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Modeling the dependence of thermistor resistance on temperature

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The results of a study on the accuracy of models describing the dependence of thermistor resistance on temperature are presented. The feasibility of using minimax (Chebyshev) approximation to calculate the model parameters is substantiated. Compared to the least squares method, the minimax approximation provides the smallest modeling error. A model in the form of an exponential function of a rational expression is proposed to describe the thermistor resistance as a function of temperature. The use of this model is based on considering the physical properties of the semiconductor resistance dependence on temperature. For the studied calibration results, the model recommended by the Consultative Committee for Thermometry (CCT) under the International Committee for Weights and Measures (CIPM) provides higher accuracy in describing the thermistor resistance–temperature dependence compared to the exponential of a rational expression. However, the accuracy of the model in the form of an exponential function of a rational expression is only slightly lower and practically comparable. Additionally, the model in the form of an exponential of a rational expression allows the use of temperature in the Celsius scale.

Keywords: *thermistor; thermometric characteristic; least squares method; Chebyshev approximation; exponential dependence; rational expression; temporal stability.*

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1. Introduction

Negative temperature coefficient (NTC) thermistors are widely used in the development of various tools for temperature measurement and control [1]. With advances in thermistor manufacturing technology, the accuracy of temperature measurements using thermistors is approaching the Standard Platinum Resistance Thermometer (SPRT) [1]. Today, thermistors are utilized for high-precision temperature measurements [1–3], cryogenic temperature measurements [4–7], and various other applications [8–10]. However, the practical application of thermistors is complicated by the significant nonlinearity of their thermometric characteristics [11]. Therefore, when developing models of the metrological characteristics of sensors, it is important to adequately account for the nonlinearity of their characteristics, as it directly affects the accuracy of physical quantity measurements [12].

Many studies have been dedicated to the investigation of optimal models for thermometric characteristics of thermistors [5, 11, 13]. In the design of high-precision temperature measurement devices, individual calibration dependencies are often used [2]. The study [14] suggests improving the accuracy of thermistor thermometric characteristic models by using the dependence of resistance on temperature. The authors of [14] justify their proposal by the high sensitivity of the resistance dependence on temperature, a viewpoint also supported by the authors of [3]. In practice, the dependence of resistance on temperature is particularly useful in designing automatic temperature-tracking systems using electronic temperature control devices.

According to the physical properties of semiconductors, the dependence of thermistor resistance on temperature is determined by the concentration of charge carriers [1]:

$$R_T = R_\infty \exp\left(\frac{E_a}{2kT}\right), \tag{1}$$

where T is absolute temperature, R_T is electrical resistance at temperature T [Ω], R_∞ is electrical resistance at temperature $T = \infty$, E_g is bandgap energy of the semiconductor, the energy required for charge carriers to transition to higher-energy states [eV], k is Boltzmann constant [eV/K].

For practical applications, instead of the dependence described by equation (1), the approximation given in [1] is proposed:

$$R(T) = R(T_0)\exp\left[\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right], \tag{2}$$

where T_0 is a convenient reference temperature, often 298.15 K (25°C), β is a parameter characterizing the thermistor material, typically ranging from 2000 to 6000.

Model (2) provides a sufficiently accurate description of the dependence of resistance on temperature for small temperature ranges [1]. For a more precise description of this dependence over a broader temperature range, Consultative Committee for Thermometry under the auspices of the International Committee for Weights and Measures [1] proposes the use of the model

$$\ln\left(\frac{R(T)}{R(T_0)}\right) = a_0 + \frac{a_1}{T} + \frac{a_2}{T^2} + \frac{a_3}{T^3}, \tag{3}$$

where a_i, $i = \overline{0,3}$ are unknown parameters, and $R(T_0)$ is the resistance at temperature T_0. The values of the unknown parameters of model (3) in [1, 15] are recommended to be calculated using the least squares method.

In this study, the accuracy of describing the dependence of thermistor resistance on temperature based on high-precision calibration results was investigated not only for model (3) in the form

$$R(T) = R(T_0)\exp\left(a_0 + \frac{a_1}{T} + \frac{a_2}{T^2} + \frac{a_3}{T^3}\right) = R(T_0)\exp\left(\sum\nolimits_{i=0}^{3} a_i T^{-i}\right), \tag{4}$$

but also for a model in the form of an exponential function of a rational expression

$$R_{21}(T) = \exp\left(\frac{a_0 + a_1 T + a_2 T^2}{b_0 + T}\right), \tag{5}$$

which depends on the unknown parameters a_0, a_1, a_2, and b_0. Parameter values of models (4) and (5) are recommended to be calculated using the minimax approximation method [16, 17]. Compared to other approximation methods, such as the least squares method, the minimax approximation ensures the minimum possible error in reproducing the dependence of thermistor resistance on temperature within the studied range [16, 18].

