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# CYBER-PHYSICAL MODELING OF IMPLANTABLE DEVICES FOR INTERSTITIAL THERAPEUTIC SYSTEMS

*Andrii Chuhai* [1,2]

[1] *Simon Kuznets Kharkiv National University of Economics, Nauky Ave. 9A, Kharkiv, Ukraine,*
[2] *Anatolii Pidhorniy Institute of Power Machines and Systems, Komunalnykiv Str., 2/10, Kharkiv, Ukraine.*
Author's e-mail: *chugay.andrey80@gmail.com*

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***Abstract*: This paper proposes a cyber-physical modeling framework that enables a planner to optimally position implantable medical devices in therapeutic systems that operate interstitially. To inspire a model, we use applications such as interstitial photodynamic therapy and brachytherapy, and frame the problem as constrained packing of cylindrical objects with spatial and angular feasibility conditions. An approach combines anatomical geometry, device orientation restrictions, and tissue-specific feasibility to create an integrated optimization model. The proposed modeling framework enables a planner to devise device configurations that temporally maximize therapeutic coverage for the targeted anatomy while minimizing the risks of overlap, thermal injury, and mechanical interference by employing incremental sequential algorithms. Numerical simulations will demonstrate the model's ability to support increased precision of treatment and geometry-aware clinical decisions.**

***Index terms*: cyber-physical modeling, implantable devices, interstitial therapy, optimization, geometry, biosensors, orientation.**

## I. INTRODUCTION

The advancement of mathematical modeling and computational optimization has given rise to the first in a new generation of tools for modern medicine. As clinical procedures become more precise, personalized, and less invasive, the demand for solid geometry-driven planning platforms is rapidly increasing. These frameworks will enable clinicians and engineers to simulate, optimize, and validate complex procedures before surgical intervention, thereby improving outcomes and reducing risk. One especially interesting topic for mathematical modeling is the placement of elongated medical devices into biological tissues, such as optical fibers, electrodes, catheters, and implantable sensing devices. These devices are often modeled as cylindrical objects under the constraint of being inserted into anatomically constrained shapes, including tumors, brain regions or vascular spaces. One of the most visible examples of interstitial photodynamic therapy (iPDT). In the iPDT treatment, optical fibers are placed within the tumor tissue to guide light energy, activating a previously loaded drug in the patient. To ensure the displacement of light throughout the tumor, we consider this arrangement to allow sufficient distance and angles between the fiber optics, thereby avoiding "hot dosages" and ensuring each fiber achieves the same coverage and effective angle of light. A more involved conception of this scenario results in a limited packing problem of cylindrical objects established by a minimum specified acute angle, where vertical lines of each fiber remain outside the angular region of overlapping light fields, which may also affect thermal damage [1].

## II. LITERATURE REVIEW AND PROBLEM STATEMENT

Brachytherapy is another well-established application in which radioactive sources are placed inside or near a tumor. The goal is to achieve a uniform dose while minimizing the dose to nearby normal tissues. This requires careful source singulations and orientations planning, and is typically modeled as a packing problem considering dosimetric and geometric constraints. Optimizing packing problems is often accomplished through optimization approaches such as simulated annealing or inverse planning [2]. Implantable biosensors provide examples of applications well-suited for packing optimization. The devices themselves typically take the form of a cylinder or a capsule and are placed in soft tissues to measure physiological parameters, such as glucose or pressure.

The biosensor for monitoring parameters must account for tissue deformation. Optimization tools define feasible layouts that consider interference, location, and effectiveness of the sensor [3]. Drug delivery systems and methods, specifically those that utilize multi-compartment capsules or implantable microchips for drug delivery, are another area of application for packing optimization. In these applications, it is often necessary to consider the packing of multiple drug reservoirs or release channels for delivery, in a confined space, as a function of desired release kinetics, miniaturization of the delivery system, and location and targeting [4]. Drug delivery systems based on reservoirs (e.g., electromechanical devices or polymer-based devices) rely on the device's internal layout, rather than its exterior shape, to function reliably and deliver therapy in a controlled manner.

