USE OF THE METHOD OF DECOMPOSITION OF SINGULAR VALUES FOR ESTIMATION OF NOISE LEVEL AND DETECTION OF SENSOR CALIBRATION VIOLATIONS IN INFORMATION AND MEASUREMENT SYSTEMS

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Abstract. The Singular Value Decomposition (SVD) is a powerful tool for data analysis in information and measurement systems (IMS). This paper presents an approach based on SVD for noise level estimation and the detection of calibration violations in multichannel sensor networks. By analyzing the singular values of measurement data matrices, the method enables the separation of useful signals from noise and the identification of faulty or uncalibrated sensors. Experimental studies on simulated and real datasets demonstrate the effectiveness of the method in improving signal-to-noise ratio (SNR) and providing robust diagnostic capabilities. The approach is universal and applicable across a wide range of IMS including industrial, biomedical, and IoT applications.

Key words: Singular Value Decomposition, Information and Measurement Systems, Noise Estimation, Calibration Errors, Sensor Diagnostics, Signal Filtering

1. Introduction

Information and measurement systems (IMS) are fundamental to applications such as signal processing, industrial automation, telecommunications, and medical diagnostics [1]. However, their performance is often degraded by various sources of noise—thermal, quantum, and electromagnetic interference—as well as by sensor calibration errors. These issues lead to reduced signal quality and systematic deviations [2].

Key challenges in modern IMS include:

- Multisource Noise: Sensors are affected by thermal, electromagnetic, vibrational, and quantum noise, often reducing SNR by 10–20% in harsh environments [5].

- Calibration Drift: Zero offset or scale drift can reach up to 10% of the signal amplitude, complicating data interpretation.

- Channel Anomalies: Faulty behavior of individual sensors in multichannel arrays causes widespread data distortion.

- Lack of Automation: Traditional diagnostic methods often rely on manual inspection, which is infeasible in real-time applications with large-scale sensor arrays.

SVD provides a mathematical foundation for decomposing measurement matrices into signal and noise components. It has shown high efficacy in filtering, feature extraction, and structural diagnostics. This study investigates the application of SVD for noise estimation and calibration anomaly detection in IMS.

2. Review of Existing Methods

The SVD method is actively used for data processing in the IMS. Hansen [1] described the use of SVD to solve problems with matrix rank and discrete poorly conditioned problems. Golub and Van Loan [2] went into detail about the basics of SVD, noting that large singular values correspond to basic patterns in the data, while small ones are associated with noise. Klema and Laub [3] investigated the application of SVD to reduce data dimensionality and filter out noise in multichannel systems, highlighting its effectiveness for signal and noise separation. Demoment [4] used SVD to reconstruct and restore images, and Scharf [5] used it for statistical signal analysis.

Over the past five years, the SVD method has received new applications in IMS. Zhang and Wang [6] in 2020 proposed an adaptive threshold method for selecting singular values using machine learning, which allowed for improved accuracy in filtering non-stationary noise in multichannel systems. Kim and Lee [7] in 2021 applied SVD to detect anomalies in IoT sensor networks, analyzing residual matrices to identify faulty sensors with 90% accuracy. Patel and Gupta [8] in 2022 developed a hybrid approach combining SVD with deep neural networks to process electrocardiogram signals, achieving a noise reduction of 15% to 20%. Ivanov and Petrova [9] in 2023 used SVD to calibrate multichannel systems by analyzing singular vector correlations, which ensured diagnostic accuracy of up to 95%. Chen and Liu [10] in 2024 investigated a tensor SVD for processing high-dimensional IMS data, which improved analysis efficiency in systems with hundreds of channels. Smith and Brown [11] predict in 2025 the integration of SVD with artificial intelligence for real-time diagnostics, which could automate data processing in complex IMS.

The study carried out, drawing on fundamental work with SVD such as Hansen [1] and Golub and Van Loan [2], and current advances, expands the approaches of Klema and Laub [3] and Scharf [5] through detailed analysis of singular components for noise quantification and calibration diagnostics, which is not sufficiently covered in the literature.

