ENHANCING THE METROLOGICAL AUTONOMY OF LOCAL MEASUREMENT SYSTEMS

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Abstract. This article presents a study on the prospects of ensuring measurement traceability through the implementation of methods aimed at increasing the metrological autonomy of measurement systems for various branches of industry and science.

Key words: Measurement traceability, metrological support, metrological autonomy, local measurement systems.

1. Introduction

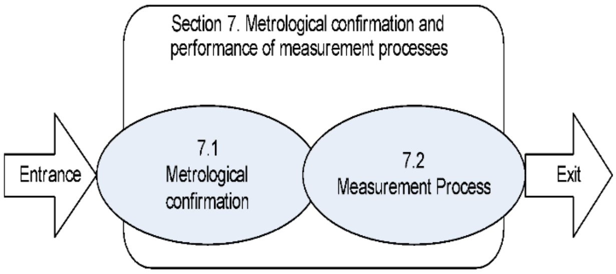
In modern production processes, the accuracy, correctness, and timeliness of measurements and control of technological parameters directly affect the efficiency and validity of decision-making regarding the operation of processes that have a direct impact on the quality of manufactured products. Therefore, the quality of measurements in industry determines the level of product competitiveness [1, 2].

The quality of measurements largely depends on the degree to which measurement traceability is ensured, both at the national level and within individual enterprises. Measurement traceability is achieved through an accurate realization and maintenance of national measurement standards of physical quantities, the transfer of their values, as well as through the provision of appropriate conditions for the correct preservation of these values during the operation of measuring instruments (MIs).

Under the conditions of significant wear of the reference standard base and the rising costs of metrological services, the role of a set of organizational and technical measures aimed at ensuring a certain level of metrological autonomy of individual production facilities increases significantly. This approach allows for the reduction of costs for metrological support while maintaining the required level of measurement traceability, accuracy, and reliability. Consequently, in the context of growing market competition, the task of ensuring measurement traceability in industry becomes highly relevant.

The purpose of activities related to the development, improvement, and maintenance of systems for the realization of physical quantity units and the transfer of their values to industrial measuring instruments is to ensure the metrological reliability of MIs. Therefore, when analyzing possible ways to increase the metrological reliability of MIs, it is necessary to search for methods to optimize the system for realizing, maintaining, and transferring reference units of physical quantities (PQs) to industrial MIs [1].

One of the principles proclaimed in ISO 9000 relates to the process approach. Measurement processes should be considered as special processes aimed at ensuring product quality within an organization. The model of the measurement management system, in accordance with the standard [2], is shown in Figure.



Model of the measurement management system

As illustrated in the presented model, any measurement process consists of two interrelated components:

– metrological confirmation – a set of operations aimed at ensuring measurement traceability: realization, maintenance, and transfer of physical quantity units, as well as the creation of conditions to maintain the metrological serviceability of measuring instruments (MIs);

– measurement process – a set of operations aimed at obtaining information about the value of a physical quantity: interaction with the measurement object, transformation of measured signals into a form suitable for further use, and processing of measurement results.

According to the requirements of [2], organizations are obliged to determine the necessary level of control measures and establish the requirements for the measurement management system to be implemented as part of their overall management system. However, while numerous innovative approaches have been adopted in recent years for organizing measurement processes (such as the use of intelligent measuring instruments, and the development of efficient systems for the transmission, storage, and processing of measurement data), metrological confirmation procedures are still largely based on outdated traditional methods of metrological support. These include the hierarchical system for the transfer of physical quantity units, the establishment of uniform recalibration intervals for similar types of measuring instruments, and the removal of instruments from the technological process to confirm their metrological serviceability.

