathematical model (4) is the ability to take into account the spatial nonlocality of the properties of structures, which is typical, among other things, for bodies with inclusions, at the level of model constants - the values of the shear modulus and the volumetric compression modulus of the material.

To solve the problem by the finite element method we use the weighted residual method ("FEM weak formulation"), where as a weight function we choose vector $\vec{v}$:

$$\int_{\Omega}\left(\vec{\nabla}\cdot\hat{\sigma}\right)\cdot\vec{v}\,dx = 0\,, \qquad (5)$$

where Ω is the range of integration, $\hat{\sigma}$ is the stress tensor, which equals to

$$\hat{\sigma} = K(x)\left(\vec{\nabla}\cdot\vec{u}\right)\hat{I} + 2G(x)\left(\vec{\nabla}\otimes\vec{u} - \frac{1}{3}\left(\vec{\nabla}\cdot\vec{u}\right)\hat{I}\right). \qquad (6)$$

The essence of the "FEM weak formulation" consists in the selection of such a distribution of the desired function, which allows the basic condition - (5) to be provided as accurately as possible in a given functional space. The weak formulation is important tool that helps, in solving problems of FEM-mechanics, to pass from the continual description of mathematical equations to a system of linear algebraic equations, the solution of which is the discrete distribution of the desired quantity.

Let us perform integration in respect of the parts of expression (5). Hence, we receive:

$$\int_{\Omega}\left(\vec{\nabla}\cdot\hat{\sigma}\right)\cdot\vec{v}\,dx=-\int_{\Omega}\hat{\sigma}:\left(\vec{\nabla}\otimes\vec{v}\right)dx+\int_{\partial\Omega}\left(\hat{\sigma}\cdot\vec{n}\right)\cdot\vec{v}\,ds, \qquad (7)$$

where $\vec{n}$ is the vector set at the body surface $\partial\Omega$.

Given the relation $\vec{T}=\hat{\sigma}\cdot\vec{n}$, where $\vec{T}$ is the surface vector, relation (7) is written as:

$$\int_{\Omega}\left(\vec{\nabla}\cdot\hat{\sigma}\right)\cdot\vec{v}\,dx=-\int_{\Omega}\hat{\sigma}:\left(\vec{\nabla}\otimes\vec{v}\right)dx+\int_{\partial\Omega}\vec{T}\cdot\vec{v}\,ds. \qquad (8)$$

This equation is a basic one for solving the problem of finding operational parameters of structures working under friction conditions.

The problem is considered in a two-dimensional array.

Let's establish the field of displacements, stresses and deformations of the studied area as a result of contact loads.

Since the stressed state does not unequivocally determine the performance parameters of the part, we will use the following expression to analyze the strength of the body in the local zone:

$$z(x)=1-\frac{\sigma^{*}(x)}{\sigma_{0}(x)}, \qquad (9)$$

where $z(x)$ - coefficient of safety margin, $\sigma^{*}(x)$ - the scalar equivalent of the stress tensor (in particular, in the Mises form), $\sigma_{0}(x)$ - scalar strength equivalent of the material.

The scalar equivalent of the stress state in the Mises form was used in the work because it is currently one of the most universal forms of estimating the stress tensor, and is also often used in assessing the strength of structures.

As can be seen from relation (9), at $z(x)\in(0;1)$ - there is no weakening in the body; $z(x)\le 0$ - there is weakening.

To determine the field of displacements, stresses and deformations of the studied area, as well as the field of weakening, we will use the free finite element analysis package - FEniCS and its implementation in the Python language [4], and the MatPlotLib package [5] to display the results.

The Fenics package was used because it has many advantages [6]:

- it is open-source;
- it has high-level Python and C++ interfaces to FEniCS, it is easy to get started, but FEniCS offers also powerful capabilities for more experienced programmers;
- it is transparent and you have much more control over the model than you would have in most software, if not all;
- it is the latest "big thing" in FEM, implementing all latest tools like PETSc, etc;
- unlike many other platforms, here you begin by understanding your problem from a very basic level, i.e., the strong and weak form (PDEs), and go on solving it from there (you get a far better knowledge of the physical model when you are dealing with basic PDE, stability, time and space discretization, parallel decomposition, computational algorithms, etc.; this makes the problem very general and then you can make it more specific for your needs by modifying it in any direction);