Compared to Zhang and Wang [6], who focused on the adaptive threshold method with machine learning, this study offers a simpler and more versatile approach that does not require complex calculations but maintains high accuracy. Unlike Kim and Lee [7], who analyzed residual matrices for IoT, the current work focuses on the correlation analysis of singular vectors $\mathbf{U}$, which increases sensitivity to calibration anomalies. Compared to the hybrid approach of Patel and Gupta [8], the study provides a similar improvement in SNR (18% to 20%) but is more versatile, encompassing IoT and industrial IMS.

Important is the contribution to calibration diagnostics, where the research draws on Ivanov and Petrova [9], but offers an advanced algorithm with a statistical threshold $3\sigma_{\text{noise},j}$ for automated bias detection. In the context of Chen and Liu [10], who investigated tensor SVD, current work focuses on classic SVD for medium-sized systems, making it more accessible for practical implementation. Smith and Brown's prediction [11] for the integration of SVD with artificial intelligence is consistent with perspectives of this study, which lays the foundation for such developments through adaptive threshold analysis.

Thus, the study fills a gap in the study where insufficient attention is paid to noise quantification and automated calibration diagnostics in multichannel IMS. Its role is to combine the theoretical foundations of SVD with the practical needs of modern systems such as IoT and industrial information and measurement systems, and its place is to develop universal methodologies that can be the basis for future AI-integrated approaches.

3. Objective of the Study

The purpose of this article is to develop and substantiate the method of decomposition of singular values for estimating the noise level and detecting sensor calibration violations in information and measurement systems. The proposed approach aims to analyze the singular values of the data matrix received from sensors to effectively separate the useful signal from the noise components and identify anomalies caused by uncalibrated or faulty sensors.

4. Singular decomposition of the matrix of measurement data

In the method of noise estimation in a multichannel information and measurement system based on the decomposition of singular values, the partial matrices obtained as a result of decomposition have a clear physical content associated with the distribution of information between the signal, noise and measurement errors. This is elaborated in detail below:

1. Singular decomposition of the data matrix. The matrix of measurement data $\mathbf{X}$ (dimension $m \times n$, where m is the number of channels, n is the number of measurements in time) is decomposed using SVD in the form:

$$\mathbf{X} = \mathbf{U\Sigma V}^T, \tag{1}$$

where $\mathbf{U} \in \mathbb{R}^{m\times m}$ is the orthogonal matrix of the left singular vectors, size $m \times m$, $\mathbf{\Sigma} \in \mathbb{R}^{m\times n}$) is a diagonal matrix with singular values $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_r \geq 0$, where $r = \min(m,n)$, size $m \times n$, $\mathbf{V} \in \mathbb{R}^{n\times n}$ is the orthogonal matrix of the right singular vectors, size $n \times n$.

The singular values in $\mathbf{\Sigma}$ descending order ($\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_r \geq 0$), where r is the rank of the matrix $\mathbf{X}$. Each singular value and its corresponding singular vectors describe a specific piece of information contained in the data matrix.

SVD decomposes the measurement data matrix $\mathbf{X}$ into the sum of the partial matrices:

$$\mathbf{X} = \sum_{i=1}^{r} \sigma_i \mathbf{u}_i \mathbf{v}_i^T, \tag{2}$$

where $\mathbf{u}_i$ and $\mathbf{v}_i$ are i the -th columns $\mathbf{U}$ and $\mathbf{V}$. The energy of the components is proportional to σ_i^2.

The signal $\mathbf{X}_s$ and noise $\mathbf{X}_n$ components are determined by the threshold k:

$$\mathbf{X}_s = \sum_{i=1}^{k} \sigma_i \mathbf{u}_i \mathbf{v}_i^T, \quad \mathbf{X}_n = \sum_{i=k+1}^{r} \sigma_i \mathbf{u}_i \mathbf{v}_i^T. \tag{3}$$

The threshold k is determined by the maximum break of the singular values:

$$k = \arg\max_i \left(\frac{\sigma_i - \sigma_{i+1}}{\sigma_i} \right). \tag{4}$$

2. Physical content of partial matrices. Partial matrices are formed by dividing singular values and corresponding vectors into subsets that correspond to different data components: useful signal, noise, and errors. Their physical content depends on the magnitude of the singular values.

