UNCERTAINTY BUDGET OF TEMPERATURE MEASUREMENT BY THE METHODS OF MONOCHROMATIC ONE- AND TWO-COLOR THERMOMETRY WITH EMISSIVITY CORRECTION

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Abstract. The work is directed on accuracy increasing of the thermodynamic temperature measurement of heated objects by their own thermal radiation, utilizing no more than two monochromatic channels for the spectral density of radiance measurement.

The classification of the methods of one- and two-color thermometry with emissivity correction have been proposed. It has been determined that these methods are based on generalized measurement equation originated from the spectral density of radiance approximation by the Wien's formula. Temperature measurement equations have been reformulated basing on the set of common input quantities. This set includes conditional one-color (radiance) temperatures, wavelengths and correction parameters, that account for spectral emissivity of controlled object. The number of input quantities for one-color methods is 4, and for the most part of two-color methods – 6.

Taking into consideration the specific of hardware realization of thermometry system (microspectrometer with multielement CCD-sensor of radiation), uncertainty components of input quantities have been researched and systematized, as well as the formulas for input quantities uncertainties evaluation basing on these components have been obtained. The uncertainty budget of indirect temperature measurement by the methods of monochromatic one- and two-color thermometry has been formed on the base of the input quantities set. The formulae for evaluation of combined standard and expanded, with confidence level 0.95, uncertainties of temperature measurement result have been obtained. They account for the possible correlation between correction parameters. For expanded uncertainty evaluation the kurtosis method has been utilized. It has been shown, that under conditions of the same uncertainties of input quantities, the accuracy of thermometry methods depends on the corresponding sensitivity coefficients only. The necessity of obtaining analytical expressions for sensitivity coefficients have been argued. This will allow analyzing the complex influence of input quantities uncertainties on temperature measurement accuracy by investigated thermometry methods, in order to determine the method, which demonstrates minimal possible uncertainty under specified measurement conditions.

Key words: thermodynamical temperature, methods of monochromatic one- and two-color thermometry, conditional temperatures, correction parameters, uncertainty budget, sensitivity coefficients.

1. Introduction

When spectral density of radiance [1] is not being measured on the multitude of spectral channels (wavelengths), in order to minimize instrumental component of uncertainty of thermodynamic temperature measurement, it is reasonable to utilize thermometry methods with minimal number of wavelengths – one- and two-color. On the modern stage of development of science and technology, sensors of thermal radiation with monochromatic (width up to 10 nm) and quasimonochromatic (10–25 nm) spectral channels are rather widespread [2, 3]. Due to the simplicity of construction and comparatively low cost, thermometers with mentioned sensors consist of considerable part of the market of contactless means of temperature measurements for industry applications, in particular, for metallurgy [4]. Also, it is worth saying, that one- and two-color operation regimes are available by the all multiwavelength thermometers (one can choose pairs of two wavelengths on the registered part of spectrum). This possibility is important when strong wavelength selection is required [5]. For example, under conditions of unstable transmission of intermediate medium due to the existence intervals of selective absorption of substances-pollutants (water vapor, gases, smoke from metal melts surface, etc.).

2. Drawbacks

Nowadays, despite of the numerous studies in the field of contactless high-temperature measurements, systematization of methods of monochromatic one- and two-color thermometry with emissivity correction is absent. They belong to the methods of indirect measurements, because the thermodynamic temperature being determined on the base of measurement equations. These equations include some input quantities, particularly, conditional temperatures, proportional to the spectral density of radiance on corresponding wavelengths. The questions, dealing with measurement equations reduction to the set of joint input quantities, systematization of the sources of these quantities uncertainties, as well as metrological analysis of input uncertainties transformation into uncertainty of the result of temperature measurement by known thermometry methods are being still not solved.

The consideration of instrumental and systematic errors of some methods of monochromatic one- and two-color thermometry, given in publications [6-8] is not sufficient for comprehensive characterization of their accuracy. To achieve this, it is necessary to estimate root-mean-square or limit errors of temperature measurement or standard / expanded uncertainties, under the same conditions. So, in the work [6] the dependencies of instrumental root-mean-square temperature measurement error on errors of dimensionless intensities of radiation on one / two wavelengths, for the methods of radiance (brightness), spectral ratio and cross-correlation conditional temperatures without emissivity correction have been obtained. Mentioned methods are relevant only for the objects, which are "absolute black body" (ABB) with values of spectral emissivity ε_1 and ε_2 on corresponding wavelengths to be identically equal to 1. For thermometry methods, corrected on the emissivity of a colored ("non-black") object, it is claimed that the estimates of instrumental error remain the same. Obviously, it is not correctly, due to the sensitivity coefficients, calculated at $\varepsilon_1 \neq 1$, $\varepsilon_2 \neq 1$ differs from those ones, obtained at $\varepsilon_1 = \varepsilon_2 \equiv 1$.

Formulae for instrumental and methodological components of temperature measurement errors, for some methods of two-color thermometry with emissivity correction, presented in paper [7], take into account the signs of absolute errors of intensity measurement (ΔI_1; ΔI_2) and emissivity determination ($\Delta\varepsilon_1$; $\Delta\varepsilon_2$). It is acceptable only in that case, when these errors are systematical. Moreover, ($\Delta\varepsilon_1$; $\Delta\varepsilon_2$) can be considered as systematical, but for the (ΔI_1; ΔI_2) such assumption is incorrect due to their random nature, caused by the sensor and measurement channels noise.

In paper [8] systematical methodological errors of temperature measurement by the methods of radiance (brightness), spectral ratio and two-color compensative thermometry with emissivity correction have been studied. It has been found that the last method has minimal methodological error if significant (>>1) relations of systematic errors of selectivity coefficient and average emissivity are observed.

Presented in work [9] comprehensive analysis of uncertainty sources of temperature measurements by infrared radiation thermometers is performed for the method of one-color thermometry with wide (not monochromatic) channel of spectral density of radiance measurement. So, this analysis can be adopted only for monochromatic one-color methods of temperature measurement, but not two-color. In addition, the transfer to the uncertainty of temperature measurement result (in Kelvins) is not obvious from obtained generalized formula for standard uncertainty of infrared thermometer measurement signal.

3. Goal of the article

The goal of the current article is to form the uncertainty budget for temperature measurement by the methods of monochromatic one- and two-color thermometry with emissivity correction. To achieve this, it is necessary to reduce measurement equations to the set of common input quantities and systematize the uncertainty sources of these input quantities. The uncertainty budget characterizes contributions of uncertainties of the input quantities in total uncertainty of temperature measurement result by one or another method. It is the base for estimation (by the B-type) of combined standard and expanded uncertainties of temperature measurements by different thermometry methods.

