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# COMPUTER SIMULATION AND EXPERIMENTAL VERIFICATION OF RECTILINEAR MOTION TRAJECTORIES OF A SINGLE-MASS OSCILLATORY SYSTEM ACTUATED BY AN INERTIAL PLANETARY-TYPE VIBRATION EXCITER

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Abstract. Problem statement and purpose of research. This study investigates whether a planetary-type inertial exciter can synthesize rectilinear motion trajectories in a single-mass oscillatory system and how the orientation of that trajectory can be programmed mechanically. Methodology. A three-dimensional CAD assembly was created and analyzed using multibody simulations to compute planar displacements, velocities, and accelerations under representative stiffness, damping, and mass properties. The initial carrier-arm angle was used as the primary design variable. A laboratory rig with orthogonal potentiometric sensing measured horizontal and vertical displacements; corresponding trajectories in the vertical plane were reconstructed for multiple geometric configurations. Findings. Simulations predict, and experiments confirm, a one-to-one mapping between the angle being studied and the direction of the straight-line path: horizontal, vertical, or inclined. In all cases, the displacement components are nearly sinusoidal and largely in phase; the component aligned with the target direction dominates in amplitude, while the orthogonal component remains small, causing the Lissajous figure to collapse toward a line. Minor nonsmoothness in measured trajectories indicates high-frequency content from non-idealities (e.g., transmission compliance, local resonances), suggesting model extensions but not affecting the primary orientation control. Originality. The work demonstrates mechanism-level trajectory programming of rectilinear motion using a single-DOF planetary exciter, validated experimentally, thereby avoiding multi-actuator synchronization or semi-active control. Practical value. The results provide a simple, reproducible design lever – the initial arm angle – for setting line orientation in vibratory equipment requiring directional impulse transfer with minimal transverse motion. Future scope. Recommended directions of further research include tolerance and robustness analyses, incorporation of gear and belt compliance, and closed-loop trimming strategies to maintain rectilinearity under parameter drift.

**Keywords:** planetary-type vibration exciter, rectilinear trajectory, single-mass oscillatory system, trajectory orientation control, multibody simulation; experimental validation.

#### Introduction

Controlling the trajectory of the working member in vibratory machinery is a prerequisite for efficient transport, classification, and compaction processes. In practice, three canonical motion modes are typically used – linear, circular, and elliptical – and each mode leads to distinct particle kinematics and process outcomes on screens and conveyors. Linear trajectories favor directed transport and precise impulse transfer; circular and elliptical motions can improve stratification and reduce aperture blinding in screening applications [1, 2]. Against this backdrop, inertial vibration exciters based on planetary mechanisms have emerged as a promising class of actuators because they can synthesize rich, time-varying excitation patterns while remaining compact and mechanically robust. Recent studies have argued that planetary-type exciters are relatively novel and prospective for technological equipment and can be configured to generate circular, elliptical, and even rectilinear paths of the oscillating body [3, 4].

#### **Problem Statement**

Vibration-driven machinery – screens, conveyors, compactors, and feeders – derive their process performance largely from the law of motion of the working member. Among the many trajectory patterns used in practice (linear, elliptical, circular), a strictly rectilinear path is frequently desired because it simplifies transport kinematics, mitigates transverse drift, and can reduce sensitivity to loading. Foundational results in vibrational mechanics and averaging methods explain how trajectory shape, phase relations, and frequency content govern effective forcing and response of single- and multi-DOF oscillatory systems [5]. In industrial conveying, even small misalignments of the resultant excitation force relative to the body's mass center perturb the intended straight-line motion and cause material flow irregularities, underscoring the need for excitation schemes that robustly yield rectilinear trajectories [6].

For screening operations, trajectory matters just as much: extensive experimental and modeling studies show that bed stratification, dewatering, and anti-blinding behavior depend on the vibration path and its controllability [7, 8]. Linear trajectories are favored for uniform advance and sharp separations in many wet and sticky feeds, whereas circular or flip-flow motions can improve de-agglomeration at the expense of transport directionality. Hence, the ability to prescribe and maintain a targeted path – ideally with minimal actuators and simple mechanics – remains a central research objective.

