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Hilbert Transform of biperiodically nonstationary random signals

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An analysis of the covariance and spectral structure of the Hilbert transform of biperiodically nonstationary random processes, which model signals with double rhythmicity, is presented here. The obtained relations connect the cross-covariance and cross-spectral characteristics of the signal and its Hilbert transform with the characteristics of the signal itself. We examine the properties of the analytic signal and present characteristic special cases determined by the spectral features of carrier-harmonic modulation.

Keywords: *biperiodically nonstationary random process; Hilbert transform; covariance and spectral components; analytic signal.*

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1. Introduction

In the analysis of signals of both natural and artificial origin, cases often occur when the stochastic repeatability of one period interacts with the repeatability of another [1,2]. In communication systems, for example, repeatability is manifested, determined by the periodicity of the carrier, as well as the rhythmic variability of the modulating signal [3–5]. In electrical power systems, this is daily, weekly, and annual repeatability [1]. In vibration signals generated by mechanical systems, birhythmic variability is caused by different rotation speeds of elements of rotating units [6–12]. A probabilistic model of double rhythmicity is a biperiodically nonstationary random processes (BPNRP) [1, 7, 8]. The covariance-spectral structure of BPNRP is determined by jointly stationary processes that model the carrier harmonics, whose frequencies are linear combinations of two basic frequencies. It has been shown in the works [12–15] that for the analysis of stochastic modulation in the case when the carrier harmonics are characterized by one basic frequency and its multiples, the Hilbert transform can be used. It was found that the properties of the Hilbert transform of both single-component and multi-component periodically nonstationary random signal (PNRS) depend on the frequency properties of the modulating processes and significantly differ from each other under low- and high-frequency modulation of carriers. The latter occurs with the appearance of local defects in rotating mechanisms [7,13,14]. The studies carried out in [12–15] showed that the envelope method, which is still widely used for detecting and analyzing defects [16–22], was found to be inaccurate. The sum of the square of the signal and its Hilbert transform, i.e., the square of the modulus of the analytic signal, cannot be considered as the square of the envelope, since the properties of the Hilbert transform and the signal itself are the same. The obtained results have changed the principles governing the use of the Hilbert transform in vibration diagnostics [13, 15]. Since stochastic variability with double rhythmicity is characteristic of vibrations of many defective mechanisms, the problem of establishing the characteristic features of the Hilbert transform of BPNRP that describe such vibrations is important. The general properties of the Hilbert transform of a BPNRP signal are analyzed in this article.

2. Covariance and spectral properties of BPNRP

The mean function of a BPNRP $m_\xi(t) = E\xi(t)$ and its covariance function $b_\xi(t,u) = E\overset{\circ}{\xi}(t)\overset{\circ}{\xi}(t+u)$, $\overset{\circ}{\xi}(t) = \xi(t) - m_\xi(t)$, where E is the mathematical expectation operator, are biperiodic functions of time and can be represented by Fourier series [1,7]:

$$m_\xi(t) = \sum_{k,l\in\mathbb{Z}} m_{kl}^{(\xi)} e^{i\omega_{kl}t} = m_{00}^{(\xi)} + \sum_{k=0}^{\infty}\sum_{l=1}^{\infty}\left(m_{kl}^c \cos\omega_{kl}t + m_{kl}^s \sin\omega_{kl}t\right) + \sum_{k=1}^{\infty}\sum_{l=0}^{\infty}\left(m_{k,-l}^c \cos\omega_{k,-l}t + m_{k,-l}^s \sin\omega_{k,-l}t\right), \tag{1}$$

$$b_\xi(t,u) = \sum_{k,l\in\mathbb{Z}} B_{kl}^{(\xi)}(u) e^{i\omega_{kl}t} = B_{00}^{(\xi)}(u) + \sum_{k=0}^{\infty}\sum_{l=1}^{\infty}\left[B_{kl}^c(u) \cos\omega_{kl}t + B_{kl}^s(u) \sin\omega_{kl}t\right] + \sum_{k=1}^{\infty}\sum_{l=0}^{\infty}\left[B_{k,-l}^c(u) \cos\omega_{k,-l}t + B_{k,-l}^s(u) \sin\omega_{k,-l}t\right]. \tag{2}$$

Here $\mathbb{Z}$ is the set of integers, $m_{kl}^{(\xi)} = (m_{kl}^c - i m_{kl}^s)/2$ and $B_{kl}^{(\xi)}(u) = (B_{kl}^c(u) - iB_{kl}^s(u))/2$, $m_{-k,-l}^{(\xi)} = \bar{m}_{kl}^{(\xi)}$, $B_{-k,-l}^{(\xi)}(u) = \bar{B}_{kl}^{(\xi)}(u)$, where "$^-$" denotes conjugation, $\omega_{kl} = k\,2\pi/P_1 + l\,2\pi/P_2$, P_1 and P_2 are periods. The process $\xi(t)$ can be represented in the form of a stochastic series:

$$\xi(t) = \sum_{k,l\in\mathbb{Z}} \xi_{kl}(t) e^{iw_{kl}t}, \tag{3}$$

where $\xi_{kl}(t) = \frac{1}{2}\left[\xi_{kl}^c(t) - \xi_{kl}^s(t)\right]$, $\xi_{-k,-l}(t) = \bar{\xi_{kl}}(t)$, are jointly stationary random processes. From the series (3) it follows that a BPNRP can be considered as a superposition of amplitude- and phase-modulated harmonics, whose frequencies are linear combinations of the basic frequencies $\omega_{10} = 2\pi/P_1 = 2\pi f_{10}$ and $\omega_{01} = 2\pi/P_2 = 2\pi f_{01}$. The mathematical expectations of the modulating processes $\xi_{kl}(t)$ are the Fourier coefficients of the function $m_\xi(t)$: $E\xi_{kl}(t) = m_{kl}$. The cross-covariance functions of the modulating processes $R_{pqmn}(u) = E\bar{\overset{\circ}{\xi}}_{pq}(t)\overset{\circ}{\xi}_{mn}(t+u)$, where $\overset{\circ}{\xi}_{pq}(t) = \xi_{pq}(t) - m_{pq}$ determine the Fourier coefficients of the covariance function:

$$B_{kl}^{(\xi)}(u) = \sum_{p,q\in\mathbb{Z}} R_{p-k,q-l,p,q}(u) e^{i\omega_{pq}u}. \tag{4}$$