4. Review of the methods of monochromatic one- and two-color thermometry

It is known, that the task of thermodynamic temperature determination on the base of measured spectral density of radiance, without any a priori information about controlled object, is ill-posed (incorrectly formulated) [10, 11]. This fact results in existence of multitude solutions with respect to the sought thermodynamic temperature. Such conclusion can be obtained due to the number of unknowns in the system, which includes, for example, 3 one-color pyrometric equations, exceeds on unity the number of equations [12]. Respectively, for this task to be correctly formulated, it is necessary to equalize the number of unknowns and equations in the system. For one-color thermometry the only way is a priori setting of monochromatic value ε_i of spectral normal emissivity (coefficient of thermal radiation, emission coefficient, degree of blackness) [1] of the object on wavelength λ_i. As for two-color pyrometry, obtaining of the unique solution with respect to the thermodynamic temperature can be possible realized by two ways:

– with the help of setting of some model emissivity dependence on wavelength – $\varepsilon_{mod}(\lambda)$, which is required to be identical with the real dependence $\varepsilon(\lambda)$ of controlled object. Such dependence is described by the exponential-power function [6, 13], with index of power to be specified through the wavelength λ_1 and λ_2 of spectral channels;

– by means of setting of monochromatic values ε_1 and ε_2 of spectral emissivity on corresponding wavelengths.

Disadvantage of the first way is that fact, that there are only 3 thermometry methods [6], which provide equality of measured conditional temperature and sought thermodynamic temperature, for predetermined $\varepsilon_{mod}(\lambda)$. For example, the method of conditional spectral ratio temperature. This conditional temperature is equal to the thermodynamic temperature only in case of "gray" spectral distribution of the object emissivity ($\varepsilon(\lambda)$=const → $\varepsilon_1/\varepsilon_2 \equiv 1$). In other cases, the unknown systematic error of thermodynamic temperature measurement will take place. Therefore, mentioned methods have very restricted application and will not be considered in this article. The methods, which realize the second way, are more universal, by the authors opinion (obviously, they can be adapted for the objects with any $\varepsilon_1 \neq 1$, $\varepsilon_2 \neq 1$).

As a result of literature analysis [6–8, 13–15] it has been found that all methods of monochromatic one– and two-color thermometry with emissivity correction can be reduced to the next generalized equation of temperature measurement

$$T_{\mathrm{T}} = 1/(1/S_{\mathrm{1C,2C}} + \Delta_{\mathrm{1C,2C}}), \qquad (1)$$

where T_T, K – calculated, in accordance with chosen thermometry method, thermodynamic temperature of the object; $S_{1C,\,2C}$, K – conditional one- or two-color temperature, basical for chosen thermometry method; $\Delta_{1C,2C}$, K^{-1} – correction due to the systematic methodological error of chosen conditional one- or two-color temperature.

Systematic methodological error is caused by the deviation of unity of product (the result of multiplication) of spectral emissivity of controlled object and transmissivity of intermediate medium, for one or two wavelengths. Mentioned product is equal to unity only when the thermometry system is sighted on thermodynamically equilibrium radiation of ABB-model, under condition of ideally transparent intermediate medium.

The equation (1) follows from pyrometric equations for conditional one- and two-color temperatures, presented in work [12], when spectral density of radiance is approximated by the Wien's formula. It is correctly for visible and near infrared parts of heat radiation spectrum, particularly, for wavelengths 500–1100 nm and temperature of the object above 1200 K. Temperature control of ferrous-carbon melts in metallurgy usually is performed under such conditions [12]. It is necessary to say, that if Planck's formula for spectral density of radiance will be utilized, generalized equation (1) becomes incorrect.

The classification of the methods of monochromatic one– and two-color thermometry with emissivity correction is presented on Fig. 1.

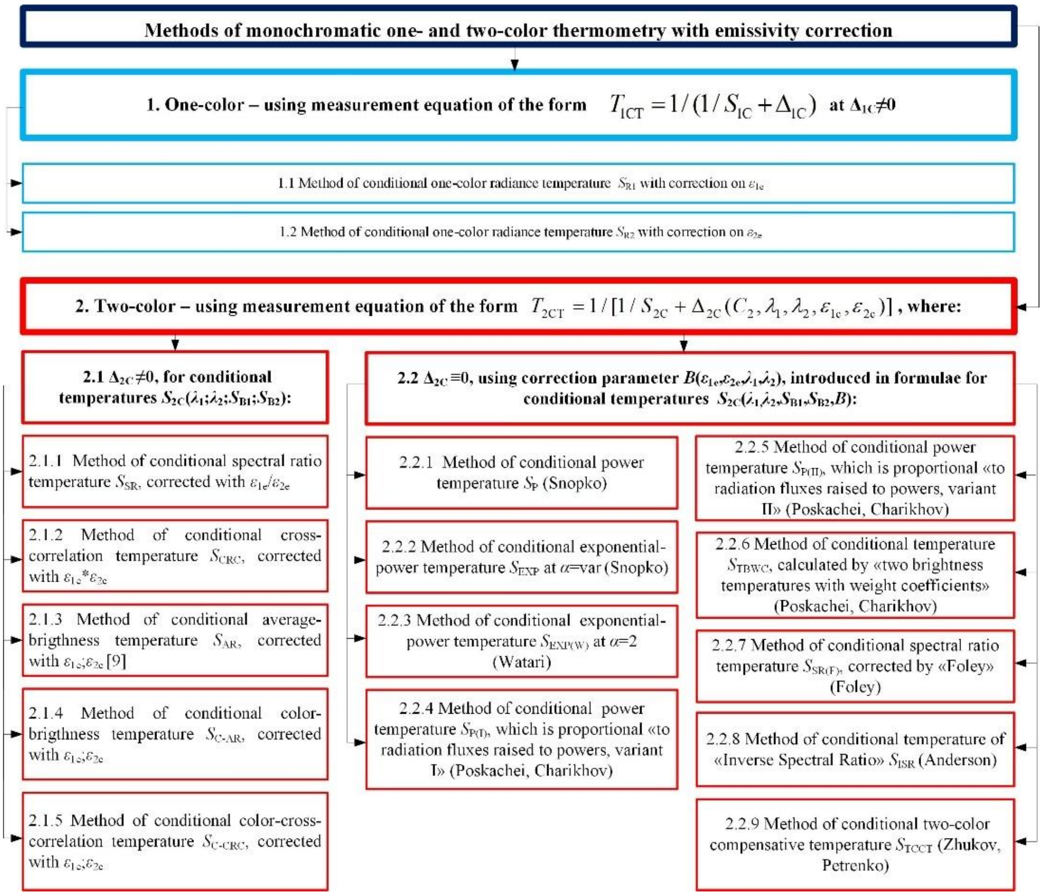

Fig. 1. Methods of monochromatic one– and two-color thermometry with emissivity correction

Classification is built on conditional temperatures nomenclature, which can be utilized in generalized equation (1). As can be seen, conditional temperatures for two-color methods are the most numerous. These methods have been divided on 2 subgroups, with 5 and 9 methods, respectively. So, for two-color thermometry it is possible to made temperature correction Δ_{2C} equal to zero (see p. 2.1.2). This is achieved due to the additional correction parameter B to be introduced in measurement equation of conditional temperature. The consequence is the measured conditional temperature is equal to the sought thermodynamic temperature.