#### Review of Modern Information Sources on the Subject of the Paper

Conventional inertial exciters based on one or more unbalanced rotors are widely used, and recent control advances (e.g., controlled synchronization of dual drives) improve phase locking and amplitude regulation to shape the trajectory envelope [9, 10]. Semi-active approaches using magnetorheological (MR) dampers placed in the suspension can even convert a circular path into a near-linear one by emulating a "virtual" secondary exciter through controlled forces, albeit at the cost of added control complexity and energy overhead [11]. These developments highlight two basic research directions: trajectory control can be achieved either by multi-actuator control or by embedding kinematic (or inertial) programmability into the exciter's mechanism.

A parallel stream of work has pursued trajectory shaping with various inertial mechanisms. Twin crank-type exciters and multi-rotor unbalanced drives have been used to switch among rectilinear, circular, and elliptical motions and to generate more complex Lissajous-like paths [12]. In screening science and conveyor engineering, trajectory control is not merely academic: changing the motion mode alters particle residence time, bed dilation, and transport velocity, which in turn affects throughput and separation quality. Recent studies show that composite/variable trajectories can equalize particle distribution, elongate particle paths, and raise efficiency compared with fixed linear motion [13, 14]. Likewise, conveyor dynamics research cautions that any drift of the resultant excitation line from the mass center degrades the desirability of an intended rectilinear path and leads to uneven material flow – underscoring the need for mechanism-level control of trajectory orientation and straightness [6].

Research on planetary-type exciters has progressed along three complementary lines. First, dynamic modeling has shown that providing adjustable (self-regulated) inertial parameters in the planetary train

enables on-the-fly tuning of operating regimes to meet technological requirements; this has been demonstrated on single-mass vibratory systems using Lagrange-d'Alembert formulations [15]. Second, kinematic synthesis has clarified how the geometry of the planetary mechanism maps to the instantaneous motion of the unbalanced masses and the resulting base excitations; dimensionless kinematic analogues have proved useful for parameter studies and scaling [4]. Third, recent force analyses have quantified reaction forces in joints and torque demands on carriers for planetary mechanisms with both internal and external gearing, providing design-level insights into stressors and energy requirements under different regimes [3]. In [16], there is shown that a symmetric planetary-type exciter can synthesize not only the canonical modes but also non-standard paths (triangular, rectangular, hexagonal) via suitable geometric and kinematic parameterization of the mechanism. Most recently, in [17], there is numerically explored how planetary-mechanism parameters map to the motion trajectories and kinematic characteristics of the oscillating body in a model single-mass system – demonstrating regions of parameter space that yield quasi-linear paths. Collectively, these works establish the feasibility of planetary-type exciters as versatile drives for vibratory machinery and motivate targeted trajectory synthesis.

Despite these advances, two gaps remain. First, while there is a rich theoretical-computational treatment of planetary-type exciters, systematic computer simulation coupled with experimental verification focused specifically on rectilinear trajectories of the single-mass system is limited. Published works on screening and conveying typically compare linear, circular, and elliptical modes or propose control strategies (e.g., MR-damper-based trajectory control) for conventional unbalanced exciters, but they do not resolve the parameter regimes in which a planetary-type inertial drive produces a straight-line path and how sensitive that rectilinearity is to geometric tolerances and phase relations [1, 11]. Second, from a design standpoint, there is a need to translate mechanism parameters into practical guidelines (e.g., amplitude ratio and phase conditions that collapse the Lissajous figure to a line, orientation control of that line, and robustness to model-plant mismatch), bridging the language of planetary gear geometry with the performance metrics used by machine designers and process engineers [6, 18].