These placement challenges are fundamentally geometric and often involve conceptualizing packing challenges, an established optimization problem in operations research and computational geometry. Packing problems involve placing a collection of objects (e.g., cylinders, spheres, or irregular shapes) within a container in a non-overlapping manner, typically with additional constraints regarding orientation, accessibility, or mechanical stability [5][6]. Though extensively studied in industrial applications, their application to the placement of medical devices has not brought the same level of attention. In medical contexts, packing challenges are further complicated by anatomy and procedural considerations. Mutual orientation constraints between devices, which are often necessary, for example, in interstitial photodynamic therapy, to place optical fibers parallel, can lead to dose hotspots, degradation of efficient light distribution, and thermal damage to healthy tissues [1]. These examples help to motivate a new type of packing model, which can include not only minimum distance-against-overlap constraints but mutual angular separation constraints (e.g., minimum angle between axes of cylinders) and orientation constraints in relation to the boundary of the placement domain (e.g., insertion angles to tissue surfaces or other anatomical landmarks). Traditional packing problems address non-overlap in the spatial domain and packing efficiency in total volume. Medical applications require special consideration of orientation constraints to avoid a long manipulator system subject to coincident or overlapping trajectories, thereby preventing physical, thermal, or functional interactions.

In interstitial photodynamic therapy, optical fibers must be inserted into tumor tissue such that their light fields overlap constructively, but not to an extreme degree. Prior studies have identified thermal damage to healthy tissue or under-irradiation of tumor margins due to the parallel placement of optical fibers, which generates dose hotspots (9). To avoid this situation, clinicians often implement angular separation between optical fibers by constraining the angle of insertion of the fibers into the tumor surface to a minimum inclination, thereby minimizing the overlap of their light fields. Although these angular separations between optical fibers have been well-documented in the literature, they are not represented in the mathematical models used to measure light or treat cancer. Angular penalties have recently been applied to inverse brachytherapy planning algorithms (8). Rather than avoiding overlap from the insertion angles, the cost function used to optimize the placement of source trajectories specifically penalizes sources that are parallel to or overlap with each other.

Regardless of these examples, no unified geometric framework currently integrates mutual orientation constraints as a first-class aspect of the packing problem. This void calls for new models designed to incorporate angular feasibility expressly.

## III. SCOPE OF WORK AND OBJECTIVES

The research presented here concerns the development of a mathematical and computational framework for siting and designing implantable devices in therapeutic systems that deliver treatment interstitially. The specific course of action for the research is to construct a constrained optimization problem generated using geometric design to facilitate the incorporation of the physical and technological constraints associated with implantable devices for medical applications. The modeling approach integrates algorithmic methods to develop a device placement, utilizing numerical simulations to assess the physical feasibility and performance of the device under relevant boundary conditions.

The objectives of the study are as follows:

- to construct a mathematical model for implant positioning that reflects geometric constraints, tissue limits of interest, and practical technological limitations of interstitial therapy;
- to develop an optimization-based design technique to seed the systematic exploration of feasible implant configurations within an anatomically realistic space;
- to conduct numerical simulations to prove the approach on synthetic test cases showing the potential robustness and adaptability of the method; analyze the influence of boundary effects and packing constraints on the achievable implant distributions, with emphasis on clinical relevance and manufacturability;

By integrating geometric optimization with cyber-physical modeling, the research aims to develop a reproducible and scalable method for enhancing the accuracy and safety of targeted therapeutic procedures.

This study presents a formal optimization framework for packing cylindrical shapes with mutual orientation constraints, as it applies to interstitial medicine. This proposed method extends classical packing theory by incorporating angular feasibility and is presented in a constrained mathematical programming format. The complex nature of constrained packing problems lends itself to an incremental sequential optimization framework.