In order to correct systematic methodological error of any conditional temperature in accordance with generalized equation (1), we should apriori know estimates of the product of spectral emissivity of the object and spectral transmissivity of intermediate medium, for one and/or two wavelengths – i.e. $\varepsilon_{1e}\cdot\tau_{1e}$ and/or $\varepsilon_{2e}\cdot\tau_{2e}$. To simplify the analysis in this paper, let us consider, that $\tau_{1e}=\tau_{2e}\equiv1$. Reasoning from the practice of contactless temperature measurements, the estimates of spectral emissivity can be either fixed, or being calculated in accordance with special algorithms, adapted for chosen object of control. Moreover, for some controlled objects, for example, metal melts, ε_1 and ε_2 are usually correlated with coefficient, equal to 1 (in other words, there are some dependencies of the form $\varepsilon_2=f(\varepsilon_1)$) [6]. Respectively, if we apply the algorithms of correction, which account for these dependencies, we will obtain correlated estimates of ε_1 and ε_2 – i.e., ε_{1e} and ε_{2e}. As a result of significant nonlinearity of such dependencies, the average $\varepsilon_{av.e}=(\varepsilon_{1e}+\varepsilon_{2e})/2$ and ratio $Ks_e=\varepsilon_{1e}/\varepsilon_{2e}$ can be either mutually correlated or no. So, proposed quantities are more informative, than ε_{1e} and ε_{2e}, for comprehensive uncertainties analysis, where influence of different algorithms of correction parameters calculation should be considered. Further, we will use just averaged level of emissivity $\varepsilon_{av.e}$ and selectivity coefficient Ks_e as a parameters of correction of systematic methodological error of conditional temperatures. In the simplest case (apriori averaged adjustment [8]), they are determined as arithmetical means of boundaries ($\varepsilon_{av.\min}$, $\varepsilon_{av.\max}$) and ($Ks_{\min}$, $Ks_{\max}$) and mutual correlation is absent. The mentioned boundaries belong to the apriori information about controlled object.

Formulae for conditional temperatures calculation, as well as corrections for them are presented in the Tables 1–3. At the same time, the mathematical models, presented in works [6–8, 13–15] have been transformed in such a way, that all thermometry methods utilize the set of common input quantities, including proposed two correction parameters.

Table 1. Mathematical models of the methods of one-color thermometry with emissivity correction

Method in accordance with the Fig. 1	Measurement equation for conditional temperature by the spectral density of radiance	Formula for Δ_{1C}
1.1	$S_{R1}=-C_2/[\lambda_1\cdot\ln(\pi\cdot\lambda_1^5\cdot E_1/C_1)]$	$\Delta_{R1}=-(\lambda_1/C_2)\cdot\ln[\ \cdot\varepsilon_{av.e}\cdot Kc_e/\ Kc_e+\ )$
1.2	$S_{R2}=-C_2/[\lambda_2\cdot\ln(\pi\cdot\lambda_2^5\cdot E_2/C_1)]$	$\Delta_{R2}=-(\lambda_2/C_2)\cdot\ln[\ \cdot\varepsilon_{av.e}\cdot Kc_e/\ Kc_e+\ )$

Table 2. Mathematical models of the methods of two-color thermometry with emissivity correction in accordance with p.2.1 ($\Delta_{2C}\neq0$)

Method in accordance with the Fig. 1	Measurement equation for conditional temperature	Measurement equation for conditional two-color temperature by one-color temperature	Formula for Δ_{2C}
2.1.1	$S_{SR}=-[\frac{\lambda_2}{\lambda_2-\lambda_1}\cdot\frac{\lambda_1}{C_2}\cdot\ln(\frac{\pi\cdot\lambda_1^5\cdot E_1}{C_1})+\frac{\lambda_1}{\lambda_1-\lambda_2}\cdot\frac{\lambda_2}{C_2}\cdot\ln(\frac{\pi\cdot\lambda_2^5\cdot E_2}{C_1})]$	$S_{SR}=1/(\frac{\lambda_2}{\lambda_2-\lambda_1}\cdot\frac{1}{S_{R1}}+\frac{\lambda_1}{\lambda_1-\lambda_2}\cdot\frac{1}{S_{R2}})$	$\Delta_{SR}=\frac{\lambda_1\cdot\lambda_2}{\lambda_2-\lambda_1}\cdot\ln(Kc_e)$
2.1.2	$S_{CRC}=-[\frac{\lambda_2}{\lambda_2+\lambda_1}\cdot\frac{\lambda_1}{C_2}\cdot\ln(\frac{\pi\cdot\lambda_1^5\cdot E_1}{C_1})+\frac{\lambda_1}{\lambda_1+\lambda_2}\cdot\frac{\lambda_2}{C_2}\cdot\ln(\frac{\pi\cdot\lambda_2^5\cdot E_2}{C_1})]$	$S_{CRC}=1/(\frac{\lambda_2}{\lambda_2+\lambda_1}\cdot\frac{1}{S_{R1}}+\frac{\lambda_1}{\lambda_1+\lambda_2}\cdot\frac{1}{S_{R2}})$	$\Delta_{CRC}=\frac{\lambda_1\cdot\lambda_2}{\lambda_1+\lambda_2}\cdot\ln[\frac{4\cdot\varepsilon^2{}_{av.e}\cdot Kc_e}{(Kc_e+1)^2}]$
2.1.3	$S_{AR}=-[\frac{1}{2}\cdot\frac{\lambda_1}{C_2}\cdot\ln(\frac{\pi\cdot\lambda_1^5\cdot E_1}{C_1})+\frac{1}{2}\cdot\frac{\lambda_2}{C_2}\cdot\ln(\frac{\pi\cdot\lambda_2^5\cdot E_2}{C_1})]$	$S_{AR}=1/(\frac{1}{2\cdot S_{R1}}+\frac{1}{2\cdot S_{R2}})$	$\Delta_{AR}=\frac{1}{2\cdot C_2\cdot(\lambda_2-\lambda_1)}\cdot[\lambda_1\cdot\ln(\frac{2\cdot\varepsilon_{av.e}\cdot Kc_e}{Kc_e+1})+$ $+\lambda_2\cdot\ln(\frac{2\cdot\varepsilon_{av.e}}{Kc_e+1})]$
2.1.4	$S_{C\text{-}AR}=-[\frac{3\cdot\lambda_2-\lambda_1}{4\cdot(\lambda_2-\lambda_1)}\cdot\frac{\lambda_1}{C_2}\cdot\ln(\frac{\pi\cdot\lambda_1^5\cdot E_1}{C_1})+\frac{\lambda_2-3\cdot\lambda_1}{4\cdot(\lambda_2-\lambda_1)}\cdot\frac{\lambda_2}{C_2}\cdot\ln(\frac{\pi\cdot\lambda_2^5\cdot E_2}{C_1})]$	$S_{C\text{-}AR}=1/(\frac{3\cdot\lambda_2-\lambda_1}{4\cdot(\lambda_2-\lambda_1)}\cdot\frac{1}{S_{R1}}+\frac{\lambda_2-3\cdot\lambda_1}{4\cdot(\lambda_2-\lambda_1)}\cdot\frac{1}{S_{R2}})$	$\Delta_{C\text{-}AR}=\frac{1}{4\cdot C_2\cdot(\lambda_2-\lambda_1)}\cdot[\lambda_2\cdot(3\cdot\lambda_1-\lambda_2)\cdot\ln(\frac{2\cdot\varepsilon_{av.e}}{Kc_e+1})-$ $-\lambda_1\cdot(3\cdot\lambda_2-\lambda_1)\cdot\ln(\frac{2\cdot\varepsilon_{av.e}\cdot Kc_e}{Kc_e+1})]$
2.1.5	$S_{C\text{-}CRC}=-[\frac{\lambda_2^2}{\lambda_2^2-\lambda_1^2}\cdot\frac{\lambda_1}{C_2}\cdot\ln(\frac{\pi\cdot\lambda_1^5\cdot E_1}{C_1})+\frac{\lambda_1^2}{\lambda_1^2-\lambda_2^2}\cdot\frac{\lambda_2}{C_2}\cdot\ln(\frac{\pi\cdot\lambda_2^5\cdot E_2}{C_1})]$	$S_{C\text{-}CRC}=1/(\frac{\lambda_2^2}{\lambda_2^2-\lambda_1^2}\cdot\frac{1}{S_{R1}}+\frac{\lambda_1^2}{\lambda_1^2-\lambda_2^2}\cdot\frac{1}{S_{R2}})$	$\Delta_{C\text{-}CRC}=-\frac{\lambda_1\cdot\lambda_2}{C_2\cdot(\lambda_2^2-\lambda_1^2)}\cdot[\lambda_1\cdot\ln(\frac{2\cdot\varepsilon_{av.e}}{Kc_e+1})-$ $-\lambda_2\cdot\ln(\frac{2\cdot\varepsilon_{av.e}\cdot Kc_e}{Kc_e+1})]$