2. Drawbacks

There are many classical approaches to the metrological confirmation of measuring instruments, which are widely covered in scientific and regulatory literature and well known to specialists. In general, it can be stated that traditional methods of metrological confirmation of MIs have the following drawbacks: First, MIs used in technological processes (TP) operate under specific conditions characteristic of that particular TP, which are usually different from the conditions under which metrological verifications are performed. This discrepancy leads to additional measurement errors caused by the mismatch between the conditions of metrological serviceability confirmation and actual operating conditions. These errors are random in nature and are often not taken into account [3].

Second, due to the common practice of specifying the permissible limits of errors for working standards (especially lower-tier standards) without distinguishing between systematic and random components, there is a possibility that a significantly “distorted” unit value may be transferred to the same MI – particularly when different standards are used during periodic metrological verifications.

Therefore, it is important to approach the issue of ensuring the metrological compliance of industrial MIs from a different perspective.

3. Goal

The purpose of this article is to investigate the prospects of ensuring measurement traceability through the implementation of methods aimed at enhancing the metrological autonomy of measurement systems for various sectors of industry and science.

4. A method for enhancing the metrological autonomy of local measurement systems

In modern mass production processes, which primarily require improved accuracy in controlling technological modes and, secondly, a reduction in losses due to measurement uncertainty, the need arises for timely confirmation of the metrological compliance of measuring instruments during the recalibration interval [4, 5]. One of the promising ways to achieve this timely confirmation is by increasing the metrological autonomy of the measurement traceability system in production. This will contribute to enhancing the reliability of interconnected measurement results, for example, in quality control during the manufacturing processes.

Metrological autonomy is understood as the ability to maintain the required accuracy of industrial measuring instruments over an extended period of time without the use of higher-level standards for transferring the unit of physical quantity. Utilizing metrological autonomy will allow, in addition to reducing the costs associated with metrological compliance confirmation procedures, the optimization of recalibration intervals.

In each technological process, a relatively stable set of MIs is used, and the measurement results of these instruments are employed over a prolonged period to control the technological modes of the TP. This set of MIs possesses certain metrological characteristics, which are in a determined relationship with technological parameters. Therefore, this set of MIs can be considered a local measurement system (LMS) characteristic of the given TP. Consequently, controlling the measurement error of MIs used in a particular LMS within specified limits is a necessary condition for ensuring process control, the parameters of which are measured by these MIs.

For each local group of identical MIs, considering their metrological uniformity, there exists a certain state of metrological serviceability at each moment in time t_i, which determines the level of measurement traceability in the LMS. It is known that the metrological reliability of individual MIs can vary significantly. Since specific MIs are used for measurements, the reliability of measurement results will depend on the individual metrological properties of these specific instruments rather than on the averaged properties of the entire set of instruments of this type. Therefore, using averaged metrological reliability characteristics when planning measures to ensure measurement traceability leads to increased costs to ensure the required level of measurement reliability in production [6].

In manufacturing, situations often arise where, during the period between calibrations, it is necessary to verify the metrological compliance of MIs, but it is impossible to remove them from the TP for calibration using traditional methods. To solve this problem, it is suggested to create an autonomous group standard for the unit of physical quantity for the selected LMS, using the MIs within it, and to assess the measurement errors of individual MIs based on comparisons with the most probable value of this group standard, which is obtained through periodic verifications.

To obtain the most probable estimate of the value of the unit of physical quantity (PQ) stored in the autonomous group standard (AGS), we consider the following mathematical model of measurements. Let at time t_1 the calibration of MIs included in the LMS be carried out using a standard with a value of X_0, resulting in a series of measurement outcomes for the MIs – $X_{1/1}, X_{2/1}, \ldots, X_{i/1}, \ldots, X_{n/1}$ from which the measurement errors are determined:

$$\Delta_{i/1} = X_{i/1} - X_0, i = 1, n \quad (1)$$

where $\Delta_{i/1}$ – is the measurement error of the *i*-th MIs from the LMS at time t_1; $X_{i/1}$ – is the reading of the *i*-th MIs from the LMS at time t_1; X_0 – is the value of the physical quantity unit reproduced by the working standard; *n* – is the number of MIs in the LMS.