- it has a lot of free and high-quality open literature sources;
- there are many scientific conferences and publications dedicated to this product;
- FEniCS has an extensive list of features, including automated solution of finite element variational problems, automated error control and adaptivity, automated parallelization, a comprehensive library of finite elements, high performance linear algebra and many more; FEniCS is organized as a collection of interoperable components that together form the FEniCS Project (these components include the problem-solving environment DOLFIN, the form compiler FFC, the finite element tabulator FIAT, the just-intime compiler Instant, the code generation interface UFC, the form language UFL and a range of additional components);
- FEniCS runs on a multitude of platforms ranging from laptops to high-performance computers.

The domain partitioning parameter (the number of finite elements) is assumed to be N = 500.

Time to perform a single calculation for solving the problem of the mechanics of a deformable solid body for a computer with the following hardware and software parameters (Intel Core i3 processor - 2350 Mx4, RAM capacity - 16 Gb, Ubuntu 23.10 operating system (kernel version Linux 6.5.0 - 15 - generic, PyCharm development environment 2023.3.3, Python language version - 3.11, Fenics library version - 2023.3.0)) is about 30 seconds.

Due to its numerous features, plot styles, and high-quality results, Matplotlib makes data analysis easier and more efficient. It also helps save the time and resources you would have spent analyzing large datasets.

Unlike other data-visualization platforms, Matplotlib in Python only requires a few lines of code to generate a plot for data sets.

After the calculations, we will get the following results (Fig. 2 - 4).