Table 3. Mathematical models of the methods of two-color thermometry with emissivity correction in accordance with p.2.2 (Δ_{2C}=0)

Method in accordance with the Fig. 1	Measurement equation for conditional two-color temperature by one-color temperature and additional correction parameter *B*	Formula for *B*
2.2.1	$S_P = 1/[\frac{B_P}{C_2}\cdot\frac{\lambda_1\cdot\lambda_2}{\lambda_2-\lambda_1}\cdot\ln(\frac{\lambda_2}{\lambda_1}) + \frac{\lambda_2}{\lambda_2-\lambda_1}\cdot\frac{1}{S_{R1}} + \frac{\lambda_1}{\lambda_1-\lambda_2}\cdot\frac{1}{S_{R2}}]$	$B_P = \log_{\lambda_2/\lambda_1}(Kc_e)$
2.2.2	$S_{EXP} = 1/(\frac{\lambda_2^{B_{EXP}+1}}{\lambda_2^{B_{EXP}+1}-\lambda_1^{B_{EXP}+1}}\cdot\frac{1}{S_{R1}} + \frac{\lambda_1^{B_{EXP}+1}}{\lambda_1^{B_{EXP}+1}-\lambda_2^{B_{EXP}+1}}\cdot\frac{1}{S_{R2}})$	$B_{EXP} = \log_{\lambda_2/\lambda_1}[\ln(\frac{2\cdot\varepsilon_{av.e}}{Kc_e+1}) / \ln(\frac{2\cdot\varepsilon_{av.e}\cdot Kc_e}{Kc_e+1})]$
2.2.3	$S_{EXP(W)} = 1/(\frac{\lambda_2^3}{\lambda_2^3-\lambda_1^3}\cdot\frac{1}{S_{R1}} + \frac{\lambda_1^3}{\lambda_1^3-\lambda_2^3}\cdot\frac{1}{S_{R2}})$	$B_{EXP(W)}=2$
2.2.4	$S_{P(I)} = 1/(\frac{\lambda_2-\lambda_1\cdot B_{P(I)}}{\lambda_2-\lambda_1}\cdot\frac{1}{S_{R1}} + \frac{\lambda_1\cdot(1-B_{P(I)})}{\lambda_1-\lambda_2}\cdot\frac{1}{S_{R2}})$	$B_{P(I)} = [\ln(\frac{2\cdot\varepsilon_{av.e}\cdot Kc_e}{Kc_e+1}) - \ln(\frac{2\cdot\varepsilon_{av.e}}{Kc_e+1})] / [\frac{\lambda_1}{\lambda_2}\cdot\ln(\frac{2\cdot\varepsilon_{av.e}\cdot Kc_e}{Kc_e+1}) - \ln(\frac{2\cdot\varepsilon_{av.e}}{Kc_e+1})]$
2.2.5	$S_{P(II)} = 1/[(1+\frac{\lambda_1\cdot B_{P(II)}}{\lambda_2})\cdot\frac{1}{S_{R1}} - \frac{\lambda_1\cdot B_{P(II)}}{\lambda_2}\cdot\frac{1}{S_{R2}}]$	$B_{P(II)} = [\ln(\frac{2\cdot\varepsilon_{av.e}\cdot Kc_e}{Kc_e+1}) - \ln(\frac{2\cdot\varepsilon_{av.e}}{Kc_e+1})] / [\frac{\lambda_1}{\lambda_2}\cdot\ln(\frac{2\cdot\varepsilon_{av.e}\cdot Kc_e}{Kc_e+1}) - \ln(\frac{2\cdot\varepsilon_{av.e}}{Kc_e+1})]$
2.2.6	$S_{TBWC} = 1/[\frac{B_{TBWC}\cdot\lambda_2}{B_{TBWC}\cdot\lambda_2-\lambda_1}\cdot\frac{1}{S_{R1}} - \frac{\lambda_1}{\lambda_1-B_{TBWC}\cdot\lambda_2}\cdot\frac{1}{S_{R2}}]$	$B_{TBWC} = [\ln(\frac{2\cdot\varepsilon_{av.e}}{Kc_e+1})] / [\ln(\frac{2\cdot\varepsilon_{av.e}\cdot Kc_e}{Kc_e+1})]$
2.2.7	$S_{SR(F)} = 1/[\frac{\lambda_1\cdot\lambda_2}{\lambda_2-\lambda_1}\cdot(\frac{1-B_F\cdot\lambda_1}{\lambda_1\cdot S_{R1}} - \frac{1-B_F\cdot\lambda_2}{\lambda_2\cdot S_{R2}})]$	$B_{SR(F)} = -\ln(Kc_e) / [\lambda_2\cdot\ln(\frac{2\cdot\varepsilon_{av.e}}{Kc_e+1}) - \lambda_1\cdot\ln(\frac{2\cdot\varepsilon_{av.e}\cdot Kc_e}{Kc_e+1})]$
2.2.8	$S_{ISR} = 1/[(1-B_{ISR})\cdot(\frac{\lambda_2}{\lambda_2-\lambda_1}\cdot\frac{1}{S_{R1}} + \frac{\lambda_1}{\lambda_1-\lambda_2}\cdot\frac{1}{S_2}) + \frac{B_{ISR}}{S_{R2}}]$	$B_{ISR} = \frac{-1}{\lambda_1/[((\lambda_1\cdot\lambda_2)/(\lambda_2-\lambda_1))\cdot(1-\ln(\frac{2\cdot\varepsilon_{av.e}}{Kc_e+1}) / \ln(\frac{2\cdot\varepsilon_{av.e}\cdot Kc_e}{Kc_e+1}))]-1}$
2.2.9	$S_{TCCT} = 2/(\frac{1}{S_{SR}} + \frac{1}{B_{TCCT}})$	$B_{TCCT} = \frac{S_{R2}\cdot\ln(\frac{2\cdot\varepsilon_{av.e}}{Kc_e+1})\cdot(\lambda_2-\lambda_1)+C_2}{\lambda_1\cdot S_{R2}\cdot C_2} + \frac{S_{R1}\cdot\ln(\frac{2\cdot\varepsilon_{av.e}\cdot Kc_e}{Kc_e+1})\cdot(\lambda_2-\lambda_1)-C_2}{\lambda_2\cdot S_{R1}\cdot C_2}]$

In Table 1–3 the next notes are introduced:

λ_1, λ_2, m – effective wavelengths of monochromatic spectral channels;

E_1, E_2, $(W/m^3)\cdot sr^{-1}$ – spectral density of radiance on wavelengths λ_1, λ_2;

$S_{R1}(E_1)$, $S_{R2}(E_2)$, K – conditional one-color (radiance) temperatures to be measured on wavelengths λ_1, λ_2;

$C_1=(3.74177107\pm0.00000029)\cdot10^{-16}$ W·M^2 – the first radiation constant;

$C_2=(1.4387752\pm0.0000025)\cdot10^{-2}$ K·m – the second radiation constant.