This class of methods can update the solution after each evaluation, providing an efficient means of optimization when evaluating in groups, as this is often impractical or costly to do [14]. The model can incorporate spatial non-overlap with minimum angular separation of objects and with orientation constraints related to anatomical boundaries. Numerical examples illustrate how the proposed framework can be applied to mathematical modeling and treatment planning. The proposed model is not intended to replicate complete clinical workflows or patient-specific anatomical data, although it highlights the real-world medical problems that iPDT and brachytherapy aim to address. The modeling framework of the proposed model is solely focused on the geometric and mathematical aspects of the placement problem and is meant to function as a type of BS-R, or bony-connective tissue relation. The proposed model could provide a practical geometrical foundation for various clinical planning systems.

## IV. COMPUTATIONAL APPROACH TO SOLVING THE PLACEMENT MODEL

To apply established normalized $\Phi$-functions, originally developed for convex polyhedra with arbitrary orientation, each cylindrical object $C_i(u_i)$ is currently approximated by a prism. This approach is necessary because direct formulations for true cylindrical geometries have not yet been developed and will be addressed in future research.

Geometrically, the circular cross-section of a cylinder is replaced by a regular polygon. This substitution results in a prism $P_i(u_i)$ whose base approximates the cylinder's footprint, and whose lateral surface is composed of flat quadrilateral panels. The top and bottom faces are modeled as regular polygons, ensuring continuity and symmetry in the approximation. The prism's vertex coordinates are computed with respect to both the cylinder's local coordinate system, incorporating the cylinder in global space, and with respect to the cylinder's local space. The cylinder's local coordinates are transformed to its coordinates in the global coordinate system using rotation matrices based on the azimuthal and inclination angles. Thus, both a consistent spatial placement and angular orientation are captured in the problem formulation.

The coordinates of the vertices of this polyhedral approximation are computed as:

$$\begin{aligned}
x_{ij} &= (x_i + r_i^0 \cos\alpha_j)\cos\omega_i - (x_i - r_i^0 \sin\alpha_j)\sin\varphi_i \sin\omega_i + \\
&+ (z_i \pm h_i^0)\cos\varphi_i \sin\omega_i, \\
y_{ij} &= (x_i - r_i^0 \sin\alpha_j)\cos\varphi_i + (z_i \pm h_i^0)\sin\varphi_i, \\
z_{ij} &= -(x_i + r_i^0 \cos\alpha_j)\sin\omega_i - (x_i - r_i^0 \sin\alpha_j)\sin\varphi_i \cos\omega_i + \\
&+ (z_i \pm h_i^0)\cos\varphi_i \cos\omega_i, \\
j &= 1, 2, ..., n_a.
\end{aligned}$$

where $\alpha_j = 2(j-1)\pi / n_a$, $j = 1, 2, ..., n_a$.

In this way, the surface of $C_i(u_i)$ is discretized using $n_a$ quadrilaterals with pairwise adjacent edges and two polygons with $n_a$ vertices discretizing the top and bottom bases of $C_i(u_i)$.

To tackle the limited packing of cylindrical objects based on spatial and angular feasibility, we develop a sequential optimization methodology. In contrast to earlier strategies that uniformly scaled objects using a homothety coefficient [12], we optimize the volume of each individual cylinder directly by treating the radius and height of the cylinder as independent design variables. The method enables a more flexible approach to layout and more clinically meaningful situations, while retaining the independence of the geometries, particularly in interstitial medical applications.

We employ a sequential optimization approach to position cylindrical items within space and angular constraints. Specifically, in this approach, we maximize the volume of each newly positioned cylinder by treating its radius and height as independent variables in the optimization. The optimization process also relates to the geometric parameters of the most recently added cylinder. In one step, the previously placed cylinders, which defined the previous orientation, are treated as variables in the optimization process, subject to the spatial and angular constraints of the system, allowing their position and orientation to be adjusted. An opportunity exists, therefore, where we can reconstruct the positioning of the previous cylinders to allow for the new cylinder placement to be accommodated into the system, while the spatial and angular constraints remain sorts of reasonable.