The quantities (4) are called covariance components. The zero$^{\text{th}}$ covariance component $B_{00}^{(\xi)}(u)$, as can be seen from (4), is determined by the autocovariance functions $R_{pq}(u) = \bar{\overset{\circ}{\xi}}_{pq}(t)\overset{\circ}{\xi}_{pq}(t+u)$:

$$B_{00}^{(\xi)}(u) = \sum_{p,q\in\mathbb{Z}} R_{pq}(u) e^{i\omega_{pq}u}. \tag{5}$$

The quantity (5) is the time-averaged value of the covariance function (2), i.e., it is the covariance function of the stationary approximation of the BPNRP. Its Fourier transform (the zero$^{\text{th}}$ spectral component)

$$f_{00}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} B_{00}^{(\xi)}(u) e^{-i\omega u} du = \sum_{p,q\in\mathbb{Z}} f_{pq}^{(m)}(\omega - \omega_{pq})$$

defines the spectral decomposition of BPNRP. Here, the quantities

$$f_{pq}^{(m)}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{pq}(u) e^{-i\omega u} du$$

are the power spectral densities of the modulating processes. The non-zero spectral components

$$f_{kl}^{(\xi)}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} B_{kl}(\xi) e^{-i\omega u} du \tag{6}$$

characterize the covariances of the spectrum, shifted by the amount ω_{kl}. These covariances are the result of the modulating processes in the stochastic series (3):

$$f_{kl}^{(\xi)}(\omega) = \sum_{p,q\in\mathbb{Z}} f_{p-k,q-l,p,q}^{(m)}(\omega - \omega_{pq}),$$

where

$$f_{pqmn}^{(m)}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{pqmn}(u)e^{-i\omega u}du.$$

From the above, it follows that the covariance-spectral structure of a BPNRP is determined by the amplitude-phase modulation of the carrier harmonics, which is described by jointly stationary random processes $\xi_k(t)$. For the analysis of stochastic modulation in the case of single-period rhythmicity, when the frequencies of the carrier harmonics are multiples of one basic frequency, the Hilbert transform is used. The properties of the signal are then described by the moment functions of periodically nonstationary random processes (PNRP). The analysis of the Hilbert transform of a multi-component PNRP in the general case, as well as separately for low- and high-frequency modulation, has been performed in the works [12–15]. In this article, we will consider the main features of this transformation of a signal that is described by a BPNRP.

3. Hilbert transform

Let us assume the random process $\xi(t)$ to have a zero$^{\text{th}}$ constant component $m_{00} = 0$. Then, there exists the Hilbert transform

$$\eta(t) = H\{\xi(t)\} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\xi(\tau)}{t-\tau}d\tau, \tag{7}$$

and for its mathematical expectation, we have:

$$m_\eta(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m_\xi(t)}{t-\tau}d\tau.$$

After substituting into this formula series (1), we obtain:

$$m_\eta(t) = \sum_{k=0}^{\infty}\sum_{l=1}^{\infty}(m_{kl}^c \sin\omega_{kl}t - m_{kl}^s\cos\omega_{kl}t) + \sum_{k=1}^{\infty}\sum_{l=0}^{\infty}\left(m_{k,-l}^c \sin\omega_{k,-l}t - m_{k,-l}^s\cos\omega_{k,-l}t\right),$$

Here it is taken into account that the Hilbert transform shifts the phases of the harmonics of the mathematical expectation (1) by $-\frac{\pi}{2}$.

The mathematical expectation of the analytic signal $\zeta(t) = \xi(t) + i\eta(t)$ then has the form:

$$m_\zeta(t) = 2\left[\sum_{k=0}^{\infty}\sum_{l=1}^{\infty} m_{kl}e^{i\omega_{kl}t} + \sum_{k=1}^{\infty}\sum_{l=0}^{\infty} m_{k,-l}e^{i\omega_{k,-l}t}\right].$$

Since the inverse Hilbert transform is

$$\xi(t) = -\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m_\eta(\tau)}{t-\tau}\,d\tau,$$

then

$$m_\xi(t) = -\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m_\eta(\tau)}{t-\tau}\,d\tau.$$

Thus

$$\overset{\circ}{\xi}(t) = \xi(t) - m_\xi(t) = -\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\overset{\circ}{\eta}(\tau)}{t-\tau}\,d\tau, \tag{8}$$

$$\overset{\circ}{\eta}(t) = \eta(t) - m_\eta(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\overset{\circ}{\xi}(\tau)}{t-\tau}\,d\tau. \tag{9}$$

For the covariance function of process (8) we then find:

$$b_\xi(t,u) = -\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{b_{\xi\eta}(t+u,\tau-t-u)}{t-\tau}\,d\tau.$$

After substituting $\tau - t - u = \tau_1$ we get:

$$b_\xi(t,u) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{b_{\xi\eta}(t+u,\tau_1)}{u+\tau_1}\, d\tau_1. \tag{10}$$

Similarly, based on (9) we obtain:

$$b_\eta(t,u) = -\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{b_{\eta\xi}(t+u,\tau)}{u+\tau}\, d\tau. \tag{11}$$

Using (8) and (9), we arrive at the following representations for the cross-covariance functions:

$$b_{\xi\eta}(t,u) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{b_\eta(t+u,\tau)}{\tau+u}\, d\tau, \tag{12}$$

$$b_{\eta\xi}(t,u) = -\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{b_\xi(t+u,\tau)}{\tau+u}\, d\tau. \tag{13}$$

Let us formulate a theorem.