2. Modeling the dependence of thermistor resistance on temperature using the model recommended by the International Committee for Weights and Measures

The modeling of the resistance-temperature dependence is illustrated using the calibration results of a thermistor presented in [2]. The authors of [2] study the properties of a thermistor designed for high-precision temperature measurement. This study includes calibration data obtained in May 2014 and February 2015 at $R_S = 1001.65\,\Omega$.

Since the ratio of the boundary values of the investigated range of thermistor resistance variation exceeds ten, relative error will be used to assess the accuracy of models describing the dependence of thermistor resistance on temperature. The appropriateness of applying models calculated with consideration of relative error is based on the fact that the fractional value of the absolute error of the model differs when approximating small and large values. The authors of the works [18, 19] recommend using models that evaluate relative error for modeling quantities whose investigated values vary by more than a factor of ten.

The accuracy of models (4) and (5) was investigated using the calibration results presented in [2] by applying both the least squares method and minimax approximation. The parameter values for model (4) using the least squares method were calculated through linearization [18], while calculation of the parameters of model (4) using the minimax approximation was performed by the method described in [20]. The distinctive feature of this method is that it employs an intermediate minimax polynomial approximation [21, 22].

Based on the calibration data obtained in May 2014, the model

$$R_{14sq}(T) = \exp\left(-7.4834302855648 + \frac{4626.55136391078}{T} - \frac{83211.004134574}{T^2} - \frac{5035940.2360629}{T^3}\right) \tag{6}$$

calculated using the least squares method, provides a relative error of 0.0039%, while the model

$$R_{14min}(T) = \exp\left(-7.490488290408 + \frac{4634.098342431868}{T} - \frac{85839.878552453}{T^2} - \frac{4736371.2496365}{T^3}\right) \tag{7}$$

calculated with minimax approximation, provides a relative error of 0.00253%.

For the data obtained in February 2015, the model

$$R_{15sq}(T) = \exp\left(-7.5206494574566 + \frac{4660.4472610159}{T} - \frac{93500.51007484}{T^2} - \frac{3995786.028146}{T^3}\right) \tag{8}$$

calculated using the least squares method, provides a relative error of 0.001458%, while the model

$$R_{15min}(T) = \exp\left(-7.52791742913 + \frac{4667.213385467}{T} - \frac{95595.69858867}{T^2} - \frac{3780007.10860779}{T^3}\right) \tag{9}$$

calculated using the minimax approximation, provides a relative error of 0.001213%.

The error graphs for models (6)–(9) are presented in Figure 1. Error graphs for models (7) and (9), whose parameter values were calculated using the minimax approximation, are shown with thicker lines.

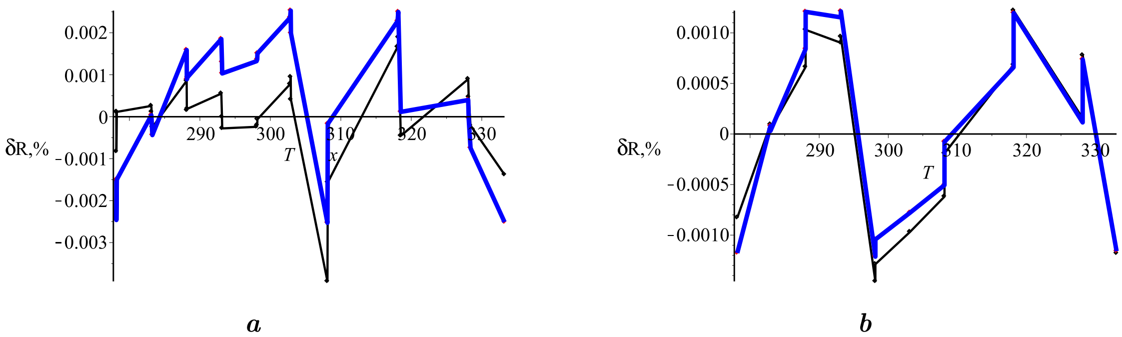


Fig. 1. Relative error graphs of: (***a***) models (6) and (7), (***b***) models (8) and (9).

Table 1. Calibration results reproduction errors for models (6)–(9).