At the moment of time t_1 we define the estimate of the value of the physical quantity and the variance of the group standard using the following expressions:

$$X^*_{E/1} = \frac{1}{n}\sum_{i=1}^{n} X_i$$
$$\sigma^2_{E/1} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X}_E)} \quad (2)$$

where $X^*_{E/1}$ – is the estimate of the value of the physical quantity of the group standard; $\sigma^2_{E/1}$ – is the variance of the value of the physical quantity of the group standard.

Then, during the period of metrological autonomy of the LMS (the recalibration period) – to estimate the value of the group standard, it is necessary to find the estimate of its error at time t_2. Let us assume that at time t_2 the MIs were verified using the method of mutual verifications. With the number of MIs in the standard being *n*, it is necessary to conduct *n(n*–1*)/*2 verifications. Taking into account (1), we can write the verification equation as:

$$X_{1/2} - X_{2/2} = \Delta_{1/2} - \Delta_{2/2} = \delta_{1-2/2} + \eta$$
$$X_{1/2} - X_{3/2} = \Delta_{1/2} - \Delta_{3/2} = \delta_{1-3/2} + \eta$$
$$\ldots\ldots\ldots \quad (3)$$
$$X_i - X_j = \Delta_{i/2} - \Delta_{j/2} = \delta_{i-j/2} + \eta$$

where – $\Delta_{i,j}, X_{i,j}, i, j=1,n$ – are the errors and values of the physical quantity of the *i,j*-th MIs at time t_2; $\delta_{i-j/2}$ – is the result of the verification at time t_2; η – are the Gaussian noise of the verifications.

To minimize the impact of multiplicative error components of individual MIs, it is recommended to verify them using a constant input quantity, the value of which is selected in the last third of the scale of the MIs being verified. This is relatively simple to implement, as identical MIs are often used in technological processes (TP).

It should be noted that the AGS can be formed from both equally accurate and differentially accurate measuring instruments. For the case of uneven accuracy measurements, a matrix of weight coefficients for mutual verifications is introduced. The elements of this matrix (the weights of the corresponding verifications) are determined by the following expression:

$$W_{ij} = \frac{\Delta_{d\,min}}{\Delta_{di}\cdot\Delta_{dj}} \quad (4)$$

where Δ_{dmin} – is the main permissible error of the most accurate MIs in the LMS; Δ_{di}, Δ_{dj} – are the main permissible errors of the *i*-th and *j*-th MIs involved in the respective verification.

Then, equation (3), taking into account the respective verification weights $W_{ij}; j = 1, n$, in accordance with (4), can be represented as:

$$(\Delta_2 - \Delta_i)\cdot W_{i(2)} = \delta_{i(2)}\cdot W_{i(2)}$$
$$(\Delta_3 - \Delta_i)\cdot W_{i(3)} = \delta_{i(3)}\cdot W_{i(3)}$$
$$\ldots\ldots\ldots \quad (5)$$
$$(\Delta_j - \Delta_i)\cdot W_{i(j)} = \delta_{i(j)}\cdot W_{i(j)}$$

Let us assume that the results of the verifications $\delta_{i-j/k}$ are independent random variables distributed according to the normal distribution law, It can be assumed that they are grouped around a certain value $X_0\ (t_2)$, which is close to X_0.

The quality of the estimate of the value $X^-_{E/i}$ is related to the estimation error $\varepsilon(X^*_{E/i}/X_0) = X^-_{E/1} - X_0$. This error is of a random nature, and for the Bayesian approach, its optimal value is achieved when the average loss $R(\varepsilon) = R(X^*_{E/i}/X_0)$ which it leads to, is minimized. The losses will be minimal if, when processing experimental data – $X_{1/1}, X_{2/1}, \ldots, X_{i/1}, \ldots, X_{n/1}$ the posterior probability density is determined $p(X^*_{E/i}/X_0)$ and the optimal estimate is chosen at the point of maximum $p(X^-_{E/1}/X^-_0)$.