Theorem 1. *A BPNRP $\xi(t)$, whose covariance function is represented by the series (2), and its Hilbert transform (7) are jointly biperiodically nonstationary processes, and their auto- and cross-covariance components are related by the ratios:*

$$B_{kl}^{(\xi\eta)}(u) = \int_{-\infty}^{\infty} h(u-\tau)B_{kl}^{(\xi)}(\tau)d\tau, \tag{14}$$

$$B_{kl}^{(\eta\xi)}(u) = -\int_{-\infty}^{\infty} h(u-\tau)B_{kl}^{(\eta)}(\tau)d\tau,$$

$$B_{kl}^{(\xi)}(u) = -\int_{-\infty}^{\infty} h(u-\tau)B_{kl}^{(\xi\eta)}(\tau)d\tau, \tag{15}$$

$$B_{kl}^{(\eta)}(u) = \int_{-\infty}^{\infty} h(u-\tau)B_{kl}^{(\eta\xi)}(\tau)d\tau.$$

where $h(\tau) = (\pi\tau)^{-1}$ is the impulse response of the Hilbert transform, which means that the covariance components $B_{kl}^{(\xi)}(u)$ and $B_{kl}^{(\xi\eta)}(u)$, as well as $B_{kl}^{(\eta\xi)}(u)$ and $B_{kl}^{(\eta)}(u)$, are Hilbert pairs.

Proof. Substitute into equation (13) the Fourier series (2). Then we get:

$$b_{\eta\xi}(t,u) = \sum_{k,l\in\mathbb{Z}} e^{i\omega_{kl}t}\left[-\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{B_{kl}^{(\xi)}(\tau)}{\tau+u}d\tau\right]e^{i\omega_{kl}u}$$

From this it follows that the cross-covariance components $B_{kl}^{(\eta\xi)}(u)$ are determined by the formula:

$$B_{kl}^{(\eta\xi)}(u) = -\frac{e^{i\omega_{kl}u}}{\pi}\int_{-\infty}^{\infty}\frac{B_{kl}^{(\xi)}(\tau)}{\tau+u}\, d\tau,$$

and then

$$B_{kl}^{(\eta\xi)}(-u) = \frac{e^{-i\omega_{kl}u}}{\pi}\int_{-\infty}^{\infty}\frac{B_{kl}^{(\xi)}(\tau)}{u-\tau}\, d\tau,$$

Since $b_{\eta\xi}(t,u) = E\overset{\circ}{\eta}(t)\overset{\circ}{\xi}(t+u) = b_{\xi\eta}(t+u,-u)$, then $B_{kl}^{(\eta\xi)}(-u) = B_{kl}^{(\xi\eta)}(u)e^{-i\omega_{kl}u}$. This means that

$$B_{kl}^{(\xi\eta)}(u) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{B_{kl}^{(\xi)}(\tau)}{u-\tau}d\tau = \int_{-\infty}^{\infty} h(u-\tau)B_{kl}^{(\xi)}(\tau)\, d\tau. \tag{16}$$

From relation (12), it follows that

$$B_{kl}^{(\xi\eta)}(u) = \left[\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{B_{kl}^{(\eta)}(\tau)}{\tau+u}d\tau\right]e^{i\omega_{kl}u}, \quad \text{or} \quad B_{kl}^{(\xi\eta)}(u)e^{-i\omega_{kl}u} = B_{kl}^{(\eta\xi)}(-u) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{B_{kl}^{(\eta)}(\tau)}{\tau+u}\, d\tau.$$

Hence

$$B_{kl}^{(\eta\xi)}(u) = -\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{B_{kl}^{(\eta)}(\tau)}{u-\tau}\, d\tau = -\int_{-\infty}^{\infty} h(u-\tau)B_{kl}^{(\eta)}(\tau)\, d\tau. \tag{17}$$

Taking into account expressions (10) and (11), we find:

$$B_{kl}^{(\xi)}(u) = \frac{e^{i\omega_{kl}u}}{\pi} \int_{-\infty}^{\infty} \frac{B_{kl}^{(\xi\eta)}(\tau)}{\tau + u}\, d\tau,$$

$$B_{kl}^{(\eta)}(u) = -\frac{e^{i\omega_{kl}u}}{\pi} \int_{-\infty}^{\infty} \frac{B_{kl}^{(\xi\eta)}(\tau)}{\tau + u}\, d\tau,$$

and from here

$$B_{kl}^{(\xi)}(u) = -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{B_{kl}^{(\xi\eta)}(\tau)}{u - \tau} d\tau = \int_{-\infty}^{\infty} h(u-\tau) B_{kl}^{(\xi\eta)}(\tau)\, d\tau, \tag{18}$$

$$B_{kl}^{(\eta)}(u) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{B_{kl}^{(\eta\xi)}(\tau)}{u - \tau} d\tau = \int_{-\infty}^{\infty} h(u-\tau) B_{kl}^{(\eta\xi)}(\tau)\, d\tau. \tag{19}$$