An algorithm for solving the problem is as follows.

Step 1. Set $n := 0$ (initialization of the placement process).

Step 2. Set $n := n+1$ (current number of cylinders to be placed).

Step 3. Set $r_n := 0.01$, $h_n := 0.01$ (initialization of metric characteristics of the cylinder to be placed).

Step 4. Choose $\mathbf{u}_n$ meeting the following system (see (3)):

$$\begin{aligned}
&\mathbf{n}_n \cdot \mathbf{n}_j - \gamma_{\min} \geq 0, \\
&\gamma_{\max} - \mathbf{n}_i \cdot \mathbf{n}_j \geq 0, \\
&j \in I_{n-1} = \{1, 2, ..., n-1\}, \\
&\mathbf{n}_n \cdot \mathbf{m}_l - \lambda_{\min} \geq 0, \\
&\lambda_{\max} - \mathbf{n}_i \cdot \mathbf{m}_l \geq 0, \ l \in J.
\end{aligned}$$

Step 5. Solve the following problem:

$$V^* = \pi \max_{\tau_n} \sum_{i \in I_n} r_n^2 h_n, \tag{1}$$

$$\tau = (\mathbf{u}_1, ..., \mathbf{u}_n, r_n, h_n, \boldsymbol{\alpha}_{1n}, ..., \boldsymbol{\alpha}_{(n-1)n}) \in W,$$

where

$$\begin{aligned}
W = \{&\tau \in \mathbf{R}^{6(n-1)+7} : \Phi_i(\mathbf{u}_i) - d > 0, i \in I_{n-1}, \\
&\Psi_n(\mathbf{u}_n, r_n, h_n) - d > 0, \\
&\Psi_{in}(\mathbf{u}_i, \mathbf{u}_n, \boldsymbol{\alpha}_{in}, r_n, h_n) - d \geq 0, \ i \in I_{n-1}, \\
&\mathbf{n}_n \cdot \mathbf{n}_i - \gamma_{\min} \geq 0, \ \gamma_{\max} - \mathbf{n}_n \cdot \mathbf{n}_i \geq 0, j \in I_{n-1}, \\
&\mathbf{n}_n \cdot \mathbf{m}_l - \lambda_{\min} \geq 0, \ \lambda_{\max} - \mathbf{n}_i \cdot \mathbf{m}_l \geq 0, \ l \in J, \\
&0 \leq r_i \leq r_i^0, \ 0 \leq h_i \leq h_i^0, \ i \in I_{n-1}\}
\end{aligned}, \tag{2}$$

where $\Psi_n(\mathbf{u}_n, r_n, h_n)$ is a normalized $\Phi$-function of for $C_n$ and $P^*$, $\Psi_{in}(\mathbf{u}_i, \mathbf{u}_n, \boldsymbol{\alpha}_{in}, r_n, h_n)$ is a normalized $\Phi$-function of for $C_i$ and $C_n$. Here, $\boldsymbol{\alpha}_{in} \in \mathbf{R}^3$, $i \in I_{n-1}$, is a vector of parameters specifying separating planes between $C_i$ and $C_n$.

Step 6. If $V^* = (r_n^0)^2 h_n$, then go to Step 2, otherwise, go to Step 7.

Step 7. Set $n^* := n-1$ and stop the algorithm.

The previous method consists of the multi-objective nonlinear programming approach (1)-(2) with both geometric and angular constraints. Due to the high dimensionality of the variables and the numerous constraint pairs that must be imposed to ensure avoidance and angular feasibility, solving this problem simultaneously for all cylinders is infeasible. To avoid this issue, we employ a decomposition approach that yields a significant reduction in computation time and cost, while maintaining a similar quality of solution. At each iteration, we formulate a local subproblem by restricting the movement of each cylinder to a bounding region, a rectangular (cuboidal) or spherical container, within which its position and orientation can vary. This localized formulation eliminates constraints between cylinders since two containers do not intersect it must follow that those two objects cannot interfere with one another. If cuboidal containers are used, we introduce at worst a constant number of additional constraints proportional to $6n$ , but those are linear constraints.