#### **Objectives and Problems of Research**

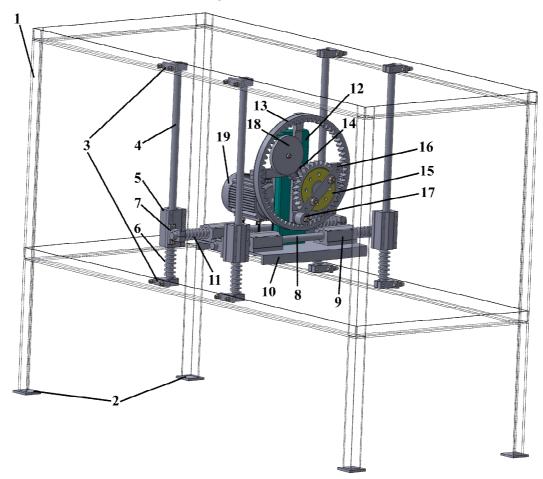
The present paper addresses these gaps by: 1) formulating and validating a computational pipeline (multibody/rigid-flexible modeling and time-domain simulation) for a single-mass oscillatory system actuated by an inertial planetary-type exciter; 2) defining quantitative rectilinearity metrics (e.g., aspect ratio and eccentricity of the trajectory locus, phase misalignment between orthogonal components) and mapping them over the space of mechanism geometry and operating speed; 3) experimentally verifying the predicted rectilinear regimes on a dedicated test bench with orthogonal displacement sensing, thereby closing the loop between model and hardware. The experimental methodology, including the layout of the planetary exciter mounted on the oscillating mass and the two-axis potentiometric sensing, follows and extends the authors' prior bench setup.

By focusing on straight-line trajectories and by combining simulation with controlled experiments, this study complements foundational vibrational-mechanics theory (averaging-based treatments of vibratory effects) with actionable design charts for planetary-type drives. The results are intended to support designers of conveyors, screens, and compactors who require directional impulse delivery with minimal transverse components and who seek mechanism-level levers – rather than suspension-level addons – to achieve and stabilize rectilinear motion.

## Simulation Model of the Single-Mass Oscillatory System

A three-dimensional digital mock-up of a single-mass oscillatory unit with a planetary-type inertial exciter was built in SolidWorks software and used as the basis for all simulations (Fig. 1). The structure consists of a rigid base frame 1 anchored to the foundation 2; vertical guide columns 4 carry linear bearings 5 preloaded by vertical springs 6 to define the out-of-plane compliance. The clamps 7 attached to these vertical carriages 5 supports a pair of horizontal rails 8 with additional linear bearings 9 and horizontal springs 11. The oscillating plate 10 – representing the working member of a vibratory machine –

is mounted on the horizontal carriages 9, so the plate 10 can undergo planar motion with well-defined elastic constraints in both axes. A bracket 12 fixed to the plate 10 holds the annulus (ring gear) 13 of the planetary mechanism. The carrier 14 rotates about its axis and drives a planet gear 15; a short lever 16 on the planet gear 15 carries an eccentric (unbalanced) mass 17. An optional counterweight 18 may be placed on the opposite side of the carrier 14 to statically trim the planet assembly. Rotary power from an electric motor 19 is delivered to the carrier 14 through a toothed-belt transmission.



**Fig. 1.** Three-dimensional digital mock-up of a single-mass oscillatory unit with a planetary-type inertial exciter: 1 – frame; 2 – foundation; 3, 7 – clamps; 4, 8 – guides; 5, 9 – linear bearings; 6, 11 – spring sets; 10 – oscillating plate; 12 – bracket; 13 – ring gear; 14 – carrier; 15 – planet gear; 16 – lever; 17 – eccentric (unbalanced) mass; 18 – counterweight; 19 – electric motor

During operation, the motor spins the carrier. The planet gear executes compound motion relative to the annulus, causing the eccentric mass to trace a path whose inertia forces are transmitted through the lever and carrier to the exciter bracket and, ultimately, to the oscillating plate. The plate, therefore, experiences a planar excitation whose resultant trajectory – linear, circular, elliptical, or polygonal – depends on the geometric ratios of the planetary set (gear radii, carrier arm length, eccentricity), the mass properties of the rotating elements, and the stiffness/damping of the supporting springs.