Calibration Data	Model (6)	Model (7)
May 2014	$\delta R_{14sq} = 0.0039\%$	$\delta R_{14min} = 0.00253\%$
	Model (8)	Model (9)
February 2015	$\delta R_{15sq} = 0.001458\%$	$\delta R_{15min} = 0.001213\%$

From the graphs presented in Figure 1, it follows that the error of models whose parameters were calculated using the minimax approximation is uniformly distributed across the entire studied temperature range. This particular feature of the minimax approximation when processing high-precision data makes it preferable compared to the least squares method [16]. The least squares method minimizes the sum of the squares of deviations from the observed results. In this case, within the range of the studied data, there may be isolated points where the error value significantly exceeds its average. Therefore, to determine the accuracy of models obtained by the least squares method, their uncertainty is calculated [11, 13, 21]. Models whose parameters are calculated using the minimax approximation based on precision data ensure the achievement of the smallest possible reproduction error within the studied range [16, 18]. The approximation error of calibration results using such a model corresponds to the measurement accuracy.

A comparison of the thermistor calibration reproduction errors for models (6)–(9) is provided in Table 1.

Models (7) and (9), whose parameters were calculated using the minimax approximation, provide higher accuracy in calculating thermistor resistance than models (6) and (8), whose parameters were obtained by the least squares method. Thus, using the minimax approximation to calculate the parameters of models describing the dependence of thermistor resistance on temperature ensures greater accuracy compared to the least squares method. Furthermore, the error of the model whose parameters were calculated according to the minimax criterion is comparable to the accuracy of reproducing calibration results [15].

3. Modeling the dependence of thermistor resistance on temperature using an exponential of a rational expression

The idea of using an exponential function of a rational expression (5) to model the dependence of thermistor resistance on temperature takes into account the physical properties of the temperature dependence of semiconductor resistance (1). According to (1), the dependence of semiconductor resistance on temperature is described by an exponential function of a rational expression. Models (2) and (4) describing the dependence of thermistor resistance on temperature are expressed as exponentials of polynomials in negative powers of temperature. The proposed model for describing the dependence of thermistor resistance on temperature in the form of an exponential function of a rational expression (5) is a logical improvement over models (2) and (4), since approximation using a rational expression generally provides higher accuracy than a polynomial approximation with the same number of parameters [16, 17, 24]. Based on these considerations, we deem it appropriate to apply the exponential of a rational expression (5) for modeling the dependence of thermistor resistance on temperature.

The parameter values for the model in the form of an exponential of a rational expression (5) are calculated using the least squares method by linearizing the exponential and the rational expression [18]. The parameters of model (5) using the minimax approximation are computed using the method described in [25]. This method involves the use of an intermediate minimax approximation with a rational expression [24, 26].

Model

$$R_{2,1\_14sq}(T) = \exp\left(\frac{0.001474233930189\,T^2 - 9.24844950622\,T + 5032.88282446}{T + 50.21452247792}\right) \tag{10}$$

calculated using the least squares method describes the May 2014 calibration data with a relative error of 0.00413%. Model

$$R_{2,1\_14min}(T) = \exp\left(\frac{0.001349146280136\,T^2 - 9.12449091186\,T + 4999.261163968}{T + 49.0889266384871}\right) \tag{11}$$

with parameters calculated using minimax approximation describes the same data with a relative error of 0.002598%.

The choice of the model in the form of an exponential of a rational expression (5) is justified by the fact that the exponential of this specific rational expression provides the highest accuracy for the calibration data from May 2014. The results of the study on the accuracy of the dependence of thermistor resistance on temperature using an exponential of a rational expression with four parameters are presented in Table 2.

Table 2. Error values for reproducing resistance based on the thermistor calibration results from May 2014.

Error	$k=0, l=3$	$k=1, l=2$	$k=2, l=1$
Least squares method	0.022%	0.00377%	0.00413%
Minimax approximation	0.0125%	0.002947%	0.002598%

In this table, k represents the degree of the numerator, and l represents the degree of the denominator. From the results presented in the table, it follows that the model with the smallest relative error in reproducing the thermistor resistance for the May 2014 calibration data is the one with a rational expression where the numerator is second-degree, and the denominator is a first-degree polynomial.