■

The relations (16) and (18), (17) and (19) indicate that the auto- and cross-covariance components $B_{kl}^{(\xi)}(u)$ and $B_{kl}^{(\xi\eta)}(u)$, as well as $B_{kl}^{(\eta\xi)}(u)$ and $B_{kl}^{(\eta)}(u)$ are Hilbert pairs. Based on the convolution theorem in the frequency domain, we have:

$$f_{kl}^{(\xi\eta)}(\omega) = H(\omega) f_{kl}^{(\xi)}(\omega), \tag{20}$$

$$f_{kl}^{(\xi)}(\omega) = -H(\omega) f_{kl}^{(\xi\eta)}(\omega), \tag{21}$$

$$f_{kl}^{(\eta)}(\omega) = H(\omega) f_{kl}^{(\eta\xi)}(\omega), \tag{22}$$

$$f_{kl}^{(\eta\xi)}(\omega) = -H(\omega) f_{kl}^{(\eta)}(\omega). \tag{23}$$

Here $H(\omega)$ is the transfer function: $H(\omega) = -i$ for $\omega > 0$ and $H(\omega) = i$ for $\omega < 0$, and the auto- and cross-covariance components are determined by formula (6), as well as

$$f_{kl}^{(\eta)}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} B_{kl}^{(\xi)}(u) e^{-i\omega u} du, \qquad f_{kl}^{(\xi\eta)}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} B_{kl}^{(\xi\eta)}(u) e^{-i\omega u} du, \tag{24}$$

$$f_{kl}^{(\eta\xi)}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} B_{kl}^{(\xi\eta)}(u) e^{-i\omega u} du. \tag{25}$$

Since the auto- and cross-covariance components satisfy the conditions

$$B_{kl}^{(\xi,\eta)}(-u) = B_{kl}^{(\xi,\eta)}(u) e^{i\omega_{kl}u}, \qquad B_{kl}^{(\xi\eta)}(-u) = B_{kl}^{(\eta\xi)}(u) e^{i\omega_{kl}u}, \tag{26}$$

then for the auto- and cross-spectral components we obtain:

$$f_{kl}^{(\xi,\eta)}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} B_{kl}^{(\xi,\eta)}(-u) e^{-i\omega u} du = \frac{1}{2\pi} \int_{-\infty}^{\infty} B_{kl}^{(\xi,\eta)}(u) e^{-i(\omega+\omega_{kl})u} du = f_{kl}^{(\xi,\eta)}(\omega + \omega_{kl}),$$

$$f_{kl}^{(\xi\eta)}(-\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} B_{kl}^{(\xi\eta)}(-u) e^{-i\omega u} du = \frac{1}{2\pi} \int_{-\infty}^{\infty} B_{kl}^{(\eta\xi)}(u) e^{-i(\omega+\omega_{kl})u} du = f_{kl}^{(\eta\xi)}(\omega + \omega_{kl}).$$

From relations (6), (24) and (25) after considering the equalities

$$B_{-k,-l}^{(\xi,\eta)}(u) = \bar{B}_{kl}^{(\xi,\eta)}(u), \quad B_{-k,-l}^{(\xi\eta)}(u) = \bar{B}_{kl}^{(\xi\eta)}(u).$$

It also follows that:

$$f_{-k,-l}^{(\xi,\eta)}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{B}_{kl}^{(\xi,\eta)}(u) e^{-i\omega u} du = \bar{f}_{kl}^{(\xi,\eta)}(-\omega) = \bar{f}^{(\xi,\eta)}(\omega + \omega_{kl}),$$

$$f_{-k,-l}^{(\xi\eta)}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{B}_{kl}^{(\xi\eta)}(u) e^{-i\omega u} du = \bar{f}_{kl}^{(\xi\eta)}(-\omega) = \bar{f}^{(\eta\xi)}(\omega + \omega_{kl}). \tag{27}$$

We use the properties of the spectral components and relations (20)–(23) to establish the links between the covariance and spectral characteristics of the signal and its Hilbert transform.

Theorem 2. *The zero*$^{\text{th}}$ *covariance component of a BPNRP signal and its Hilbert transform are equal, and their zero*$^{\text{th}}$ *cross-covariance components differ only by sign, they are odd functions and are determined by the formula:*

$$B_{00}^{(\xi\eta)}(u) = 2 \int_{0}^{\infty} f_{00}^{(\xi)}(\omega) \sin \omega u \, d\omega.$$

Proof. From equality (20), and considering that $f_{00}^{(\eta\xi)}(\omega)=\bar{f}_{00}^{(\xi\eta)}(\omega)$, we have:

$$f_{00}^{(\eta)}=H(\omega)\bar{f}_{00}^{(\xi\eta)}(\omega).$$

Substituting into this formula equation (20), we arrive at the equality:

$$f_{00}^{(\eta)}=H(\omega)\bar{H}(\omega)f_{00}^{(\xi)}(\omega)=|H(\omega)|^2\,f_{00}^{(\xi)}(\omega).$$

Since $|H(\omega)|^2=1$, then $f_{00}^{(\eta)}(\omega)=f_{00}^{(\xi)}(\omega)$, hence $B_{00}^{(\eta)}(u)=B_{00}^{(\xi)}(u)$.