$B=f(\lambda_1,\lambda_2,\varepsilon_{av.e},Kc_e)$ – the additional correction parameter, which provides $S_{2C}=T$.

5. Sources of uncertainties of temperature measurement equations input quantities

As can be seen from Table 1–3, all temperature measurements equations include the set of common input quantities. For one-color methods there are 4 input quantities (radiance temperature S_{R1} or S_{R2}, wavelength λ_1 or λ_2 and correction parameters $\varepsilon_{av.e}$, Ks_e), for two-color methods – 6 (S_{R1}, S_{R2}, λ_1, λ_2, $\varepsilon_{av.e}$, Ks_e). Among the last methods exception is only for methods according p. 2.1.1 and 2.2.1, because of their measurement equations include 5 input quantities, due to the only one correction parameter Ks_e.

The Ishikawa diagram for the uncertainty components of temperature measurement by the methods of monochromatic one– and two-color thermometry is presented on the Fig. 2. It takes into account hardware realization of thermometry system – microspectrometer-based, with multielement radiation sensor on the base of charge coupled devices (CCD) [16].

Microspectrometer application is explained by the requirement of high degree of monochromatizating of spectral channels (width up to 10–12 nm) to be guaranteed [17]. The chain of input quantities transformation in CCD sensor is following: spectral density of radiation→ → charge → direct voltage → digital value of voltage→ →radiance temperature [16]. This process is accompanied by accumulation of the uncertainties, caused by the specific sensor noises. So, in papers [16, 18], for the CCD-based radiation sensors the next types of noise are determined:

– readout noise, caused by the integrated schemes of signal processing in each spectral channel of the sensor (charge and voltage amplifiers) and independent on the integration time;

– photoelectron or shot noise, caused by the statistical variation of the photon accumulation speed due to the quantum nature of falling electromagnetic emission;

– dark noise, caused by the thermal energy of electrons in semiconductor structures and, correspondingly, proportional to the integration time and temperature of the sensor;

– quantization noise, caused by the finite resolution of the built-in analog-to-digital converter of the microspectrometer.

For quantitative characterization of mentioned types of noise we will use the corresponding root mean square values – $\sigma_{rd.n}(N)$, $\sigma_{sh.n}(N)$, $\sigma_{d.n}(N)$, $\sigma_{q.n}(N)$. Probability density function for readout and dark noise is described by normal law, photoelectron noise – by Poisson law and quantization noise – by uniform law [16].

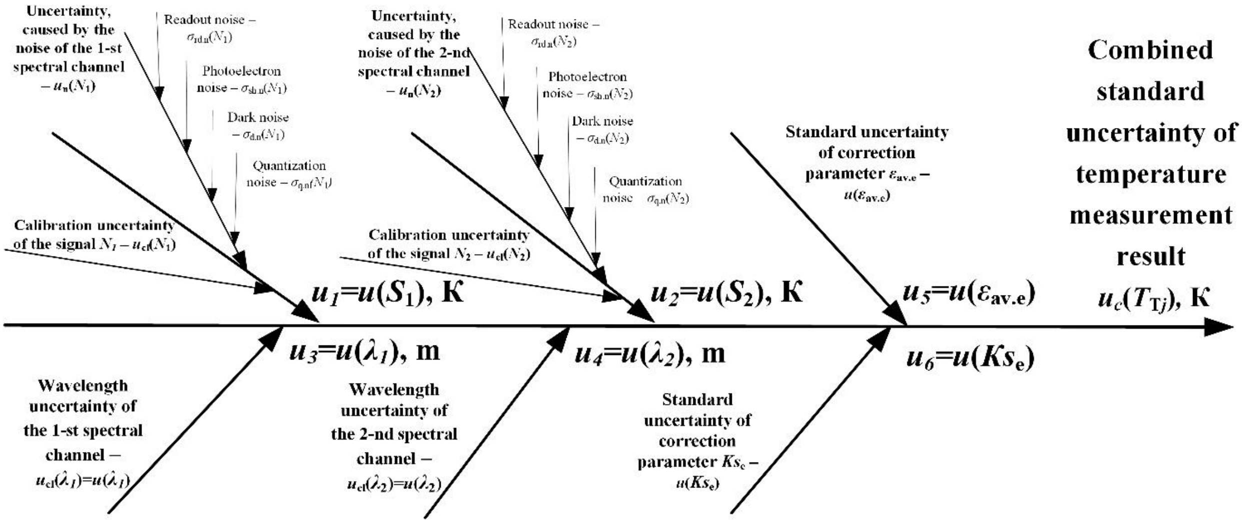

Fig. 2. Ishikawa diagram for the uncertainty components of temperature measurement by the methods of monochromatic one– and two-color thermometry with emissivity correction

Yet another source of radiance temperature measurement uncertainty is calibration procedure of microspectrometer output signal with the help of ABB-model. This component of uncertainty depends on the calibration scheme for radiance temperature obtaining [17]. Usually, the impact of next destabilizing factors of ABB-model should be taken into consideration:

– instability of the spectral emissivity of the cavity;

– uncertainty of the measurement or instability of the setting of the cavity temperature;

– non-isothermality and time instability of the cavity temperature.

Typical estimations ("normal" and "best") of calibration uncertainties of radiation temperatures for different calibration schemes are presented in report [17].

The standard uncertainties of radiance temperatures measurement for each spectral channel being calculated using the following formulas (type B evaluation)

$$u_1 = u(S_{R1}) = \sqrt{(\frac{\partial S_{R1}}{\partial N_1})^2 \cdot [\sigma^2(N_{rd.n}) + \sigma^2(N_{ph.n}) + \sigma^2(N_{d.n}) + \sigma^2(N_{q.n})] + u_{cl}^2(S_{R1})}\,, \quad (2)$$

$$u_2 = u(S_{R2}) = \sqrt{(\frac{\partial S_{R2}}{\partial N_2})^2 \cdot [\sigma^2(N_{rd.n}) + \sigma^2(N_{ph.n}) + \sigma^2(N_{d.n}) + \sigma^2(N_{q.n})] + u_{cl}^2(S_{R2})}\,, \quad (3)$$

where $S_{R1}(N_1) = \frac{C_2}{\lambda_1 \cdot \ln(C_1 \cdot K_1 \cdot t / ((N_1 - B_1) \cdot \pi))}$, $S_{R2}(N_2) = \frac{C_2}{\lambda_2 \cdot \ln(C_1 \cdot K_2 \cdot t / ((N_2 - B_2) \cdot \pi))}$ – measurement equations for radiance temperatures of the first and the second spectral channels of the CCD-sensor; N_1, N_2 – obtained, during integration time t, digital counts of voltage signal for the corresponding spectral channels of CCD-sensor, which are proportional to the measured values of spectral density of radiation; K_1, K_2, B_1, B_2 – calibration constants for the corresponding spectral channels of the CCD-sensor, obtained at ABB-model [16]; t – integration time; $u_{cl}(S_{R1})$, $u_{cl}(S_{R2})$ – calibration uncertainties for the corresponding spectral channels, which can be evaluated (by A type) on the base of calibration procedure in accredited metrological institution.