For the February 2015 calibration data, the model that provides the smallest thermistor resistance reproduction error is also a four-parameter model $R_R(T)$, where the numerator is of degree $k=2$, and the denominator is of degree $l=1$. This model was calculated using the least squares method,

$$R_{2,1\_15sq}(T) = \exp\left(\frac{0.0012890622726766\,T^2 - 9.07218980088\,T + 4986.415085805}{T + 48.7143121397354}\right) \tag{12}$$

describes calibration data for February 2015 with a relative error of 0.00159%, and model calculated using the minimax approximation

$$R_{2,1\_15min}(T) = \exp\left(\frac{0.001274822267976\,T^2 - 9.0583682873\,T + 4982.720865367}{T + 48.5929164310684}\right) \tag{13}$$

describes the same data with a relative error of 0.001426%.

The error graphs for reproducing the calibration results using models (11) and (13) are presented in Figure 2.

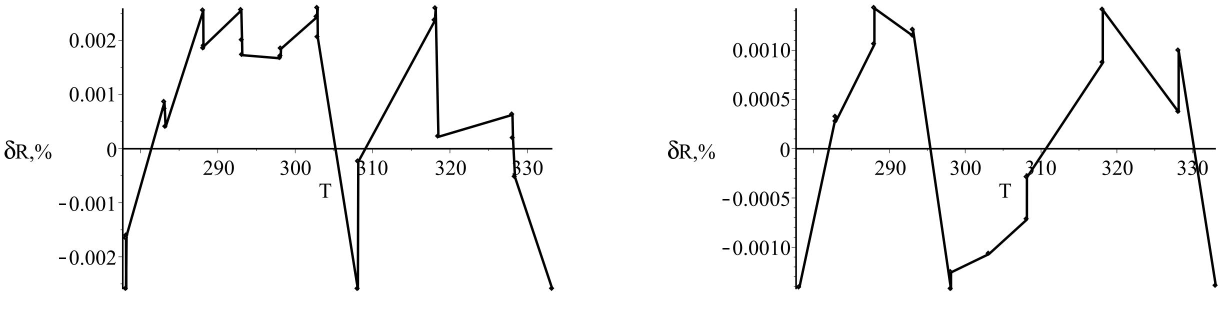


Fig. 2. Relative error graphs of: (***a***) model (11), (***b***) model (13).

From the graphs presented in Figure 2, it follows that the errors of models (11) and (13) are uniformly distributed across the studied temperature range and, in absolute value, do not exceed 0.002598% and 0.001426%, respectively.

A comparison of the errors of models of types (4) and (5), parameters of which were calculated using the minimax approximation, is presented in Table 3.

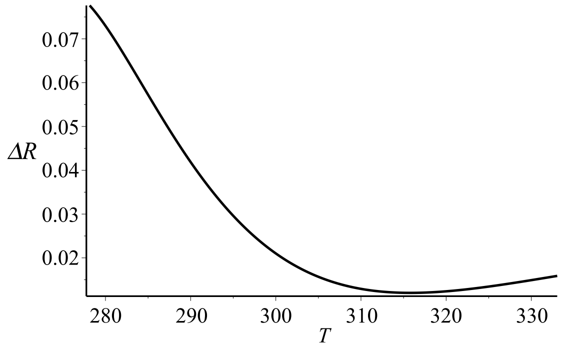
Table 3. Errors of models whose parameters were calculated using the minimax approximation.

Model	Calibration data	
	May 2014	February 2015
$R(T_0)\exp\left(\sum_{i=0}^{3} a_i T^{-i}\right)$	$\delta R_{14min} = 0.00253\%$ (7)	$\delta R_{15min} = 0.001213\%$ (9)
$\exp\left(\frac{a_0 + a_1 T + a_2 T^2}{b_0 + T}\right)$	$\delta R_{2,1\_14min} = 0.002598\%$ (11)	$\delta R_{2,1\_15min} = 0.001426\%$ (13)

In Table 3, the model number is indicated alongside the provided error values. From the errors presented in this table, it follows that the model of form (4), recommended by the International Committee for Weights and Measures for the studied calibration results, provides higher accuracy in determining resistance by thermistor temperature than the exponential of a rational expression (5).

Models $R_{14min}(T)$ and $R_{2,1\_14min}(T)$, describing the dependence of thermistor resistance on temperature, whose parameters were calculated using the minimax criterion, also provide satisfactory temporal stability. The maximum discrepancy between the values of models $R_{14min}(T)$ and $R_{15min}(T)$ for the investigated thermistor is $0.077\,\Omega$, while the maximum discrepancy between the values of models $R_{2,1\_14min}(T)$ and $R_{2,1\_15min}(T)$ is $0.075\,\Omega$. Graphs of the discrepancies between the values of models $R_{14min}(T)$ and $R_{2,1\_14min}(T)$ compared to models $R_{15min}(T)$ and $R_{2,1\_15min}$ are shown in Figure 3.