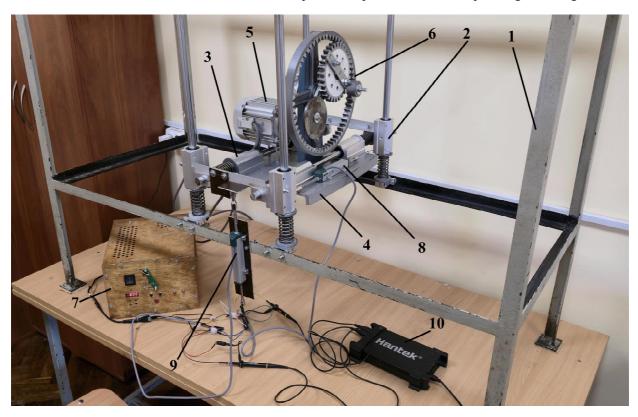
Kinematic and dynamic studies were carried out using SolidWorks Motion Analysis software. The CAD assembly was parameterized to allow rapid changes of planetary geometry (e.g., carrier arm, eccentric offset, satellite mass) and operating speeds (carrier and planet angular velocities). Spring and damping elements along the vertical and horizontal axes captured the compliance of the suspension. The solver produced time histories of plate displacement, velocity, and acceleration in both axes, as well as the corresponding Lissajous-type motion traces in the plate plane.

## Computer Simulation and Experimental Verification of Rectilinear Motion Trajectory...

Virtual experiments were organized as sweeps over key mechanism and excitation parameters. For each parameter set, the following outputs were recorded: 1) 2D trajectory of the plate's reference point (mass center); 2) synchronized time dependencies of its displacements, velocities, and accelerations x(t), y(t),  $\dot{x}(t)$ ,  $\dot{y}(t)$ ,  $\ddot{x}(t)$ ,  $\ddot{y}(t)$ ; 3) qualitative assessment of trajectory class (rectilinear, circular, elliptical, or polygonal). This workflow was used to substantiate the feasibility of synthesizing targeted motion paths of the working member with a single-DOF planetary exciter.

## **Experimental Test Bench of the Single-Mass Oscillatory System**

The experimental test bench (Fig. 2) is intended to investigate the dynamics of a single-mass oscillatory system driven by an inertial planetary-type vibration exciter. The structural basis is a massive frame 1, which ensures the stability and rigidity of the entire system during experiments. Mounted on this frame is the oscillating mass 4, the primary object of study. This mass is supported by a set of elastic elements that allow oscillations in two mutually perpendicular directions. The vertical elastic suspension 2, implemented by a set of vertically oriented helical cylindrical springs, enables motion along the vertical axis. At the same time, the horizontal motion elastic system 3, which likewise consists of similar spring elements, allows oscillations within the horizontal plane and provides the corresponding restoring force.



**Fig. 2.** Experimental test bench for studying the dynamics of a single-mass oscillatory system with a planetary-type vibration exciter: 1 – frame; 2 – vertical-motion elastic suspension; 3 – horizontal-motion elastic system; 4 – oscillating mass; 5 – drive electric motor; 6 – planetary-type vibration exciter; 7 – control unit; 8, 9 – potentiometric displacement sensors of the oscillating mass (horizontal and vertical); 10 – oscilloscope

Excitation of the oscillations of the mass 4 is provided by an inertial planetary-type vibration exciter 6 installed directly on the oscillating mass. The exciter is driven by an electric motor 5, also mounted on the oscillating mass and connected to the exciter by a toothed-belt (timing-belt) transmission. Control of the electric motor – specifically the rotational speed of the drive shaft, which determines the vibration frequency and intensity – is performed from the control unit 7. Potentiometric sensors of horizontal 8 and

vertical 9 displacement are used to register the motion parameters of the oscillating mass 4. These sensors convert the mechanical displacements of the mass into corresponding electrical signals. The acquired signals are fed to an oscilloscope 10, which enables real-time visualization of the displacement time histories of the oscillating mass, as well as recording and subsequent analysis of the experimental data. In this way, the test bench allows a comprehensive study of the dynamic behavior of a single-mass system under a controlled vibrational load generated by the planetary-type exciter.