It is necessary to say, that uncertainty of radiance temperature measurement can be additionally affected by such factors, as instability of spectral coefficient of transmissivity of optical system, instability of converting coefficients for the spectral channels due to the changes of ambient temperature, etc. [13]. Influence of these factors are usually minimized by stabilizing conditions of radiance temperatures measurement. In cases, when impact of mentioned factors is significant, the total standard uncertainties of radiance temperatures measurement should be evaluated "in-situ" (by A type).

Uncertainties $u_3=u_{cl}(\lambda_1)$ and $u_4=u_{cl}(\lambda_2)$ of corresponding wavelengths setting is determined as a result of calibration procedure. For this purpose, standard lamp with gaseous operating environment is usually utilized – for example, Ocean Optics HG-1 Mercury-Argon Calibration Source [19]. Such lamp generates emission with accurately known wavelengths, which correspond to the narrow spectral lines of the gases. Typical uncertainty of wavelength setting for Ocean Optics microspectrometers consists 0.4…0.6 nm [19].

Let us consider, that quantities $\varepsilon_{av.}$, Ks, which characterize spectral emissivity of the object, have uniform law of distribution with boundaries [$\varepsilon_{av.\min}$, $\varepsilon_{av.\max}$] and [$Ks_{\min}$, $Ks_{\max}$] and their estimates, i.e. $\varepsilon_{av.e}$, Ks_e, are the arithmetic means of boundaries. So, the standard uncertainties of $\varepsilon_{av.e}$, Ks_e can be expressed as

$$u_5 = u(\varepsilon_{\text{av.e}}) = \frac{\varepsilon_{\text{av.max}} - \varepsilon_{\text{av.min}}}{2 \cdot \sqrt{3}} \ , \tag{4}$$

$$u_6 = u(Ks_{\text{e}}) = \frac{Ks_{\text{max}} - Ks_{\text{min}}}{2 \cdot \sqrt{3}} \ . \tag{5}$$

Also, if only $\varepsilon_{1\text{e}}$, $\varepsilon_{2\text{e}}$ with their standard uncertainties $u(\varepsilon_{1\text{e}})$, $u(\varepsilon_{2\text{e}})$ are predetermined, the uncertainties of correction parameters will be calculated as following [20]

$$u_5 = u(\varepsilon_{\text{av.e}}) = \sqrt{\frac{1}{4} \cdot u^2(\varepsilon_{1\text{e}}) + \frac{1}{2} \cdot r(\varepsilon_{1\text{e}}, \varepsilon_{2\text{e}}) \cdot u(\varepsilon_{1\text{e}}) \cdot u(\varepsilon_{2\text{e}}) + \frac{1}{4} \cdot u^2(\varepsilon_{2\text{e}})} \ , \tag{6}$$

$$u_6 = u(Ks_{\text{e}}) = \sqrt{\frac{1}{\varepsilon_{2\text{e}}^2} \cdot u^2(\varepsilon_{1\text{e}}) - 2 \cdot r(\varepsilon_{1\text{e}}, \varepsilon_{2\text{e}}) \cdot \frac{\varepsilon_{1e}}{\varepsilon_{2e}^3} \cdot u(\varepsilon_{1\text{e}}) \cdot u(\varepsilon_{2\text{e}}) + \frac{\varepsilon_{1\text{e}}^2}{\varepsilon_{2\text{e}}^4} \cdot u^2(\varepsilon_{2\text{e}})} \ , \tag{7}$$

where $r(\varepsilon_{1\text{e}},\varepsilon_{2\text{e}})$ – correlation coefficient between $\varepsilon_{1\text{e}}$ and $\varepsilon_{2\text{e}}$, dependent on algorithm of correction parameters calculation.

6. Uncertainty budget of temperature measurement result

On the base of Fig. 2, it is possible to form the uncertainty budget of temperature measurement by the methods of monochromatic one- and two-color thermometry with emissivity correction (Table 4).

For convenience of the next studies, it was decided to separate instrumental and methodological components of uncertainty. It is known, that the first component is determined only by the hardware part (radiation sensor) of thermometry system. The second is dependent only on applied algorithm of correction parameters calculation. It was supposed that all input quantities, excepted correction parameters, are mutually independent.

Table 4. Uncertainty budget of temperature measurement result

<table>
<tr><th>i</th><th>Estimate of i-th input quantity</th><th>Standard uncertainty of i-th input quantity</th><th>Probability distribution law</th><th>Kurtosis of probability distribution law</th><th>Sensitivity coefficient C_{ij} of j-th thermometry method to the uncertainty of i-th input quantity</th><th colspan="2">Contribution of uncertainty of i-th input quantity</th></tr>
<tr><td colspan="8">1. Instrumental component of temperature measurement uncertainty</td></tr>
<tr><td>1</td><td>S_{R1}, K</td><td>u_1, K</td><td rowspan="4">Normal</td><td>$\eta_1=0$</td><td>$C_{1j} = \partial T_{\text{T}j} / \partial S_{\text{R1}}$</td><td colspan="2">$C_{1j} \cdot u_1$</td></tr>
<tr><td>2</td><td>S_{R2}, K</td><td>u_2, K</td><td>$\eta_2=0$</td><td>$C_{2j} = \partial T_{\text{T}j} / \partial S_{\text{R2}}$</td><td colspan="2">$C_{2j} \cdot u_2$</td></tr>
<tr><td>3</td><td>λ_1, m</td><td>u_3, m</td><td>$\eta_3=0$</td><td>$C_{3j} = \partial T_{\text{T}j} / \partial \lambda_1$</td><td colspan="2">$C_{3j} \cdot u_2$</td></tr>
<tr><td>4</td><td>λ_2, m</td><td>u_4, m</td><td>$\eta_4=0$</td><td>$C_{4j} = \partial T_{\text{T}j} / \partial \lambda_2$</td><td colspan="2">$C_{4j} \cdot u_4$</td></tr>
<tr><td colspan="8">2. Methodological component of temperature measurement uncertainty</td></tr>
<tr><td>5</td><td>$\varepsilon_{av.e}$</td><td>u_5</td><td rowspan="2">Uniform</td><td>$\eta_5=–1,2$</td><td>$C_{5j} = \partial T_{\text{T}j} / \partial \varepsilon_{\text{av.e}}$</td><td colspan="2">$C_{5j} \cdot u_5$</td></tr>
<tr><td>6</td><td>Ks_e</td><td>u_6</td><td>$\eta_6=–1,2$</td><td>$C_{6j} = \partial T_{\text{T}j} / \partial Ks_{\text{e}}$</td><td colspan="2">$C_{6j} \cdot u_6$</td></tr>
<tr><td colspan="8">Correlation coefficient between correction parameters: $-1.0 \le r_{5\text{-}6} = r(\varepsilon_{av.e}, Ks_{\text{e}}) \le 1.0$</td></tr>
<tr><td colspan="8">3. Characteristics of accuracy of temperature measurement result by the <i>j</i>-th thermometry method</td></tr>
<tr><td colspan="2">Estimate of temperature measurement result by the j-th thermometry method, K</td><td colspan="2">Combined standard uncertainty, K</td><td>Resulting kurtosis</td><td>Confidence level $P_{conf.}$</td><td>Coverage factor k</td><td>Expanded uncertainty, K</td></tr>
<tr><td colspan="2">$T_{Tj}=f(S_{R1},S_{R2},\lambda_1,\lambda_2,\varepsilon_{av.e}, Ks_{\text{e}})$</td><td colspan="2">$u_{\text{c}}(T_{Tj})=f(C_{1j},\ldots,C_{6j}, u_1,\ldots,u_6,r_{5\text{-}6})$</td><td>$\eta_{\Sigma j}=f(\eta_5,\eta_6,C_{5j},C_{6j},u_5,u_6, r_{5\text{-}6},u_{\text{c}}(T_{\text{T}j}))$</td><td>0.95</td><td>$k_j=f(\eta_{\Sigma j}, P_{\text{conf.}})$</td><td>$U(T_{Tj})= =f(u_{\text{c}}(T_{Tj}),k_j)$</td></tr>
</table>