For the zero$^{\text{th}}$ cross-covariance component, taking into account relation (20) we obtain:

$$\begin{aligned}B_{00}^{(\xi\eta)}(u)&=\int_{-\infty}^{\infty}f_{00}^{(\xi\eta)}(\omega)e^{i\omega u}d\omega=\int_{-\infty}^{\infty}H(\omega)f_{00}^{(\xi)}(\omega)e^{i\omega u}d\omega\\&=i\int_{-\infty}^{0}f_{00}^{(\xi)}(\omega)e^{i\omega u}d\omega-i\int_{0}^{\infty}f_{00}^{(\xi)}(\omega)e^{i\omega u}d\omega\\&=i\int_{0}^{\infty}\left[f_{00}^{(\xi)}(-\omega)e^{-i\omega u}-f_{00}^{(\xi)}(\omega)e^{i\omega u}\right]d\omega.\end{aligned}$$

Since $f_{00}^{(\xi)}(-\omega)=f_{00}^{(\xi)}(\omega)$, then

$$B_{00}^{(\xi\eta)}(u)=2\int_{0}^{\infty}f_{\xi}(\omega)\sin\omega u\,d\omega. \tag{28}$$

From the formula

$$B_{00}^{(\eta\xi)}(u)=\int_{-\infty}^{\infty}f_{00}^{(\eta\xi)}(\omega)e^{i\omega u}d\omega$$

and the equalities

$$f_{00}^{(\eta\xi)}(\omega)=-H(\omega)f_{00}^{(\eta)}(\omega),\quad f_{00}^{(\eta)}(\omega)=f_{00}^{(\xi)}(\omega),$$

we have

$$B_{00}^{(\eta\xi)}(u)=-\int_{-\infty}^{\infty}H(\omega)f_{00}^{(\xi)}(\omega)e^{i\omega u}\,d\omega,$$

and from here

$$B_{00}^{(\eta\xi)}(u)=-2\int_{0}^{\infty}f_{00}^{(\xi)}(\omega)\sin\omega u\,d\omega. \tag{29}$$

From formulas (28) and (29) it is clear that the zero$^{\text{th}}$ cross-covariance components are odd functions and $B_{00}^{(\xi\eta)}(u)=-B_{00}^{(\eta\xi)}(u)$. ■

Theorem 3. *The cross-covariance function of a BPNRP signal and its Hilbert transform, as well as the autocovariance function of the latter, vary biperiodically with time, and their Fourier coefficients are determined by the formulas:*

$$B_{kl}^{(\eta)}(u)=\int_{-\infty}^{0}f_{kl}^{(\xi)}(\omega)e^{i\omega u}d\omega-\int_{0}^{\omega_{kl}}f_{kl}^{(\xi)}(\omega)e^{i\omega u}d\omega+\int_{\omega_{kl}}^{\infty}f_{kl}^{(\xi)}(\omega)e^{i\omega u}d\omega,$$

$$B_{kl}^{(\xi\eta)}(u)=i\int_{0}^{\infty}\left[f_{kl}^{(\xi)}(\omega+\omega_{kl})e^{-i\omega u}-f_{kl}^{(\xi)}(\omega)e^{i\omega u}\right]d\omega.$$

Proof. Taking into account the equalities $H(\omega)=-H(\omega)$, $f_{kl}^{(\xi\eta)}(-\omega)=f_{kl}^{(\xi\eta)}(\omega+\omega_{kl})$ and (22) for the covariance component $B_{kl}^{(\eta)}(u)$ we obtain:

$$B_{kl}^{(\eta)}(u)=-\int_{-\infty}^{\infty}H(\omega)f_{kl}^{(\eta\xi)}(-\omega)e^{-i\omega u}d\omega=-\int_{-\infty}^{\infty}H(\omega)f_{kl}^{(\xi\eta)}(\omega+\omega_{kl})e^{-i\omega u}d\omega.$$

Introduce a new integration variable $\nu=\omega+\omega_{kl}$ and take into account the equality $f_{kl}^{(\xi\eta)}(\omega)=H(\omega)f_{kl}^{(\xi)}(\omega)$. Then

$$B_{kl}^{(\eta)}(u)=-e^{i\omega_{kl}u}\int_{-\infty}^{\infty}f_{kl}^{(\xi)}(\omega)H(\omega)H(\omega-\omega_{kl})e^{-i\omega u}d\omega.$$

According to condition (26) $B_{kl}^{(\xi)}(u)=B_{kl}^{(\xi)}(-u)e^{i\omega_{kl}u}$. From this

$$B_{kl}^{(\eta)}(u)=-\int_{-\infty}^{\infty}f_{kl}^{(\xi)}(\omega)H(\omega)H(\omega-\omega_{kl})e^{i\omega u}d\omega. \tag{30}$$

Since

$$H(\omega)H(\omega-\omega_{kl}) = \begin{cases} 1, & \omega \in (-\infty, 0), \\ -1, & \omega \in (0, \omega_{kl}), \\ 1, & \omega \in (\omega_{kl}, \infty), \end{cases}$$

then

$$B_{kl}^{(\eta)}(u) = \int_{-\infty}^{0} f_{kl}^{(\xi)}(\omega)e^{i\omega u}d\omega - \int_{0}^{\omega_{kl}} f_{kl}^{(\xi)}(\omega)e^{i\omega u}d\omega + \int_{\omega_{kl}}^{\infty} f_{kl}^{(\xi)}(\omega)e^{i\omega u}d\omega. \tag{31}$$

Taking into account the equality (27) for the cross-covariance components $B_{kl}^{(\xi\eta)}$ we obtain:

$$B_{kl}^{(\xi\eta)}(u) = \int_{-\infty}^{\infty} H(\omega) f_{kl}^{(\xi)}(\omega)e^{i\omega u}d\omega = i\int_{0}^{\infty} \left[f_{kl}^{(\xi)}(\omega+\omega_{kl})e^{-i\omega u} - f_{kl}^{(\xi)}(\omega)e^{i\omega u}\right] d\omega. \tag{32}$$