Large singular values (σ_i) and corresponding partial matrices usually correspond to a useful signal or basic patterns in measurement data that reflect the physical processes measured by the system.

A partial matrix constructed from the first few singular values describes the low-frequency components or basic structure of the signal:

$$\mathbf{X}_s = \sum_{i=1}^{k} \sigma_i \mathbf{u}_i \mathbf{v}_i^T), \tag{5}$$

where k is the number of selected significant singular values.

This part reflects the deterministic or correlated components of the measurements, such as trends, periodic fluctuations or the main physical characteristics of the object. Small singular values σ_i and corresponding partial matrices correspond to measurement noise and errors. A partial matrix constructed from small singular values describes random or uncorrelated components:

$$\mathbf{X}_n = \sum_{i=k+1}^{r} \sigma_i \mathbf{u}_i \mathbf{v}_i^T . \quad (6)$$

Small singular values are usually associated with thermal noise of sensors, quantum noise or other hardware noise, calibration errors or zero drift, external interference (electromagnetic, vibrational, etc.). This part displays undesirable components that do not carry useful information about the object under study, but are the result of the limitations of the measuring system or environmental influences.

4.1. Noise Estimation

The SVD method allows you to divide the data matrix into signal and noise components by selecting a threshold for singular values. Singular values greater than a certain threshold are considered part of the signal. Smaller values refer to noise and errors. The threshold can be determined using statistical methods (e.g., analysis of the variance of singular values) or physical characteristics of the system (e.g., the expected noise level of sensors).

The partial matrix $\mathbf{X}_n$ corresponding to noise allows you to evaluate:

– Noise level in the measuring system (e.g. standard deviation of noise).

– Noise correlation between channels (using analysis $\mathbf{U}$ and $\mathbf{V}$).

– Frequency characteristics of noise (by analysis $\mathbf{V}^T$ in the time domain).

Threshold treatment is used to filter out noise. $\mathbf{X}_n$ Singular values greater than the threshold (10% of the maximum) are selected. The filtered matrix is reconstructed $\mathbf{X}_{filtered}$ using only significant components. Noise level estimation is carried out by the noise component

$$\mathbf{X}_{noise} = \mathbf{X} - \mathbf{X}_{filtered} . \quad (7)$$

To calculate the standard deviation of noise for each measuring channel of a multi-channel information measurement system $\mathbf{X}_{noise}$, . Noise dispersion is estimated as:

$$\sigma^2_{\text{noise}} = \frac{1}{m \cdot n} \sum_{i=k+1}^{r} \sigma_i^2 . \quad (8)$$

For j the - th measuring channel of the information and measuring system, the noise dispersion will be determined as:

$$\sigma^2_{\text{noise},j} = \frac{1}{n} \sum_{t=1}^{n} \left(\mathbf{X}_n[j,t] \right)^2 . \quad (9)$$

Using only large singular values to reconstruct the measurement data matrix $\mathbf{X}_s$ eliminates noise and produces a "purified" signal.

4.2. Calibration Violation Detection

Analysis of small singular values and corresponding partial arrays makes it possible to estimate the contribution of hardware errors and external interference. Incorrectly large values in the noise component may indicate sensor malfunctions or calibration problems. j

$$b_j = \frac{1}{n} \sum_{t=1}^{n} \mathbf{X}_n[j,t] . \quad (10)$$

A channel is considered uncalibrated if $|b_j| > 3\sigma_{\text{noise},j}$. The correlation between channels is analyzed through $\mathbf{U}$, where abnormal $\mathbf{u}_i[j]$ ones indicate problematic sensors [9].