The data, given in Table 4, are sufficient to estimate (by B type) the uncertainty of the temperature measurement result by any of mentioned above thermometry methods. In general, the laws of distribution of input quantities uncertainties can differ from presented in Table 4. This fact is accounted by the corresponding kurtoses setting. Also, the existence of correction parameters mutual correlation can be taken into consideration by setting the coefficient $r_{5\text{-}6}$.

Accuracy characteristics [20] of temperature measurement result by the methods of monochromatic one- and two-color thermometry with emissivity correction are estimated in the next way.

1. One can obtain the temperature measurement result by the j-th thermometry method in accordance with generalized measurement equation (1), after substitution in this equation formulas for specific conditional temperature via one-color radiance temperatures, wavelengths and corresponding correction parameters (see Tables 1–3).

2. The combined standard uncertainty of temperature measurement result is calculated on the base of instrumental $u_{\text{ins}}(T_{\text{T}j})$ and methodological $u_{\text{met}}(T_{\text{T}j})$ components

$$u_{\text{c}}(T_{\text{T}j}) = \sqrt{u_{\text{ins}}^2(T_{\text{T}j}) + u_{\text{met}}^2(T_{\text{T}j})} \ , \tag{8}$$

where

$$u_{\text{ins}}(T_{\text{T}j}) = \sqrt{C_{1j}^2 \cdot u_1^2 + C_{2j}^2 \cdot u_2^2 + C_{3j}^2 \cdot u_3^2 + C_{4j}^2 \cdot u_4^2} \ , \tag{9}$$

$$u_{\mathrm{met}}(T_{\mathrm{T}j}) = \sqrt{C_{5j}^2 \cdot u_5^2 + C_{6j}^2 \cdot u_6^2 + 2 \cdot r_{5-6} \cdot C_{5j} \cdot C_{6j} \cdot u_5 \cdot u_6} \, . \quad (10)$$

3. The expanded uncertainty of temperature measurement result (at predetermined confidence level $P_{conf.}$) is calculated as a product of coverage factor on combined standard uncertainty

$$U(T_{\mathrm{T}j}) = k(P_{\mathrm{conf}}) \cdot u_{\mathrm{c}}(T_{\mathrm{T}j}) \, . \quad (11)$$

At the same time, to evaluate correctly the expanded uncertainty, it is necessary to determine in right way coverage factor at predetermined confidence level P_{conf}. In order to solve this task, let us use the kurtosis method, which was specially adapted for evaluation of expanded uncertainty of indirect measurand at confidence level 0.95 [21]. So, the coverage factor for resulting law of uncertainty distribution of measuring quantity is calculated on the base of resulting kurtosis of measuring quantity, using some proposed approximation dependency. In order to evaluate the resulting kurtosis of the measured temperature, let us use the rule of summarizing of input quantities kurtosis. If the correlation coefficient between the correction parameters is equal to zero, the following formula can be used

$$\eta_{\Sigma j} = \frac{\eta_{5-6} \cdot (C_{5j}^4 \cdot u_5^4 + C_{6j}^4 \cdot u_6^4)}{u_c^4(T_{\mathrm{T}j})} \, . \quad (12)$$

If correction parameters are mutually correlated ($r_{5\text{-}6} \neq 0$), the formula (12) becomes the form

$$\eta_{\Sigma j} = \frac{\sum_{i=1}^{4} \eta_i \cdot (C_{ij} \cdot u_i)^4 + \eta_{5-6} \cdot (C_{5j}^2 \cdot u_5^2 + 2 \cdot r_{5-6} \cdot C_{5j} \cdot C_{6j} \cdot u_5 \cdot u_6 + C_{6j}^2 \cdot u_6^2)^2}{u_c^4(T_{\mathrm{T}j})} \, . \quad (13)$$

In the formulae (12) and (13) the next notes are introduced:

$\eta_1 = \eta_2 = \eta_3 = \eta_4 = 0$ – the kurtoses of input quantities 1-4, which are normally distributed;

$\eta_{5-6} = \eta_5 = \eta_6 = -1{,}2$ – the kurtosis of correction parameters, which have the same distribution law – uniform.

Coverage factor, at confidence level 0.95, is calculated as following [21]

$$k(\eta_{\Sigma j}, P_{\mathrm{conf}} = 0{,}95) = 0{,}1085 \cdot \eta_{\Sigma j}^3 + 0{,}1 \cdot \eta_{\Sigma j} + 1{,}96 \, . \quad (14)$$

It can be concluded, that the uncertainty of temperature measurement result by one or another thermometry method will be affected by the corresponding sensitivity coefficients (partial derivatives of the measurement equation of specified thermometry method by the input quantities), as well as the presence of correlation between the correction parameters.

It is obviously, that the first stage of temperature measurement accuracy increasing is to minimize the uncertainties of the input quantities in measurement equations. To achieve this, it is necessary to more accurately measure one-color radiance temperatures, as well as to determine the correction parameters, choosing the most relevant, to the specified object, algorithm of correction parameters calculating.

On the second stage of temperature measurement accuracy increasing, it is necessary to choose the thermometry method, which demonstrates the minimal possible uncertainty under specific measurement conditions. In particular, the necessity of such choose is concluded by the author of the paper [7]. Relying on the presented above uncertainty budget, the transformation of the uncertainties of input quantities into the uncertainty of temperature measurement result by the different thermometry methods, should be investigated. Correspondingly, the analytical expressions for sensitivity coefficients C_{1j},..., C_{6j} for each j-th thermometry method, are required. Substituting these expressions into formulae (8)–(11), one can obtain quantitative and qualitative regularities of influence of one-color radiance temperatures uncertainties and the correction parameters uncertainties on the uncertainties of different thermometry methods, under the same measurement conditions. Analyzing these influence regularities, the conditions when the certain thermometry method demonstrate minimal uncertainty, should be estimated. Or, under determined measurement conditions, characterized by estimates of input quantities, their standard uncertainties and possible correlation between correction parameters, we should know, how to choose thermometry method which provides minimal uncertainty of temperature measurement – the best accuracy.