On the other hand, spherical containers require only $n$ additional constraints, and they are quadratic. In this paper, we adopt the same approach and assign each cylinder its own spherical container. This is a good compromise between the number of constraints we use and computational feasibility. We solve the subproblem to find a local extremum and use this point as the starting point for the next iteration. This iterative process enables the algorithm to scale efficiently with the number of objects, while maintaining high placement accuracy and satisfying all spatial and angular constraint requirements.

## V. NUMERICAL EXAMPLES AND ANALYSIS

The algorithm being studied was tested on several placement problems involving slim cylinders with an aspect ratio of 1:20 (i.e., a ratio of radius to height). The nonlinear programming problems were solved using the IPOPT solver [13]. The solutions to the placement problems were visualized three-dimensionally using the OpenGL graphics library. All computational experiments were conducted in C++ and executed on a machine with the following hardware specifications: Intel(R) Core(TM) i5-5300U CPU @ 2.30GHz and 8 GB RAM.

To assess the effect of angular constraints, one test case was completed with angular constraints and then repeated without them.

To assess the impact of angular constraints, a test case was completed with and without angular constraints.

Example 1. Baseline placement with spatial constraints only, with cylinders having radius 0.5 and half-height 10. The placement domain is a cuboid with sizes 19x19x19. Placement of the 24 and 30 cylinders is seen in Fig. 1, respectively. The corresponding run-times were approximately 3 and 5 minutes.

As it can be seen, there are nearly parallel cylinders and cylinders aligned with the cuboid faces. It will be demonstrated in the next example how this issue is addressed.

Example 2. Placement with angular constraints between cylinders. Sizes of cylinders and cuboid as in Example 1. Angular limitations are as follows: $\gamma_{\min} = \lambda_{\min} = 0.1$, $\gamma_{\max} = \lambda_{\max} = 0.9$ . 20 cylinders were placed. Illustration is shown in Fig. 2.