### **Results of Computer Simulation**

Building on the authors' earlier studies [3, 4, 12, 15–17] and the developed 3D model of the single-mass oscillatory system equipped with a planetary-type vibration exciter, let us carry out computer simulations of the system response. Unless noted otherwise (see Fig. 2), the baseline parameters were: carrier angular speed  $\omega \approx 50$  rad/s (8 Hz); equivalent stiffness of the vertical and horizontal spring sets  $k \approx 30000$  N/m; equivalent viscous damping for both directions  $c \approx 100$  N·s/m; total oscillating mass  $M \approx 12$  kg; unbalanced (eccentric) mass  $m \approx 0.6$  kg; carrier arm length H = 0.06 m; pitch radius of the planet gear  $R_1 = 0.6$  m; pitch radius of the ring gear (annulus)  $R_0 = 0.12$  m; the initial angle of the arm holding the unbalanced body with respect to the carrier axis  $\varphi_1 = 0$ .

In order to simulate the horizontal rectilinear path of the oscillating mass, the initial angle of the carrier arm is set to zero [4]:  $\varphi_0 = 0^\circ$ . The vertical rectilinear path is described in the case when  $\varphi_0 = 90^\circ$ . In addition, let us consider the possibility of generating the rectilinear path inclined at an angle of 45° to the horizon:  $\varphi_0 = 45^\circ$ . The corresponding simulation results are shown in Fig. 3.

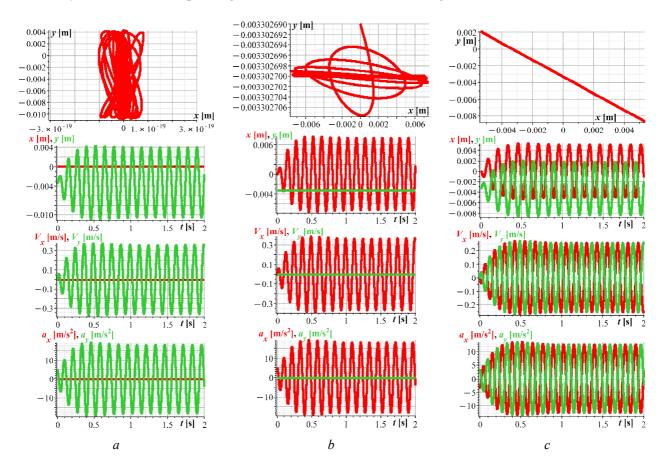


Fig. 3. Simulation results showing the motion trajectories, horizontal and vertical displacements, velocities, and accelerations of the oscillating mass under different geometrical parameters of the planetary-type excitation mechanism:  $a - \varphi_0 = 90^\circ$ ;  $b - \varphi_0 = 0^\circ$ ;  $c - \varphi_0 = 45^\circ$ 

Therefore, using the 3D model of a single-mass oscillatory system with a planetary-type exciter (see Fig. 1), the carried-out computer simulation fixed representative stiffness, damping, mass, and geometric parameters and then varied the initial carrier-arm angle  $\varphi_0$  to steer the motion path of the oscillating body. The obtained outputs include trajectories and the time histories of the horizontal and vertical displacements, as well as their first and second derivatives (velocities and accelerations).

The simulations show a clean one-to-one link between  $\varphi_0$  and the direction of the straight-line path:  $\varphi_0 = 90^\circ$  – near-vertical trajectory;  $\varphi_0 = 0^\circ$  – near-horizontal trajectory;  $\varphi_0 = 45^\circ$  – near-diagonal trajectory. This confirms that the exciter's geometry can mechanically program the trajectory orientation of the oscillatory system without additional actuators.

In each case, the horizontal (x(t)) and vertical (y(t)) components exhibit phase alignment consistent with a degenerate Lissajous figure (ellipse collapsing toward a line). The component aligned with the target direction dominates in amplitude, while the orthogonal component remains minimal, yielding a quasi-rectilinear locus. Velocities and accelerations follow the same trend, indicating consistency across kinematic orders.