The search for the maximum of the posterior probability density will be performed as follows. Let us assume that the true value of the physical quantity of the *i*-th measurement instrument in the AGS, denoted as – X_i follows a normal distribution. Then, the likelihood function can be represented as follows [7]:

$$p(X_i / X) = \frac{1}{\sqrt{2\pi\sigma_i}} exp\left\{-\frac{(X_i - X)^2}{2\cdot\sigma_i^2}\right\} \quad (6)$$

where X – is the true value of the physical quantity of the AGS; σ_i^2 – is the variance of X_i.

Since the errors of individual measurement instruments within the AGS are independent variables, the likelihood function for the group standard can be written as:

$$p(X_1, X_2, \ldots, X_n / X) = \prod_{i=1}^{n}\left[\frac{1}{\sqrt{2\pi\sigma_i}} exp\left\{-\frac{(X_i - X)^2}{2\cdot\sigma_i^2}\right\}\right] \quad (7)$$

According to the maximum likelihood criterion, the expression for the optimal estimate of the physical quantity value of the AGS is as follows [7]:

$$X^* = arg\,max\left[p(X_1, X_2, \ldots, X_n / X)\right] \quad (8)$$

Then, the estimate of the value of the physical quantity reproduced by the autonomous group standard can be found from the expression:

$$X_i^* = \frac{\sum_{i=1}^{n} X_i \cdot \prod_{\substack{j=1\ldots n \\ j\neq i}}^{n}\sigma_j^2}{\sum_{i=1}^{n}\prod_{\substack{j=1\ldots n \\ j\neq i}}^{n}\sigma_j^2} \quad (9)$$

The optimal estimate of the value of the PQ based on the previous X^*_{i-1} and current X^*_i estimates can be determined using the following expression:

$$\hat{X} = \frac{\sigma_{i-1}^2}{\sigma_{i-1}^2 + \sigma_i^2}\cdot X^*_{i-1} + \frac{\sigma_i^2}{\sigma_{i-1}^2 + \sigma_i^2}\cdot X^*_i \quad (10)$$

The proposed formula makes it possible to implement an algorithm for forming the optimal estimate of the physical quantity value, which links the previous estimate of the unit value of the physical quantity with its current value, allowing control over the stability of the autonomous group standard under given conditions.

Assuming that the error values of physical quantity units $\Delta_i, i = 1, n$, are Markovian and described by a normal distribution, the task of determining the intercalibration interval T_n for the measuring instruments that maintain the unit of physical quantity within the group standard will be solved based on the Fokker – Planck equation for random processes [7].

The forward Kolmogorov equation for *n* measuring instruments, taking into account [7], is written as follows:

$$\frac{\partial P(\Delta_1,\ldots,\Delta_n,t)}{\partial t} = \frac{K_{2n}}{2} \cdot \sum_{i-1}^{n} \frac{\partial^2 P(\Delta_1,\ldots,\Delta_n,t)}{\partial \Delta_i^2} \quad (11)$$

where P – is the probability that the error values of the physical quantity fall within the confidence interval $(-\Delta_d;+\Delta_d)$; K_{2n} – is the diffusion coefficient for the group standard.

The solution to this equation is the function [8]:

$$P(\Delta_1,\ldots,\Delta_n,t) = \prod_{i=1}^{n}\left[\frac{1}{\sqrt{2\cdot\pi\cdot K_{2(i)}\cdot t}} exp\left(-\frac{\Delta_i^2}{2\cdot K_{2(i)}\cdot t}\right)\right] \quad (12)$$

where $K_{2(i)} = \sigma_i^2$ – is the diffusion coefficient of the *i-th* MIs within the group standard.