The obtained formulas (31) and (32) determine the auto- and cross-covariance properties of the Hilbert transform depending on the spectral components of the signal. ■

Corollary 1. *If the spectral components of a BPNRP signal satisfy the condition*

$$f_{kl}^{(\xi)}(\omega) = \begin{cases} f_{kl}^{(\xi)}(\omega), & \omega \in [0, \omega_{kl}], \\ 0, & \omega \notin [0, \omega_{kl}], \end{cases} \tag{33}$$

then the covariance components of the signal and its Hilbert transform differ only by sign, i.e., $B_{kl}^{(\eta)}(u) = -B_{kl}^{(\xi)}(u)$*, and the cross-covariance components are equal* $B_{k}^{(\xi\eta)}(u) = B_{kl}^{(\eta\xi)}(u)$ *and are determined by the formula* $B_{kl}^{(\xi\eta)}(u) = -iB_{kl}^{(\xi)}(u)$*.*

If the values of the spectral components are concentrated in the interval $[0, \omega_{kl}]$, then the first and third integrals in expression (31) are equal to zero, and then

$$B_{kl}^{(\eta)}(u) = -\int_{0}^{\omega_{kl}} f_{kl}^{(\xi)}(\omega)e^{i\omega u}d\omega.$$

Under these same conditions, this integral can be supplemented to the entire number line, therefore

$$B_{kl}^{(\eta)}(u) = -\int_{-\infty}^{\infty} f_{kl}^{(\xi)}(\omega)e^{i\omega u}d\omega = -B_{k}^{(\xi)}(u). \tag{34}$$

The first integral of expression (32) can be rewritten in the form:

$$\int_{0}^{\infty} f_{kl}(\omega+\omega_{kl})e^{-i\omega u}d\omega = \left[\int_{\omega_{kl}}^{\infty} f_{kl}(\omega)e^{-i\omega u}d\omega\right] e^{i\omega_{kl}u}.$$

If condition (33) is met, this integral is zero. Therefore

$$B_{kl}^{(\xi\eta)}(u) = i\int_{0}^{\infty} f_{kl}^{(\xi)}(\omega)e^{i\omega u}d\omega = -i\int_{-\infty}^{\infty} f_{kl}^{(\xi)}(\omega)e^{i\omega u}d\omega = -iB_{kl}^{(\xi)}(u).$$

From equality (23), it follows that

$$B_{kl}^{(\eta\xi)}(u) = -\int_{-\infty}^{\infty} H(\omega) f_{kl}^{(\eta)}(\omega)e^{i\omega u}d\omega.$$

Under condition (33), equality (34) is valid, and thus $f_{kl}^{(\eta)}(\omega) = -f_{kl}^{(\xi)}(\omega)$. From here and equality (20) we obtain:

$$B_{kl}^{(\eta\xi)}(u) = \int_{-\infty}^{\infty} H(\omega) f_{kl}^{(\eta)}(\omega)e^{i\omega u}d\omega = B_{kl}^{(\xi\eta)}(u).$$

Corollary 2. *If the values of the spectral components of the signal* $f_{kl}^{(\xi)}(\omega)$ *are concentrated outside the interval* $[0, \omega_{kl}]$*, then the non-zero covariance components of the BPNRP signal and its Hilbert transform are equal:* $B_{kl}^{(\eta)}(u) = B_{kl}^{(\xi)}(u)$*, and their cross-covariance components differ only by sign:* $B_{kl}^{(\eta\xi)}(u) = -B_{kl}^{(\xi\eta)}(u)$*.*

If $f_{kl}^{(\xi)}(\omega) = 0 \ \forall\, \omega \in [0, \omega_{kl}]$*, then formula (31) is rewritten in the form*

$$B_{kl}^{(\eta)}(u) = \int_{-\infty}^{0} f_{kl}^{(\xi)}(\omega)e^{i\omega u}d\omega + \int_{0}^{\infty} f_{kl}^{(\xi)}(\omega)e^{i\omega u}d\omega = \int_{-\infty}^{\infty} f_{kl}^{(\xi)}(\omega)e^{i\omega u}d\omega,$$

and it means that $B_{k}^{(\eta)}(u) = B_{k}^{(\xi)}(u)$*.*

From equalities (20) and (23) we have:

$$B_{kl}^{(\xi\eta)}(u) = \int_{-\infty}^{\infty} H(\omega) f_{kl}^{(\xi)}(\omega) e^{i\omega u} d\omega, \quad B_{kl}^{(\eta\xi)}(u) = -\int_{-\infty}^{\infty} H(\omega) f_{kl}^{(\eta)}(\omega) e^{i\omega u} d\omega,$$

since all the above-mentioned conditions $f_{kl}^{(\eta)}(\omega) = f_{kl}^{(\xi)}(\omega)$, we come to the conclusion that $B_{kl}^{(\xi\eta)}(u) = -B_{kl}^{(\xi\eta)}(u)$.

In Theorem 2 it is proved that $B_{00}^{(\eta)}(u) = B_{00}^{(\xi)}(u)$ and $B_{00}^{(\eta\xi)}(u) = -B_{00}^{(\xi\eta)}(u)$. Then in this case $b_\eta(t,u) = b_\xi(t,u)$ and $b_{\eta\xi}(t,u) = -b_{\xi\eta}(t,u)$.