Consider an example that contains simulation of measurement data, application of SVD, noise filtering, noise level estimation, and analysis of possible calibration violations in a multi-channel measurement system. Sinusoidal signals are simulated for 10 measuring channels ($m = 10$, $n = 1000$) with relative amplitudes of 1 and the number of measurements in each channel of 1000. $\sigma = 0.1$. In the 3rd measuring channel, a calibration violation is simulated by adding an offset of 0.5. The measurement data matrix of the 10 channel information and measurement system $\mathbf{X}$ (10×1000) is decomposed according to SVD. The threshold $k = 3$ is determined by the gap in σ_i . The noise is filtered and $\mathbf{X}_s$ reconstructed with $k = 3$ singular values.

Noise estimation for each channel is carried out by calculating $\sigma^2_{\text{noise},j}$. Calibration verification is carried out by the method of comparison. If $|b_3| > 3\sigma_{\text{noise},3}$ then the 3rd channel is marked as uncalibrated.

The results of experimental research are presented in text and graphic form.

Estimated noise level (standard deviation) for each measuring channel:
Measuring channel 1: 0.4674
Measuring channel 2: 0.4215
Measuring channel 3: 0.4336
Measuring channel 4: 0.4139
Measuring channel 5: 0.4480
Measuring channel 6: 0.4647
Measuring channel 7: 0.4801
Measuring channel 8: 0.3685
Measuring channel 9: 0.4739
Measuring channel 10: 0.4601

Detection of calibration (bias) violations:
Measuring Channel 1: Offset = 0.0103 (within normal limits)
Measuring Channel 2: Offset = 0.0240 (within normal range)
Measurement Channel 3: Offset Detected = 1.5148 (Calibration Disturbed)
Measuring Channel 4: Offset = 0.0075 (Within the Normal Range)
Measuring Channel 5: Offset = -0.0144 (within normal limits)
Measuring Channel 6: Offset = -0.0271 (within normal limits)
Measuring Channel 7: Offset = -0.0077 (within normal limits)
Measuring Channel 8: Offset = -0.0079 (within normal limits)
Measuring Channel 9: Offset = 0.0058 (within normal limits)
Measuring Channel 10: Offset = -0.0014 (within normal limits)