On these graphs, the horizontal axis represents the temperature values, while the vertical axis shows the error in calculating the resistance value $\Delta R = R_{14\_min}(T) - R_{15\_min}(T)$ in Figure 3a and $\Delta R = R_{2,1\_14min}(T) - R_{2,1\_15min}(T)$ in Figure 3b.



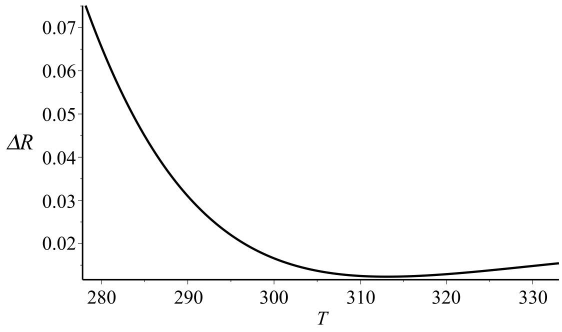


Fig. 3. Graphs of model deviations: (***a***) $R_{14min}(T)$ vs. $R_{15min}(T)$, (***b***) $R_{2,1\_14min}(T)$ vs. $R_{2,1\_15min}(T)$.

4. Modeling the dependence of thermistor resistance on temperature in degrees Celsius

Model (5) can also be used to describe the dependence of thermistor resistance on temperature in the Celsius scale. For models (11) and (13), the transition from the absolute temperature scale to the Celsius scale formally involves only recalculating their parameters and does not affect the accuracy of the models. The dependencies of resistance on temperature in degrees Celsius, constructed based on the calibration results of the investigated thermistor, are described by the following models:

$$R_{2,1\_14min}(T) = \exp\left(\frac{0.0013491158756\,t^2 - 8.387439883\,t + 2607.5655316}{t + 322.238681134}\right), \tag{14}$$

$$R_{2,1\_15min}(T) = \exp\left(\frac{0.00127545220442\,t^2 - 8.3621752135\,t + 2603.581916226}{t + 321.747697}\right). \tag{15}$$

Model (14) reproduces the thermistor resistance based on the calibration results from May 2014 with a relative error of 0.002698%, while model (15) reproduces the thermistor resistance for the calibration data from February 2015 with a relative error of 0.001424%

5. Conclusions

Expectations regarding the improvement in accuracy of describing the dependence of thermistor resistance on temperature using the exponential of a rational expression were not confirmed. For the investigated calibration results, the model proposed by the International Committee for Weights and Measures provided higher accuracy. It is advisable to calculate the parameters of models describing the dependence of thermistor resistance on temperature using the minimax criterion. The error of models obtained by the minimax criterion corresponds to the accuracy of calculating the thermistor resistance value, and in this case, there is no need to determine uncertainty. Models whose parameters are calculated using the minimax criterion also provide satisfactory temporal stability in reproducing the dependence of thermistor resistance on temperature.

To confirm the higher accuracy of describing the dependence of thermistor resistance on temperature by the model recommended by the International Committee for Weights and Measures, compared to the exponential of a rational expression, it is advisable to conduct studies using high-precision individual calibration results of other thermistors.

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Моделювання залежності опору термістора від температури

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Подано результати дослідження точності моделей залежності опору термістора від температури. Обґрунтовано доцільність використання мінімаксного (чебишовського) наближення для обчислення значень параметрів моделей. Порівняно з методом найменших квадратів мінімаксне наближення забезпечує досягнення найменшої похибки моделі. Для опису залежності опору термістора від температури запропоновано модель у вигляді експоненти від раціонального виразу. Використання цієї моделі ґрунтується на врахуванні фізичних властивостях залежності опору напівпровідника від температури. Для досліджуваних результатів калібрування модель, рекомендована Всесвітньою метрологічною організацією, порівняно з експонентою від раціонального виразу забезпечує вищу точність опису залежності опору термістора від температури. Точність моделі у вигляді експоненти від раціонального виразу дещо нижча хоча практично сумірна. Модель у вигляді експоненти від раціонального виразу допускає використання температури за шкалою Цельсія.

Keywords: *термістор; термометрична характеристика; метод найменших квадратів; чебишовське наближення; експоненціальна залежність; раціональний вираз; часова стабільність.*