Given a confidence interval $(-\Delta_d;+\Delta_d)$ and a confidence probability $P(-\Delta_d;+\Delta_d)$, the expression for the probability of a random variable falling within a finite interval can be written as:

$$P(-\Delta_d;+\Delta_d) = \int_{-\Delta_d}^{+\Delta_d} \frac{1}{2/n\sqrt{\pi\cdot K_{2n}\cdot T_n}} exp\left(-\frac{1/2(n\Delta)^2}{2\cdot K_{2n}\cdot T_n}\right)\cdot d\Delta \quad (13)$$

By substituting variables, we transform the integrand in formula (14) into the form of the error function (Laplace function), the values of which are provided in standard tables:

$$P(-\Delta_d;+\Delta_d) = \Phi\left[\sqrt{\frac{n^2}{2K_{2n}T_n}}\cdot\Delta_d\right] \quad (14)$$

The general solution of the equation for determining the recalibration interval T_n for *n* MIs included in the group standard in this case takes the following form:

$$T_n = \frac{n^2\cdot\Delta_d^2}{2\cdot\left[\Phi^{-1}(P(-\Delta_d;+\Delta_d))\right]^2\cdot K_{2n}} \quad (15)$$

where $\Phi^{-1}(P(-\Delta_d;+\Delta_d))$ is the inverse function to $\Phi(P(-\Delta_d;+\Delta_d))$.

Solving a similar problem for a single measuring instrument, we obtain the expression for the recalibration interval of an individual MI from the AGS:

$$T_{(i)} = \frac{\Delta_{d(i)}^2}{\left[\Phi^{-1}(P(-\Delta_d;+\Delta_d))\right]^2\cdot K_{2(i)}} \quad (16)$$

where $T_{(i)}$ – recalibration interval of the given measuring instrument.

The expression for the error estimation Δ_i^* MIs from the group standard AGS will take the form

$$\Delta_i^* = \sqrt{\left[\Phi^{-1}(P(-\Delta_d;+\Delta_d))\right]^2\cdot K_{2(i)}\cdot t_1} \quad (17)$$

where t_1 – the moment of time for the intra-group verifications of the AGS;

The expression for determining the diffusion coefficient of the *i*-th MIs from the AGS $K_{2(i)}$ is as follows:

$$K_{2(i)} = \frac{\Delta_{d(i)}^2}{\left[\Phi^{-1}(P(-\Delta_d;+\Delta_d))\right]^2\cdot T_{(i)}} \quad (18)$$

Thus, we will determine the estimate of the total error of the group standard.

$$\Delta_\Sigma^* = \sum_{i-1}^{n}\Delta_i^* \quad (19)$$

Taking into account (5), we write the system of *n* equations in matrix form.

$$M\cdot\Delta^* = N \quad (20)$$

where Δ_i^* – estimates of the main errors of the *i-th* MIs in the set of the group standard $i=1,n$.

The matrix of error estimates Δ^* of the measuring instruments in the set of the group standard is obtained from the equation:

$$\Delta^* = M^{-1}\cdot N \quad (21)$$

where M^{-1} – the inverse matrix of the coefficients of the system (22).

The estimate of the value of the physical quantity unit of the group standard based on mutual verifications will be obtained according to the maximum likelihood criterion.

The condition of stability of the group standard value can be determined based on the *G*-criterion (Cochran criterion):

$$G_P = \frac{\sigma^2\{X_i^*\}_{max}}{\sum_{i=1}^{N}\sigma^2\{X^*\}} \quad (22)$$

where $\sigma^2\{X_i^*\} = \frac{1}{m-1}\sum_{i=1}^{m}(X_i^* - X^-)$ – variance of the *i*-th estimate of the value of the AGS; $\sigma^2\{X^-\} = \sum_{i=1}^{m}\sigma^2\{X^*\})$ – variance of the weighted optimal estimate of the AGS for all previous comparisons; *m* – number of comparisons at the *i*-th moment in time; *N* – number of procedures for determining the optimal AGS value throughout the entire period of metrological autonomy.