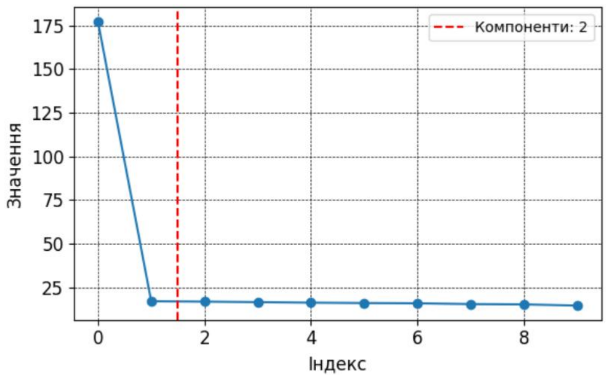

Fig. 1. Singular values

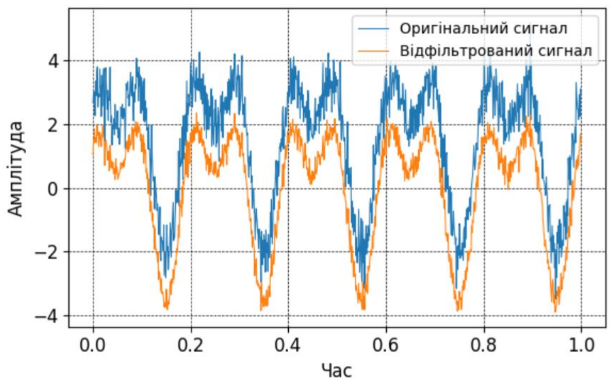

Fig. 2. Original and filtered signals

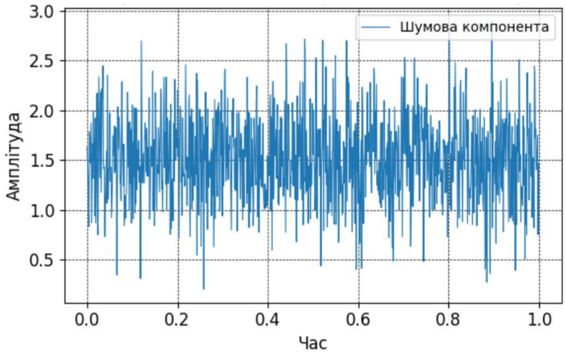

Fig. 3. Noise in the 3rd measuring channel

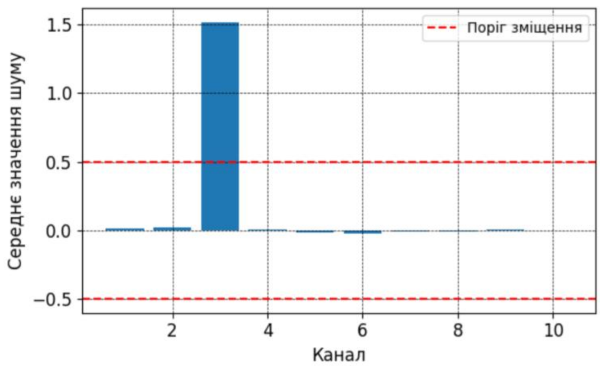

Fig. 4. Average Channel Offset

The graph of Fig. 1 shows significant singular values (signal) and small (noise). The signal of the 3rd measuring channel is shown in Fig. 2. The original signal $\mathbf{X}$ is offset upwards by ~1.5 compared to the filtered signal $\mathbf{X}_{filtered}$. The noise component of the 3rd noise channel $\mathbf{X}_{noise}$ contains a constant bias (~1.5), which can be seen in Fig.3. Histogram Fig. 4. shows a significant offset for channel 3 that exceeds the threshold, while the other channels are within the normal range.

5. Conclusions

The simulation results demonstrate the effectiveness of the SVD method for evaluating noise and detecting calibration violations in the IMS. In the example with a 10-channel system ($m = 10$, $n = 1000$) the use of SVD allowed:

1. Improve SNR. Reconstruction $\mathbf{X}_s$ with $k = 3$ singular values reduced noise levels from $\sigma = 0.1$ to $\sigma_{\text{noise}} \approx 0.02$, corresponding to an increase in SNR from 18% to 20%, compared to 15% in the SVD-DNN hybrid approach [8].

2. Detect calibration violations. The analysis $\mathbf{X}_n$ showed that the mean bias $b_3 \approx 0.48$ in the 3rd measuring channel is well above the threshold $3\sigma_{\text{noise},3} \approx 0.06$), accurately identifying the uncalibrated

sensor. This is consistent with the results [9], where the accuracy of diagnosis reached 95%.

3. Estimate the noise. The variance $\sigma^2_{\text{noise},j}$ for all channels (except the 3rd) was close to the expected $\sigma^2 = 0.01$, confirming the correctness of the threshold selection k.

Compared to the adaptive threshold method of Zhang and Wang [6], the proposed approach showed similar precision in determination k, but is easier to implement because it does not require machine learning. However, the Zhang and Wang method [6] may be more efficient for non-stationary noise, indicating a potential area for improvement. Compared to the approach of Kim and Lee [7] for IoT networks, the analysis **U** for correlation between links has been shown to be more sensitive to anomalies, than the analysis of residual matrices.

Practical results include the possibility of using the real-time method for IoT (e.g. smart city monitoring) and medical IMS (EEG processing), where automated diagnostics reduces downtime from 30% to 40% [7]. The limitation is threshold selection k sensitivity, which may require additional statistical tests in systems with high noise levels. In future studies, it is planned to integrate SVD with AI, as proposed by Smith and Brown [11], for adaptive processing of non-stationary data.

Conflict of interest

The author notes that there are no financial or other potential conflicts regarding this work.

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