7. Conclusions

Thus, it has been determined, that monochromatic methods of one and two-color thermometry with emissivity correction rely on generalized equation of temperature measurement. This equation originates from approximation of the spectral density of radiance of emitting object by the Wien's formula. Measurement equation of the specific thermometry method consists of basic (by which method is named after) conditional temperature with corresponding correction parameters to compensate the spectral emissivity of controlled object.

As a result of next analysis of temperature measurement equations, it has been confirmed, that all investigated thermometry methods utilize the set of common input quantities. To characterize radiative properties of the controlled object, two correction parameters (average level of emissivity and selectivity coefficient), to be calculated by monochromatic spectral emissivity values on two wavelengths, have been proposed. The factors, which impact on input quantities uncertainties have been determined and the formulas for their evaluation have been derived. The uncertainty budget of indirect temperature measurement has been formed. The formulae for evaluation of the combined standard and expanded, with confidence level 0.95, uncertainties of thermometry methods have been obtained. It has been concluded, that under conditions of the same input quantities uncertainties, the accuracy of thermometry methods depends on quantitative estimations of the corresponding sensitivity coefficients only. The necessity of analytical expressions obtaining for sensitivity coefficients have been showed. This will allow analyzing the complex influence of input quantities uncertainties on temperature measurement accuracy by investigated thermometry methods, in order to determine the most accurate (with minimal possible uncertainty) method under specified measurement conditions, or to determine quantitatively such measurement conditions, when the certain method is the most accurate.

Conflict of Interest

The authors state that there are no financial or other potential conflicts regarding this work.

References

[1] A. Prokhorov et al., *Spaceborne optoelectronic sensors and their radiometric calibration terms and definitions. Part 1. Calibration techniques. NIST Interagency/Internal Report 7203*. US Department of Commerce, Gaithersburg, 2021.

[2] Ratio Pyrometers Datasheet – Williamson Corporation [Online]. Available: https://info.williamsonir.com/hubfs/Offer_Docs/Pro%20Ratio%20Pyrometer%20DS.pdf

[3] Principle, advantages, limitations and applications of two-colour pyrometers in thermal processes – Keller GmbH [Online]. Available: https://www.keller.de/dl.php?f=gisul-bogoseo-two-colour-pyrometer&h=f6861e87d41f43cL3Zhci93d3cvdmhvc3RzL2tlbGxlci5kZS9odHRwZG9jcy9zdG9yYWdlL2Z0cC9pdHMvSVRTL8O2ZmZlbnRsaWNoZXIgQmVyZWljaCBmcmVpZXIgWnVnYW5nL2VuLzA0IFRlY2huaWNhbCBSZXBvcnRzLzAxIEZ1bmRhbWVudGFscy9UUiBUd28tY29sb3VyIHB5cm9tZXRlcl9JRDI1MTNfMjAxNTA3X2VuLnBkZg

[4] Pyrometers – A Global Market Overview. January, 2022. [Online]. Available: https://www.researchandmarkets.com/reports/5527563/pyrometers-a-global-market-overview?srsltid=AfmBOooFTyVlI0zbL7UfSY0nFcwHxXoHJUIHEdQyfwsGubsOsHVKuhst

[5] M. Sugiura et al., “Two-color Method for Steel Temperature Measurement Unaffected by Water-induced Obstructions”, *ISIJ International*, vol. 63, pp. 346–353, 2023.

[6] V. N. Snopko, *Fundamentals of pyrometry methods by the spectrum of thermal radiation*. Minsk: Institute of physics named after B. I. Stepanov NASB, 1999 (in Russian).

[7] N. Ye. Hots, “Analysis of temperature measurement errors by the methods of two-color pyrometry of radiation”, *Bulletin of the Lviv Polytechnic National University, series Computer Systems and Networks*, No. 688, pp. 83–90, 2010 (in Ukrainian).

[8] L. F. Zhukov, D. O. Petrenko, “Influence of metal alloys radiative characteristics on methodical errors of two-color compensative and classical thermometry”, Measuring equipment and metrology, vol. 80, No. 3, pp. 39–45, 2019 (in Ukrainian).

[9] N. Hots, “Investigation of Temperature Measurement Uncertainty Components for Infrared Radiation Thermometry” in *Recent Advances in Systems, Control and Information Technology. Series: Advances in Intelligent Systems and Computing, vol. 543, “International Conference on Systems, Control and Information Technologies–2016”*, Springer, 2016, pp. 556–566.

[10] A. Araujo, R. Silva, “Surface temperature estimation in determined multi-wavelength pyrometry systems”, *The Review of scientific instruments*, vol. 91, iss. 5, 054901, 2020.

[11] J. Chen et al., “A data processing method for two-color pyrometers in accurate temperature measurement of high-temperature flow fields”, *Measurement*, vol. 243, 116431, 2025.

[12] L. F. Zhukov, D. O. Petrenko, “Resource-saving continuous optical control of liquid metal temperature in metallurgy of energy machine building”, *Systems research in energy*, iss. 3 (74), pp. 64–77, 2023 (in Ukrainian).

[13] Z. Zhang, B. Tsai, G. Machin, *Radiometric Temperature Measurements II. Applications*. New York: Academic Press (Elsevier), 2010.

[14] B. K. Tsai, R. L. Shoemaker, D. P. DeWitt et al., “Dual-wavelength radiation thermometry: emissivity compensation algorithms”, *International Journal of Thermophysics*, vol. 11, pp. 269–281, 1990.

[15] B. K. Tsai, D. P. DeWitt, G. J. Dail, “Application of dual-wavelength radiation thermometry to the aluminum industry”, *Measurement*, vol. 11, iss. 3, p. 211–221, 1993.

[16] Y. Zhang et al., “Measurement performance analysis for a charge-coupled-device-based near-infrared multi-spectral pyrometer”, *Infrared Physics & Technology*, vol. 106, 103273, 2020.

[17] P. Saunders et al., *Uncertainty estimation of primary radiometric temperature measurements*. Technical report of International Committee for Weights and Measures, 2018.

[18] J. J. Davenport, J. Hodgkinson, J. R. Saffell, R. P. Tatam, “Noise analysis for CCD-based ultraviolet and visible spectrophotometry”, *Applied Optics*, vol. 54, iss. 27, pp. 8135–8144, 2015.

[19] USB4000 Fiber Optic Spectrometer. Installation and Operation Manual [Online]. Available: https://focenter.com/media/wysiwyg/documents/Ocean-Optics-Inc-Ocean-Optics-USB4000-Fiber-Optic-Spectrometer-Fiber-Optic-Center.pdf

[20] O. M. Vasilevsky, V. Yu. Kucheryk, *Fundamentals of the theory of measurement uncertainty*. Kherson: Oldi-plus, 2020 (in Ukrainian).

[21] I. Zakharov, P. Neyezhmakov, O. Botsiura, “Expanded uncertainty evaluation taking into account the correlation between estimates of input quantities”, *Ukrainian Metrological Journal,* No. 1, pp. 4–8, 2021.