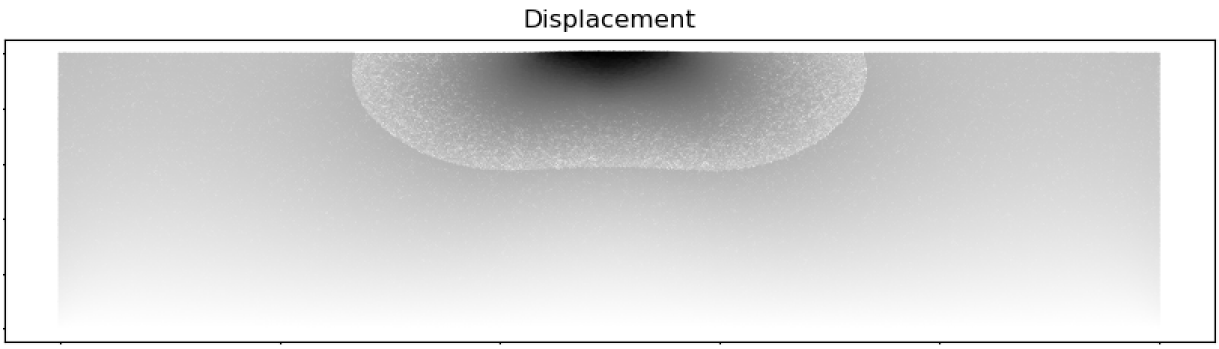

Fig. 2. The displacement field arising in the structure under study

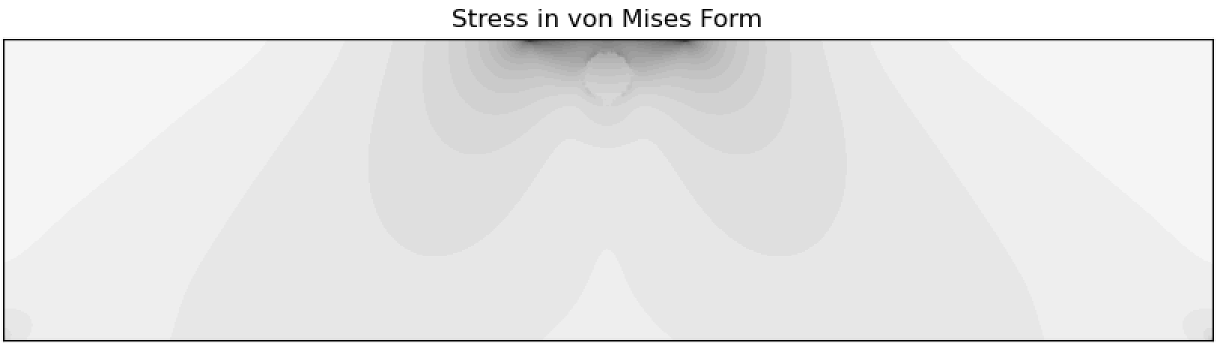

Fig. 3. The Mises-shaped stress field arising in the structure

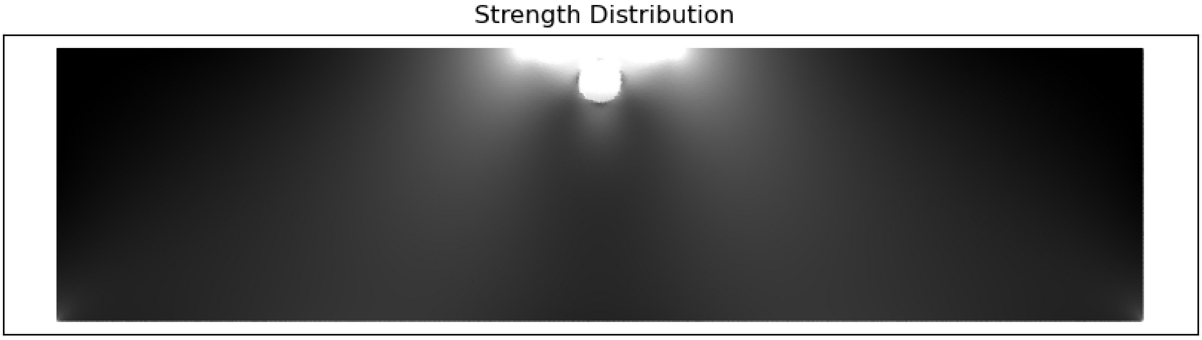

Fig. 4. Distribution of structural weakening.

The obtained studies show that the presence of an inclusion has a negative effect on the strength of the structure under study (it reduces the size of the zone with reduced strength characteristics - Fig. 4), in addition, on the surface of the inclusion - the base metal, the stress level increases (Fig. 3), which leads to the development of such negative phenomena as corrosion, diffusion saturation of this zone with impurity elements, etc.

The obtained results once again confirm the well-known fact that destruction begins from zones in which there is a sharp drop (gradient) of stress or strain. Most often, such zones are external or internal interface surfaces, but the extension of the internal interface surfaces to the external boundaries of the parts will be especially negative. In turn, the elimination (maximum possible reduction) of such zones leads to an increase in the performance properties of products.

At the same time, the field of movements of the body with the inclusion in the considered problem is slightly distorted (Fig. 2), which may indicate the fact of the formation of cracks not due to the mechanism of detachment or shear, but due to the formation of zones with lower strength parameters, in particular, zones with increased damage [7, 8].

To eliminate the negative impact of inclusions, it is recommended to technologically ensure the formation of wheel material structure parameters in zones with the possible appearance of inclusions with such gradient characteristics (one-dimensional or multidimensional) that allow to neutralize the negative influence of inclusions on the formation of the stress-strain state of the structure according to the principles proposed in the works [3, 9, 10].

Thus, for locomotive wheels with possible subsurface geometric stress concentrators of the circular type, it is proposed to increase the depth of tire reinforcement in order to reduce the amount of weakening under contact loads [3], and for pairs that work under prevailing friction-sliding conditions, to form "mosaic zones" of increased and decreased strength parameters, which corresponds to the approaches (principles) of Charpy [9-12].

Conclusions

1. Based on the approaches of contact mechanics, variational principles of mathematics, the author's mathematical model of functional-gradient elastic continuum, weak formulation of the finite element method, in the article presents a computer algorithmic scheme for studying the operational behavior of rail transport wheels with subsurface defects such as non-metallic circular inclusions.

2. The negative impact of the presence of this type of defect on the strength parameters of parts and the possibility of fracture initiation in the "base metal-inclusion surface" zone have been established.

3. The studies have shown that the displacement field in the presence of an inclusion is slightly curved, which indicates the fact of crack formation not by shear mechanisms, but due to their propagation in damage zones near the "base metal-inclusion" interface.

4. The ways of increasing the operational reliability of parts using a technological modification that ensures the formation of functionally graded structures with high resistance to the formation of scattered and localized damage in the "base metal-inclusion" transition zone have been proposed.

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