Selecting  $\varphi_0$  provides a practical "trajectory dial" for designers: once the suspension and mass properties are fixed, the initial arm orientation can be used to set – and later reproduce – the desired straight-line direction on the working member. This is valuable for conveyors, screens, and compactors that require directional impulse transfer with minimal transverse motion.

Because the rectilinearity hinges on amplitude balance and phase relation, it may be sensitive to parameter drift (e.g., damping, stiffness tolerances, or speed). Therefore, the results motivate to perform further investigations containing a tolerance/sensitivity analysis and experimental checks to quantify acceptable bounds for maintaining the straight-line regime.

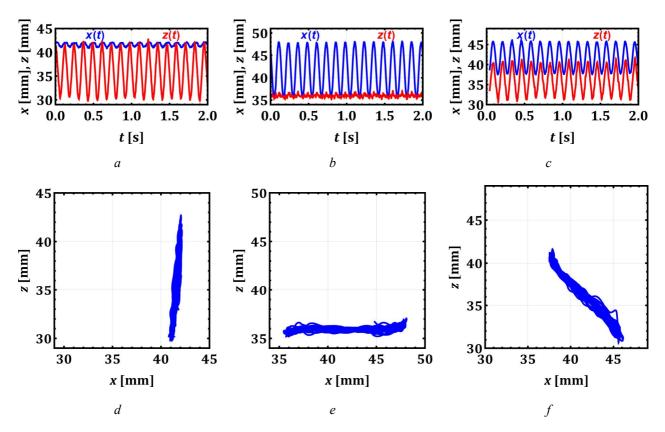
### **Results of Experimental Investigations**

Fig. 4 visualizes experimental data for the oscillations of a single-mass system excited by an inertial planetary vibration exciter under different geometric configurations of the exciter. The figure comprises two rows of plots. The top row – Fig. 4 (a-c) – shows the time histories of the horizontal displacement x(t) (blue curve) and the vertical displacement z(t) (red curve) of the oscillating mass; displacements are given in millimeters (mm) and time in seconds (s). The bottom row – Fig. 4 (d-f) – presents the corresponding motion trajectories of the mass in the vertical plane (xOz), with both axes likewise graduated in millimeters. Each trajectory plot in Fig. 4 (d-f) directly corresponds to the time histories in the panel immediately above it – Fig. 4 (a-c), respectively.

In the first case – illustrated by the plots in Fig. 4 (a) and Fig. 4 (d) – the horizontal x(t) and vertical z(t) displacements are oscillatory and close to sinusoidal, with the amplitude of the horizontal oscillations (along the Ox-axis) being substantially smaller than that of the vertical oscillations (along the Oz-axis). As a consequence, the motion trajectory (Fig. 4 (d) takes the form of a slender ellipse that approaches an almost vertical straight line, whose position is determined by the amplitude ratio and the in-phase relation of the oscillation components.

The second case – depicted in the plots of Fig. 4 (b) and Fig. 4 (e) – illustrates a situation where the amplitude of the horizontal oscillations x(t) far exceeds that of the vertical oscillations z(t), which is comparatively minimal. As a result, the trajectory (Fig. 4 (e) forms a narrow rectangle, strongly elongated along the horizontal Ox-axis – i.e., it approaches an almost horizontal straight line – indicating a predominantly horizontal character of the oscillating mass's motion.

The third case – shown in the plots of Fig. 4 (c) and Fig. 4 (f) – demonstrates that both displacement components, x(t) and z(t), have comparatively large and approximately equal amplitudes. A key feature of the time histories in Fig. 4 (c) is the absence of a pronounced phase shift between x(t) and z(t). Consequently, the motion trajectory (Fig. 4 (f) resembles a tilted, slender ellipse approaching a straight line. In this case, the ellipse's (line's) shape and orientation are directly determined by the amplitude ratio and any phase offset between the oscillatory components.