The task of evaluating the stability of the AGS value reduces to testing the hypothesis of homogeneity of variances $\sigma^2\{X_i^*\}$ and $\sigma^2\{X^-\}$. To test the hypothesis H_0 about the homogeneity of variances $\sigma^2\{X_i^*\}$ and $\sigma^2\{X^-\}$ the obtained value of G_P is compared with the critical value G_{KP}, which is found from the table of critical values of the *G*-criterion at the intersection of the column $f_{num}=m-1$ and the row $f_{demon}=N$ for the given significance level 〈. If $G_P<G_{KP}$, the hypothesis about the homogeneity of variances is accepted, confirming the stability of the AGS value.

The proposed approach for forming the optimal estimate of the physical quantity of the AGS allows setting conditions for detecting the metrological failures of individual MIs. The condition for the metrological failure of the *i*-th MIs is determined using the Fisher criterion and consists of testing the hypothesis about the equality of the measurement result of the *j*-th MIs – $X^*_{j/i}$ for the *i*-th comparison and the estimate of the AGS value – X^*_i for the *i*-th comparison: H_0: $X^*_{j/i} = X^*_i \Leftrightarrow H_1: X^*_{j/i} \neq X^*_i$ for the chosen significance level:

$$z = \frac{X^*_{j/i} - X^*_i}{\sqrt{\frac{\sigma^2\{X^*_{j/i}\}}{n_1} + \frac{\sigma^2\{X^*_i\}}{n_2}}} \quad (23)$$

where n_1, n_2 – are the sample sizes for the values of the *j*-th MIs and *i*-th comparison, respectively, when estimating the AGS value – X^*_i.

The obtained value z is compared with the critical value $z_{KP} = z_{1-\alpha/2}$ for the chosen confidence probability. If $z > z_{KP}$, the hypothesis H_1, is accepted, and a conclusion is made about the metrological failure of the *j*-th MIs, leading to its calibration.

The methodology for forming the maximum likelihood estimate of the value of a physical quantity for the AGS based on the results of internal group calibrations of the measuring instruments included in it involves the following sequence of operations:

1) after mutual calibrations, write the system of linear equations (3);

2) determine the weighting coefficients of the mutual calibrations (4) and form the weight matrix;

3) write the weighted equations for the internal group calibrations (5);

4) calculate the diffusion coefficients for each MIs included in the AGS (19).

5) calculate the estimates of the measurement errors for the MIs (18).

6) determine the estimate of the total error of the group standard according to (20).

7) according to the matrix of weighting coefficients (4), select *n*–1 linearly independent calibration equations that include the errors of all the MIs in the AGS, and write the system of *n* equations in matrix form (21) considering the total error of the group standard;

8) find the estimates of the main errors of the physical quantity values for the MIs from equation (22);

9) find the maximum likelihood estimate of the value of the physical quantity for the AGS based on the results of internal group calibrations from equation (9) and determine the weighted estimate (10);

10) perform a hypothesis test for the stability of the AGS value according to the criterion (23) and perform a hypothesis test for metrological failure (24) for each MIs.

5. Conclusions

Thus, along with improving the measurement accuracy, the proposed method for enhancing the metrological autonomy of local measurement systems will help reduce the metrological component of costs for ensuring product quality.

Improving the accuracy of industrial measurements can be achieved by using more accurate and reliable MIs in technological processes. However, this approach requires additional costs, which, in the context of fierce competition, is generally economically unfeasible. A more promising approach is the implementation of a system for the operational control of the metrological characteristics of MIs used in technological processes. The proposed approach will improve the metrological conformity of industrial MIs and reduce enterprises' costs for metrological support of their TPs.

Conflict of interest

The authors state that there are no financial orother potential conflicts regarding this work.

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