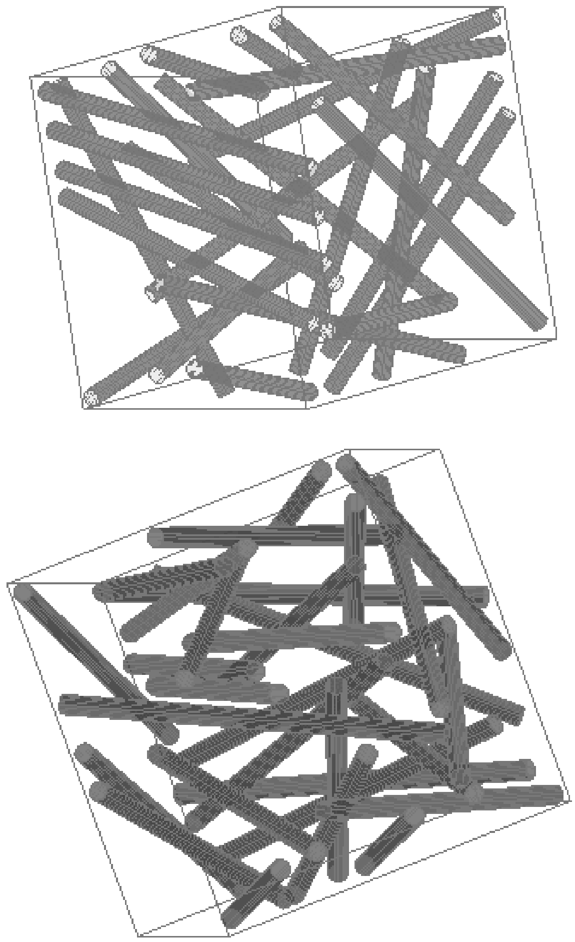

*Fig. 1. Placement of 24 and 30 cylinders.*

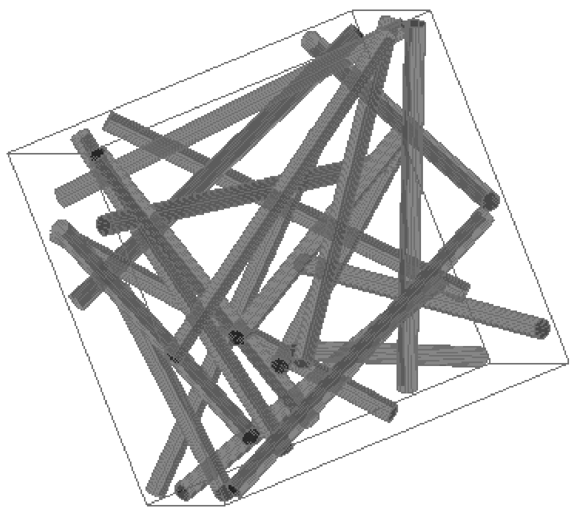

*Fig. 2. Placement of 20 cylinders with angular constraints.*

The addition of angular constraints greatly enhanced the spatial arrangement of the cylinders. Specifically, it averts the cylinders from being placed in a parallel manner with one another or the boundaries of the domain. This is an important aspect for potential interstitial medical applications, as a parallel trajectory of cylinders could lead to dose hotspots or thermal damage, affecting healthy tissue. Overall, the algorithm successfully achieved a minimum angular separation, resulting in more uniform spacings and configurations that are acceptable for clinical settings.

Example 3. Placement of 50 cylinders with radius 0.5 and height 10. Placement domain $P$ is cuboid with sizes 25x25x25. $\gamma_{\min} = \lambda_{\min} = 0.1$, $\gamma_{\max} = \lambda_{\max} = 0.9$. The illustration is shown in Fig. 3. The runtime was approximately 40 minutes.

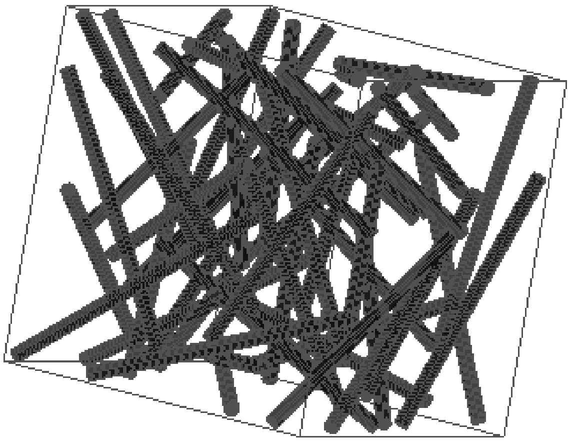

*Fig. 3. Placement of 50 cylinders with angular constraints.*

Through numerical experiments, several intriguing factors emerged. To position the cylinders at a distance apart, adding angular constraints inherently increased the distance. This finding is interesting because correctly controlling angles indirectly leads to a better distance, a critical factor in ensuring patient safety during medical procedures.

Our computational method also worked exceedingly well. The problem was simplified by using spherical containers, which broke it down into smaller areas, significantly reducing the computational requirements. The reduction enabled the algorithm to solve a large number of cylinders while maintaining a reasonable processing time, even when the cylinders were packed extremely closely together.

The verification steps for the actual placement results clearly demonstrated that the proposed method effectively avoids placing the object in problematic areas on the edges that doctors do not want. An optimization process was completed using a step-wise method to account for newly added and relocated cylinders, while maintaining high placements, despite the very restrictive geometric boundary conditions imposed during cylinder placement.