**Fig. 4.** Experimentally obtained time histories of the horizontal x(t) and vertical z(t) displacements of the oscillating mass (a-c), and the corresponding motion trajectories (d-f), under different parameter settings of the planetary-type vibration exciter

Overall, Fig. 4 effectively illustrates the flexibility of the planetary-type vibration exciter in shaping diverse motion trajectories of the oscillatory system by varying its geometric parameters. The experiments that were conducted demonstrate the feasibility of achieving quasi-linear trajectories of a single-mass oscillatory system with different orientations in the vertical plane. The slight irregularity (non-smoothness) observed in the experimental trajectories may indicate the influence of high-frequency components and merits further investigation. These results are of practical importance for the design and optimization of vibration equipment that requires precise formation of the working members' vibration trajectories to efficiently perform technological operations such as conveying, separation, and compaction.

#### **Discussion**

Agreement between model and experiment. The simulations predict that adjusting the initial carrier-arm angle  $\varphi_0$  produces quasi-rectilinear trajectories whose orientation is set mechanically:  $\varphi_0 = 90^\circ$  yields an almost vertical line;  $\varphi_0 = 0^\circ$  – an almost horizontal line;  $\varphi_0 = 45^\circ$  – a near-diagonal line. Bench tests reproduce these regimes: when one component dominates, the trajectory collapses toward the corresponding axis; when the amplitudes are comparable and phases are aligned, the locus degenerates to a tilted, slender ellipse approaching a line. This cross-validation supports the use of the planetary mechanism as a one-DOF "trajectory programmer".

*Kinematic signatures and rectilinearity.* In all three representative cases, x(t) and z(t) (y(t)) are close to sinusoidal and largely in phase, consistent with a degenerate Lissajous figure. The component aligned with the intended direction has the higher amplitude, while the orthogonal component remains small. Velocity and acceleration signals show the same structure, indicating internal consistency across kinematic orders and reinforcing the mechanism-level origin of the straight-line motion.

Sources of discrepancy. Minor roughness observed in experimental trajectories points to high-frequency content not captured in the nominal linear model (e.g., transmission compliance, local

resonances, backlash, sensor noise, etc.). These effects are small in magnitude and do not alter the primary orientation control, but they motivate further development and analysis of an extended mathematical model including non-idealities and frequency-rich excitations.

**Design implications.** Once suspension stiffness/damping and mass properties are fixed,  $\varphi_0$  acts as a reproducible "dial" for line orientation, enabling directional impulse delivery with minimal transverse components – valuable for conveyors, screens, and compactors. The sensitivity of rectilinearity to amplitude balance and phase suggests practical tolerances for manufacturing (gear radii, eccentricity, belt tension) and operating conditions (speed, load, etc.).

**Limitations and outlook.** The current model treats damping as linear and neglects gear compliance and frictional nonlinearities. Future work should quantify tolerance windows for rectilinearity, incorporate drivetrain elasticity and backlash, and explore closed-loop trimming to compensate slow parameter drift.

#### **Conclusions**

A planetary-type inertial exciter can generate quasi-rectilinear motion of a single-mass oscillatory system, with orientation set mechanically by the initial carrier-arm angle  $\varphi_0$ . Simulation and experiment are in strong qualitative agreement: vertical, horizontal, and diagonal straight-line regimes are achieved by  $\varphi_0 = 90^\circ$ ,  $\varphi_0 = 0^\circ$ , and  $\varphi_0 = 45^\circ$ , respectively. The observed phase alignment and amplitude dominance along the target direction explain the degeneracy of the Lissajous locus to a near-line, confirmed by displacement, velocity, and acceleration data. Minor trajectory roughness arises from high-frequency effects and non-idealities; these do not compromise orientation control but indicate opportunities for refined modeling. The approach provides a single-DOF, actuator-minimal strategy for trajectory programming in vibratory machinery, easing implementation compared with multi-motor synchronization or semi-active damping solutions. Recommended next steps include tolerance/sensitivity studies, incorporation of drivetrain compliance, and experimental robustness tests under load and speed variations.

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