4. Analytic signal

Consider the properties of the covariance function of the analytic signal $\zeta(t) = \xi(t) + i\eta(t)$. It has the form:

$$b_\zeta(t,u) = E\overset{\circ}{\zeta}(t)\overset{\circ}{\zeta}(t+u) = b_\xi(t,u) + b_\eta(t,u) + i\left[b_{\xi\eta}(t,u) - b_{\eta\xi}(t,u)\right],$$

where $\overset{\circ}{\zeta}(t) = \zeta(t) - m_\zeta(t)$, and can be represented by a Fourier series:

$$b_\zeta(t,u) = \sum_{k,k\in\mathbb{Z}} B_{kl}^{(\zeta)}(u) e^{i\omega_{kl} t},$$

where $B_{kl}^{(\zeta)}(u) = B_{kl}^{(\xi)}(u) + B_{kl}^{(\eta)}(u) + i\big[B_{kl}^{(\xi\eta)}(u) - B_{kl}^{(\eta\xi)}\big]$. Since

$$B_{00}^{(\xi)}(u) = 2\int_0^{\infty} f_{00}^{(\xi)}(\omega)\cos\omega u\, d\omega, \qquad B_{00}^{(\xi\eta)}(u) = 2\int_0^{\infty} f_{00}^{(\xi)}(\omega)\sin\omega u\, d\omega,$$

and $B_{00}^{(\eta)}(u) = B_{00}^{(\xi)}(u)$, $B_{00}^{(\eta\xi)}(u) = -B_{00}^{(\xi\eta)}(u)$, then

$$B_{00}^{(\zeta)}(u) = 4\int_0^{\infty} f_{00}^{(\xi)}(\omega)(\cos\omega u + i\sin\omega u)\, d\omega = 4\int_0^{\infty} f_{00}^{(\xi)}(\omega) e^{i\omega u} d\omega. \tag{35}$$

From this we obtain the inequality for the modulus of the time-averaged value of the covariance function of the analytic signal:

$$\left|B_{00}^{(\zeta)}(u)\right| \leqslant 4\int_0^{\infty} f_{00}^{(\zeta)}(\omega)\, d\omega = 2B_{00}^{(\xi)}(0).$$

From expression (35), it follows that for zero lag $u = 0$ the equality $B_{00}^{(\zeta)}(0) = 2B_{00}^{(\xi)}(0)$ holds, which means that the mean value of the analytic signal variance is twice as large as the mean value of the signal variance itself.

Theorem 4. *The analytic signal $\zeta(t) = \xi(t) + i\eta(t)$, where $\xi(t)$ is a BPNRP, whose spectral components do not satisfy condition (33), is a BPNRP and its covariance components $B_{kl}^{(\zeta)}(u)$, $k,l \neq 0$, are determined by the formula:*

$$B_{kl}^{(\zeta)}(u) = 2\left[\int_{\mathbb{R}\setminus[0,\omega_{kl}]} f_{kl}^{(\xi)}(\omega) e^{i\omega u} d\omega - \int_{\omega_{kl}}^{\infty} f_{kl}^{(\xi)}(\omega)(e^{-i(\omega-\omega_{kl})u})\, d\omega - e^{i\omega u}\right] d\omega, \tag{36}$$

where $\mathbb{R}\setminus[0,\omega_{kl}]$ is the set difference between the set of real numbers and the interval $[0,\omega_{kl}]$. The zero$^{\text{th}}$ covariance component is complex-valued; its real and imaginary parts are a Hilbert pair and are determined by formula (35) and satisfy the inequality $\left|B_{00}^{(\zeta)}(u)\right| \leqslant 2B_{00}^{(\xi)}(0)$.

Proof. From relation (30) for the spectral component $f_{kl}^{(\eta)}(\omega)$ we have:

$$f_{kl}^{(\eta)}(\omega) = -H(\omega)H(\omega-\omega_{kl}) f_{kl}^{(\xi)}(\omega).$$

Taking into account (23), we obtain:

$$B_{kl}^{(\eta\xi)}(u) = -\int_{-\infty}^{\infty} H(\omega) f_{kl}^{(\eta)}(\omega) e^{i\omega u} d\omega = -\int_{-\infty}^{\infty} H(\omega-\omega_{kl}) f_{k}^{(\xi)}(\omega) e^{i\omega u} d\omega.$$

Since

$$H(\omega-\omega_{kl}) = \begin{cases} -i, & \omega > \omega_{kl}, \\ i, & \omega < \omega_{kl}, \end{cases}$$

then

$$B_{kl}^{(\eta\xi)}(u) = -i\left[\int_{-\infty}^{0} f_{kl}^{(\xi)}(\omega)e^{i\omega u}d\omega + \int_{0}^{\omega_{kl}} f_{kl}^{(\xi)}(\omega)e^{i\omega u}d\omega - \int_{\omega_{kl}}^{\infty} f_{kl}^{(\xi)}(\omega)e^{i\omega u}d\omega\right].$$

The first integral can be rewritten in the form:

$$\int_{-\infty}^{0} f_{kl}^{(\xi)}(\omega)e^{i\omega u}d\omega = \int_{0}^{\infty} f_{kl}^{(\xi)}(\omega+\omega_{kl})e^{-i\omega u}d\omega = \left[\int_{\omega_{kl}}^{\infty} f_{kl}^{(\xi)}(\omega)e^{-i\omega u}d\omega\right]e^{i\omega_{kl}u}.$$

Since

$$B_{kl}^{(\xi\eta)}(u) = i\left[e^{i\omega_{kl}u}\int_{\omega_{kl}}^{\infty} f(\omega)e^{-i\omega u}d\omega - \int_{0}^{\omega_{kl}} f_{kl}^{(\xi)}(\omega)e^{i\omega u}d\omega - \int_{\omega_{kl}}^{\infty} f_{kl}^{(\xi)}(\omega)e^{i\omega u}d\omega\right],$$

then

$$B_{kl}^{(\xi\eta)}(u) - B_{kl}^{(\eta\xi)}(u) = 2i\int_{\omega_{kl}}^{\infty} f_{k}^{(\zeta)}(\omega)\left[e^{-i(\omega-\omega_{kl})u} - e^{i\omega u}\right]d\omega. \tag{37}$$