The unique contribution of the overall idea described is that the solution itself naturally balances several competing requirements, including maintaining safe distances, achieving the right insertion angles, and adhering to operational constraints, all of which are present in most medical procedures.

## VI. CONCLUSION

We proposed a formal optimization framework for placing multiple cylindrical shapes under both spatial and angular constraints. The motivation stems from the practical context of working under real-world constraints encountered in interstitial medical procedures, such as photodynamic therapy, brachytherapy, and the placement of implantable biosensors. The primary objective of the model is to determine a feasible placement for all cylinders within a bounded domain, while meeting certain clinically motivated constraints on minimum distance, angular separation, and a specific orientation with respect to the boundary. To accomplish this, we developed a nonlinear programming model, which accounts for both geometric and angular feasibility.

A key feature of the proposed approach is the idea that the radius and height of each cylinder are modeled as independent optimization variables, rather than treating the cylinder shapes as fixed parameters or scaling them uniformly. Of course, this flexibility in modeling the cylinders does not occur in a vacuum, as it is ultimately intended to allow feasible placement under a tight set of anatomical and procedural constraints.

The application of dimensional adaptation to each individual cylinder, based on suitable bounds, makes the creation of more feasible arrangements possible for densely packed or irregular loading domains.

To enable an efficient solution to the potentially high-dimensional and constraint-heavy optimization problem posed by this procedure, sequential placements were introduced. In the sequential placement procedure, one object enters the system at a time. For each object that enters the system, the goal is to maximize the resulting volume by rearranging all previously placed cylindrical arrangements.

Decoupling cylinder placements enables dynamic reconfiguration while maintaining an overall feasible system subject to additional placements.

To further reduce the computational burden, a decomposition method is proposed that divides the problem into individual spherical containers, decoupling the constraints of non-intersecting objects and eliminating constraints on active objects and auxiliary variables. Spherical regions yield fewer constraints than cuboidal containers, where only one constraint per object on any object, plus the quadratic forms, are perhaps still tractable within the resolved problem.

The resultant algorithm is both scalable and adaptable, effectively enabling applications that require precise geometric control within anatomical constraints. This also lays the groundwork for future efforts in automated treatment planning, as well as implant design and configuring devices in minimally invasive medicine. In future work, we aim to expand the model to incorporate true cylindrical geometries, rather than relying on polyhedral approximations. The framework may also be extended to include multi-objective optimization, such as balancing placement feasibility with dosimetric or functional performance criteria.

Furthermore, the proposed geometric framework can also be considered a baseline framework for future clinical applications. By incorporating other medical constraints, such as tissue heterogeneity, patient-specific anatomical information, or device-specific function, the model can be adapted to be relevant to both realistic treatment planning and device deployment considerations.

## VII. CONFLICTS OF INTEREST

The author declares no conflicts of interest.

## VIII. DECLARATION ON GENERATIVE AI

During the preparation of this work, the author used Grammarly in order to: Grammar and spelling check, Paraphrase and reword. After using this tool/service, the author reviewed and edited the content as needed and takes full responsibility for the content of the publication.

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**Andrii Chuhai** was born in Kharkiv, Ukraine, in 1980. He received the M.S. degree in applied mathematics in 2002 and the Ph.D. degree in applied mathematics in 2006. He received the Doctor of Technical Sciences (D.Sc.) degree in applied mathematics in 2019 at National Academy of Sciences of Ukraine, Kharkiv, Ukraine. He is currently a Professor at the Department of Cybersecurity and Information Technologies at the Simon Kuznets Kharkiv National University of Economics and a Leading Research Scientist at the Anatolii Pidhorniy Institute of Power Machines and Systems of the National Academy of Sciences of Ukraine. His research interests include applied mathematics, operations research, information technologies, cybersecurity, computational optimization methods, geometric modeling, relaxation techniques, and intelligent decision-support systems.