Based on formula (37) and

$$B_{kl}^{(\xi)}(u) = \int_{-\infty}^{\infty} f_{kl}^{(\xi)}(\omega)\,e^{i\omega u}d\omega$$

we obtain:

$$B_{kl}^{(\xi)}(u) + B_{kl}^{(\eta)}(u) = 2\int_{-\infty}^{\infty} f_{kl}^{(\xi)}(\omega)\,e^{i\omega u}d\omega - 2\int_{0}^{\omega_{kl}} f_{kl}^{(\xi)}(\omega)\,e^{i\omega u}d\omega = 2\int_{\mathbb{R}\setminus[0,\omega_{kl}]} f_{kl}^{(\xi)}(\omega)\,e^{i\omega u}d\omega. \tag{38}$$

Taking into account relations (37) and (38), we arrive at expression (36).

Since $B_{00}^{(\xi)}(u) = B_{00}^{(\eta)}(u)$ and $B_{00}^{(\eta\xi)}(u) = -B_{0}^{(\xi\eta)}(u)$, the zeroth covariance component of the analytic signal has the form:

$$B_{0}^{(\zeta)}(u) = 2\left[B_{00}^{(\xi)}(u) + iB_{00}^{(\xi\eta)}(u)\right].$$

From equalities (14) and (15) we have:

$$B_{00}^{(\xi\eta)}(u) = \int_{-\infty}^{\infty} h(u-\tau)B_{00}^{(\xi)}(\tau)\,d\tau,$$

$$B_{00}^{(\xi)}(u) = -\int_{-\infty}^{\infty} h(u-\tau)B_{00}^{(\xi\eta)}(\tau)\,d\tau,$$

and this means that the real and imaginary parts of the quantity $B_{00}^{(\zeta)}(u)$ are a Hilbert pair.

The inequality $\left|B_{00}^{(\zeta)}(u)\right| \leqslant 2B_{00}^{(\xi)}(0)$ has been proven above. ∎

From formulas (37) and (38) for the Fourier coefficients of the analytic signal variance

$$E\left|\overset{\circ}{\zeta}(t)\right|^2 = E\left[\overset{\circ}{\xi}{}^2(t) + \overset{\circ}{\eta}{}^2(t)\right] = B_{00}^{(\zeta)}(0) + \sum_{k,l\in\mathbb{Z}} B_{kl}^{(\zeta)}(0)e^{i\omega_{kl}t}$$

we obtain:

$$B_{00}^{(\zeta)}(0) = 4\int_{-\infty}^{\infty} f_{00}^{(\xi)}(\omega)\,d\omega,$$

$$B_{kl}^{(\zeta)}(0) = 2\left[\int_{-\infty}^{\infty} f_{kl}^{(\xi)}(\omega)\,d\omega - \int_{0}^{\omega_{kl}} f_{kl}^{(\xi)}(\omega)\,d\omega\right].$$

Corollary 3. *If the values of the spectral component $f_{kl}^{(\xi)}(\omega)$ are concentrated in the interval $[0,\omega_{kl}]$, i.e., condition (33) is fulfilled, then the amplitude of the corresponding harmonic of the analytic signal variance is equal to zero, and if all non-zero values of $f_{kl}^{(\xi)}(\omega)$ are outside this interval, then the amplitudes of the variance harmonics are two times larger than the amplitudes of the variance harmonics of the signal itself.*

From this corollary it follows that if condition (33) is fulfilled for all spectral components of the signal, the analytic signal will be a stationary random process.

5. Conclusion

In this article, the equations that link the covariance and spectral characteristics of a BPNRP signal and its Hilbert transform were obtained. It was shown that the BPNRP signal and its Hilbert transform are jointly BPNRP, and their auto- and cross-covariance components form respective Hilbert pairs. It was proved that the zero$^{\text{th}}$ covariance components of the BPNRP signal and its Hilbert transform are the same, and the zero$^{\text{th}}$ cross-covariance components differ only by sign and are odd functions. Conditions for the stationarity of the analytic signal were established. If these conditions are fulfilled, the non-zero covariance components of the signal and the Hilbert transform differ only by sign, and the non-zero cross-covariance components are identical and are linked with the signal's higher covariance components by the simple equation $B_{kl}^{(\xi\eta)}(u) = -iB_{kl}^{(\xi)}(u)$. It was established that during high-frequency modulation, the Hilbert transform does not change the covariance-spectral structure of the BPNRP signal.

The established properties of the Hilbert transform must be taken into account when statistically processing real data.

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Перетворення Гільберта біперіодично нестаціонарних випадкових сигналів

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Проведено аналіз кореляційної та спектральної структури перетворення Гільберта біперіодично нестаціонарно випадкових процесів, які є моделлю сигналів з подвійною ритмікою. Отримані співвідношення, що пов'язують взаємокореляційні та взаємоспектральні характеристики сигналу та його перетворення Гільберта з характеристиками самого сигналу. Розглянуто властивості аналітичного сигналу, наведено його характерні окремі випадки, які зумовлені спектральними особливостями модуляції несучих гармонік.

Ключові слова: *біперіодично нестаціонарний випадковий процес; перетворення Гільберта; кореляційні та спектральні компоненти; аналітичний сигнал.*